

## **A Term Structure Model for Defaultable European Sovereign Bonds**

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### **Abstract**

Following Ang and Piazzesi (2003), we use an arbitrage-free affine term structure model in which sovereign bond yield spreads are used as dependent variables in an equation that includes, among others, fiscal variables. Our main question is: in what extent these yield spreads can be attributed to economic fundamentals? In particular, we are interested in the contribution of deficit and debt in the expansion of sovereign spreads in the years after the onset of the economic crisis of 2007. Our idea is to distinguish the effects of fiscal shocks from the effects of shocks to other macroeconomic variables and potentially relevant indicators of risk aversion. We chose three euro-area countries for this analysis: Spain, Greece and Italy. We note that the own country's debt has been playing an important role in the recent widening of spreads, especially for Greece and Italy. For Spain, the recent rise in spreads is being driven mainly by variables related to Germany (and German debt is the most important one among them), Spain's inflation and market stress (HY). Yield spreads of Greece respond more strongly to shocks from debt and deficit. A shock of one standard deviation from the country's deficit causes an initial response of the 1 year yield spread of 30% of its standard deviation

## 1. Introduction

The recent economic crisis, triggered by a downturn in the banking system of the United States, hit economies around the world. Tax incentive programs have been implemented in many countries, especially in advanced economies, leaving behind quite fragile fiscal positions (IMF [2010]). While the world GDP has come from a decline of 0.6% in 2009 to an expansion of 4.2% in 2010, the projection for the fall of the global deficit is only 6.7% in 2009 to 6.0% of GDP in 2010.<sup>1</sup>

Increases in the countries' deficit and debt levels can bring in an increased perception of sovereign risk. In the sovereign bonds market, this movement can cause higher yields paid by these countries to finance its debt. Although a channel through which increased fiscal deficits can positively impact government bond yields occurs via an increased perception of sovereign risk, as noted by Baldacci and Kumar (2010), there are theoretical relationships between these two variables that can turn effect of fiscal variables on interest controversial. On the one hand, the increase in government spending reduces national savings and increase aggregate demand, creating an excess supply of bonds and hence increasing the yields paid for debt financing. If agents are forward looking, there may be a Ricardian Equivalence effect in the private sector: In anticipation of future tax hikes, private savings would increase in reaction to increased government spending. If this last movement does occur, the positive effect of public expenditures over bond yields may be understated.

Following Ang and Piazzesi (2003), we use an arbitrage-free affine term structure model in which sovereign bond yield spreads are used as dependent variables in an equation that includes, among others, fiscal variables.<sup>2</sup> The advantage of our model to existing empirical literature applied to Europe, as Bernoth, von Hagen and Schuknecht [2004], Schuknecht, von Hagen and Wolswijk [2008] and Georgoutsos and Migiakis [2010], is that we can study the behavior of bond yields of various maturities imposing restrictions in order to respect the hypothesis of no-arbitrage between the returns on those securities. This approach has advantages in comparison to an unrestricted VAR approach: First, it allows us to access the impact of fiscal shocks to the whole yield curve, not only on observed ones. VAR models has little to say about how yields of maturities not included in the model move. Second, in VAR models the implied movements of yields in relation to each other may not rule out arbitrage opportunities. We know of no other work that addresses in this way the relationship between fiscal variables and the term structure of bond yields for countries in the euro zone.

Describing the behavior of the whole yield curve and macroeconomic variables, especially those over which governments have greater discretion, such as fiscal variables, is important for policy and investment strategies and pricing of sovereign bonds. We also believe that Europe is an especially interesting case for undertaking the research. Unlike other studies in the literature (eg, Duffie, Pedersen and Singleton [2003] and Pan and Singleton [2008]), the European case brings no problems related to foreign exchange risk: Almost all debt issued by these countries is denominated in euro, the currency in which governments collect taxes and realize their spending.

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<sup>1</sup> World economic growth (actual and projected) from IMF's World Economic Outlook (April 2010). Deficit (actual and projected) from IMF's Fiscal Monitor (May 2010).

<sup>2</sup> Introduced by Duffie and Kan (1996). Duffie and Singleton (2000) is an important extension.

Moreover, as the monetary authority is the European Central Bank, countries cannot individually reduce the par value of its debt, which would make it cloudy, when analyzing sovereign bonds prices, to distinguish inflationary risk of default risk. These factors suggest that, particularly in the European case, fiscal variables are actually the ones that matter in the assessment of sovereign risk.

We chose three euro-area countries for this analysis: Spain, Greece and Italy. We believe that these three countries are representative of the group of European countries that has been facing debt financing problems more recently (Portugal and Ireland would also be included in this group). The evolution of debt and deficit have been different among Spain, Greece and Italy (See Table A1.2 in the Appendix 1): Spain has been incurring in high deficits (11.1% of GDP in 2009), although its debt/GDP ratio is relatively low (53.2%); Greece has been incurring in high deficits (13.6% of GDP in 2009), and has the second highest debt/GDP ratio in the euro-area (115.1% in 2009); Italy has been incurring relatively low deficits (5.7% of GDP in 2009), but has the highest debt/GDP ratio in the euro-area (116.0% in 2009). The main advantage of working with a small, but representative, group of countries is the possibility of building a simple model that describes well the economies under study. The larger the number of countries studied, certainly the more complex the model that would be needed to describe all of them.

In summary, our main question is: in what extent these yield spreads can be attributed to economic fundamentals? In particular, we are interested in the contribution of deficit and debt in the expansion of sovereign spreads in the years after the onset of the economic crisis of 2007. Our idea is to distinguish the effects of fiscal shocks from the effects of shocks to other macroeconomic variables and potentially relevant indicators of risk aversion. It is worth defining what we are calling default: default happens when a country repudiates, in whole or in part, its debt and payment of dividends related to it.

The remaining of this paper is organized as follows: In section 2 we present the related literature (both on term structure models and the relation between fiscal variables and yield spreads in Europe). In the third section, we give a first look at the data and, in the fourth section, we present the model. Data and estimation strategy are presented in sections 5 and 6, respectively. The results are shown in section 7 and section 8 concludes.

## 2. Related Literature

As mentioned earlier, our model is a special case of discrete versions of the model of the term structure arbitrage-free affine introduced by Duffie and Kan (1996). In this article, bonds of different maturities are considered default-free. The literature on pricing of defaultable securities was heavily influenced by Duffie and Singleton (1999), in which, in a span of Duffie and Kan (1996), the authors develop a tool to model this type of security, applied to corporate bonds market. Duffie, Pedersen and Singleton (2003) extend the 1999 article by modeling the spread of the different securities issued by Russia on U.S. LIBOR. Among the papers mentioned above, none uses observable variables as factors, which are treated as latent elements of the market of yields itself.<sup>3</sup>

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<sup>3</sup> In Duffie, Pedersen and Singleton (2003), for example, volatility is used as an explanatory factor.

Ang and Piazzesi (2003) introduce inflation and activity indicators, together with unobservable latent variables, as state variables of an affine term structure model. Using data from the U.S. economy, they find that macroeconomic variables account for over half the variance of forecast errors in bond yields, and its participation is greater in longer forecasts. The impulse response functions show that the impact of shocks from macroeconomic variables on bond yields is of greater magnitude in models that impose restrictions of no arbitrage than in models that ignore this restriction. Lowenkron and Bonomo (2008) apply the model of Duffie and Singleton (1999) for Brazilian, Colombian and Mexican bonds (considered as defaultable) and model the spreads relative to U.S. bonds. As Ang and Piazzesi (2003), they use observable and latent variables and note that observables related to the U.S. economy and national economies are important in the dynamics of spreads.

In the literature on the European sovereign bonds market, the papers tackling of the role of fiscal variables on the bond yield spreads are not conclusive. Focusing in the European bond market, Bernoth, von Hagen and Schuknecht (2004) and Schuknecht, von Hagen and Wolswijk (2008) studied the relationship between fiscal variables and spreads for some selected maturities.<sup>4</sup> The basic equation of the two articles has spreads as explanatory variable and fiscal variables and maturity of bonds (among others), as dependent variable, with no restriction of no-arbitrage. The authors find a positive effect on the deficit and debt spreads. In a more general approach, using a sample of 26 countries, Longstaff et al. (2010) found that the excess returns from investing in sovereign credit are largely compensation for bearing global risk, and that there is little or no country-specific credit risk premium. Our goal is to unite the literature on term structure models summarized above with the literature on the European sovereign bonds market.

### 3. A First Look at the Data

In the European case, specifically in the cases of Italy, Greece and Spain (group we will call IGS) and Germany, Figures 1-3 suggest that the relationship between deficit and sovereign yields does exist. Note in Figure 1, that from 1999 to early 2008, the 1-year yield spread between each of the IGS and Germany sovereign bonds was very low, in accordance to the interest-rate parity theory.<sup>5</sup> From mid 2008 onwards, this spread began to widen. That yield spreads continue to exist in a monetary union – i.e., after the elimination of the exchange rate risk – is mainly due to the fact that investors do not regard government bonds of the different EMU countries as complete substitutes. See in Figure 2 that the budget deficit of these selected countries has also increased significantly since 2008. Figure 3 (first column) shows the 4 quarters moving average of deficit and 1 year bond yield spreads of IGS (the last line shows the mean of these variables among the countries). The figure suggests that deficits are closely related to spreads.

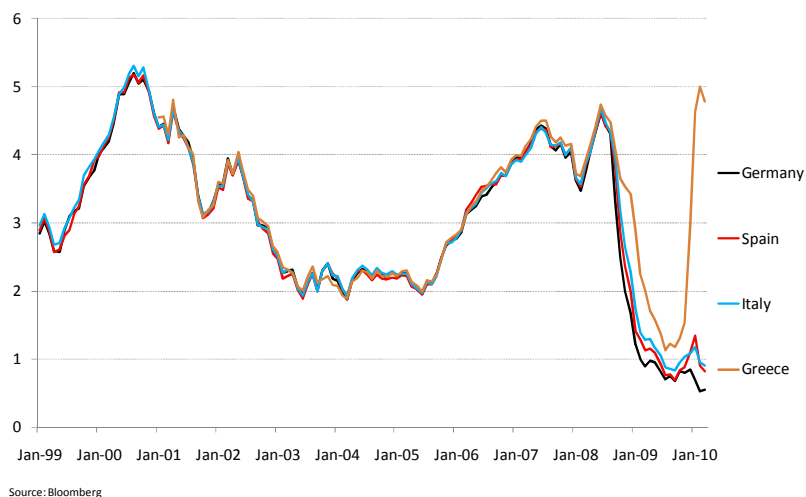
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<sup>4</sup> It is standard this literature to evaluate, instead of the bond yields themselves, the spreads in relation to the yields paid by the German securities. Schuknecht, von Hagen and Wolswijk (2008) also studied Canadian government bond market.

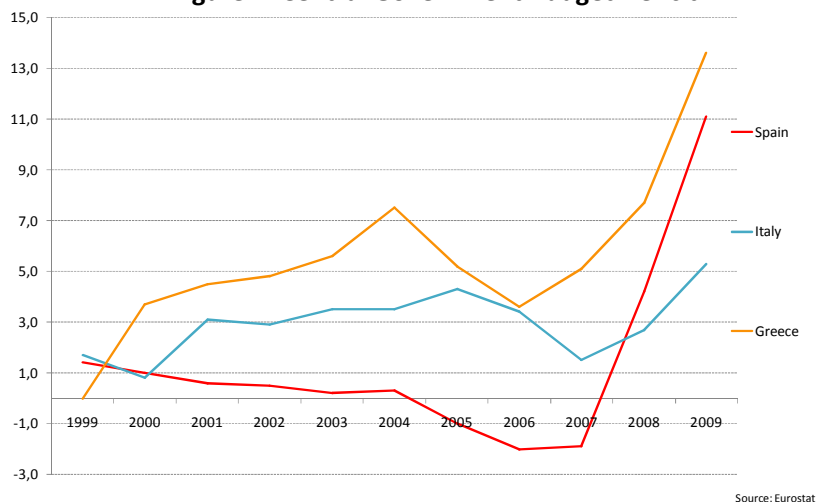
<sup>5</sup> This theorem relates the forward exchange rate to the money-market interest differential,  $(F - S)/S = r_d - r_f$ , where  $F$  = the forward exchange rate,  $S$  = the spot exchange rate,  $r_d$  = the domestic interest rate, and  $r_f$  = the foreign interest rate. See Aliber (1973).

Because part of government spending is used to pay the service of its debt, deficit and interest rates are naturally related variables. But note that this relationship exists even when we extract the interest payments from the calculation of the deficit (see Figure 2, second column, perhaps the most supportive to our argument).

**Figure 1: 1 Year Sovereign Bond Yield – Selected Countries**



**Figure 2: Central Government Budget Deficit**



#### 4. The model

##### 4.1. Basic Concepts

Consider  $V_t$  the price at  $t$  of a bond that pays no coupon and makes a payment of \$ 1 at time  $t + N$  and  $Y_t$  the yield paid by this bond until maturity. So we have:

$$(1 + Y_t)^N = \frac{1}{V_t}$$

Using lowercase letters to denote variables in *log*, we have:

$$y_t = -\frac{1}{N}v_t$$

The holding period return on a bond is the return over some holding period. When this period equals one, it is given by:

$$(1 + R_{t+1}) = \frac{V_{t+1}}{V_t} \tag{1}$$

Consider an intertemporal choice problem of an investor who can freely trade one asset *i* and maximizes the following program, conditional on a usual budget constraint:

$$\text{Max } E_t \left[ \sum_{j=0}^{\infty} \delta^j U(C_{t+j}) \right]$$

where  $U(\cdot)$  represents the utility of the agent,  $C_t$ , her consumption in *t* and  $\delta$  the discount factor of intertemporal utility. One of the Euler equations is as follows:

$$U'(C_t) = \delta E_t \left[ (1 + R_{t+1}^{(i)}) U'(C_{t+1}) \right]$$

Dividing both sides by  $U'(C_t)$ , we have:

$$1 = E_t \left[ (1 + R_{t+1}^{(i)}) M_{t+1} \right] \tag{2}$$

where  $M_{t+1} \equiv \delta \frac{U'(C_{t+1})}{U'(C_t)}$  is the stochastic discount factor. Substituting (1) in (2) yields:

$$V_t = E_t [ M_{t+1} V_{t+1} ] \tag{3}$$

It can be proved (see Duffie [2001]) that equation (3) guarantees the existence of no arbitrage. Note that, if *N* denotes the maturity of the bond, once established the processes (in *t*) governing  $V_t^{(N=1)}$  and  $M_{t+1}$ , we calculate recursively the prices of bonds of all maturities  $N = 2, 3, \dots$ .

## 4.2. Model Specification

### 4.2.1. Term Structure Model for Non-Defaultable Bonds<sup>6</sup>

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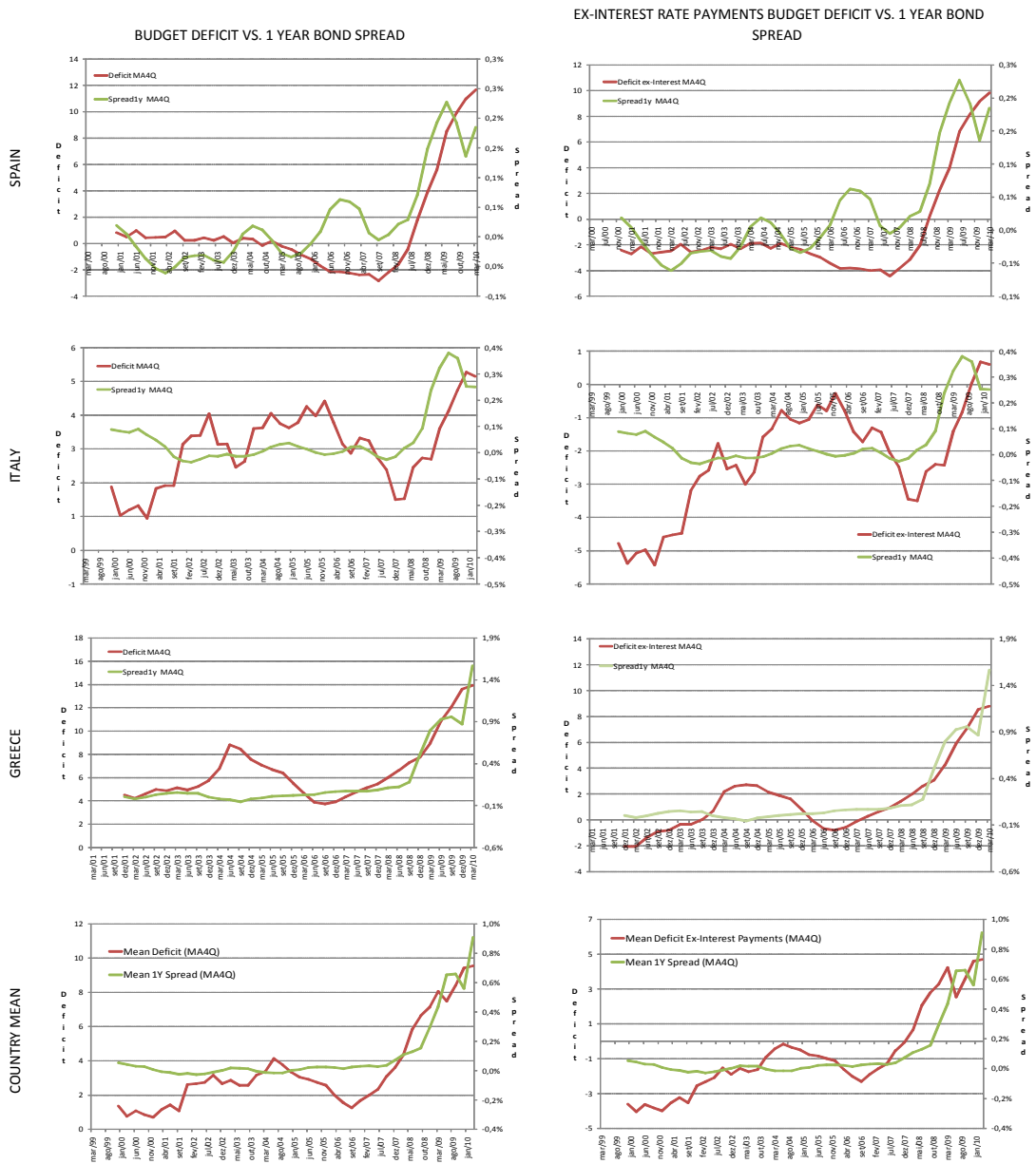
<sup>6</sup> We follow Duffie and Kan (1996) and Ang and Piazzesi (2003).

In this section we consider the model for pricing of securities issued by the German government, which, according to our assumption, are non-defaultable securities.

Assume that the short rate,  $r_t$ , follows a linear function of all state variables, grouped in the vector  $X_t$  of dimension K (Duffie and Kan [1996]):

$$r_t = \delta_0 + \delta_1' X_t \tag{4}$$

**Figure 3: Deficit vs. 1 Year Bond Yield Spreads - 4 quarters Moving Average**



Sources: Eurostat (Deficit) and Bloomberg (Spread)

Assume that  $X_t$  follows a Gaussian VAR (P):

$$A\mathbf{X}_t = \varphi + \sum_{i=1}^P \rho_i X_{t-i} + B\varepsilon_t$$

such that

$$\begin{aligned} \mathbf{X}_t &= A^{-1}\varphi + A^{-1} \sum_{i=1}^P \rho_i \mathbf{X}_{t-i} + A^{-1}B\varepsilon_t \\ &= \mu + \sum_{i=1}^P \Phi_i \mathbf{X}_{t-i} + \Sigma\varepsilon_t \end{aligned}$$

We assume the simplifying hypothesis that  $P = 1$ . So, we have:

$$\mathbf{X}_t = \mu + \Phi\mathbf{X}_{t-1} + \Sigma\varepsilon_t \quad (5)$$

The composition of  $X_t$  will be discussed bellow, but it includes fiscal variables. In a known result in the finance literature (see Ang and Piazzesi [2003]), the stochastic discount rate  $M_{t+1}$  can be written as:

$$M_{t+1} = \exp\left\{-\frac{1}{2}\lambda'_t\lambda_t - \delta_0 - \delta'_1\mathbf{X}_t - \lambda'_t\varepsilon_{t+1}\right\} \quad (6)$$

where  $\lambda_t$  is the market price of risk (time variant) associated with the sources of uncertainty  $\varepsilon_t$ .  $\lambda_t$  is parameterized as an affine function of the state variables:

$$\lambda_t = \lambda_0 + \lambda'_1\mathbf{X}_t \quad (7)$$

for a  $K \times 1$  vector  $\lambda_0$  a  $K \times K$  matrix  $\lambda_1$ . From Ang e Piazzesi (2003), we know:

**Result 1:** If  $M_{t+1}V_{t+1}^{(N)}$  follows a lognormal distribution, the restriction of no arbitrage implies that the price at  $t$  of a bond (in case we are evaluating a bond issued by the German government) to maturity  $(N + 1)$  can be written as:

$$V_t^{(N+1)Ge} = \exp(\bar{A}_{N+1} + \bar{B}'_{N+1}\mathbf{X}_t^{Ge}) \quad (8)$$

where  $\bar{A}_{N+1}$  e  $\bar{B}'_{N+1}$  follow the differential equations:

$$\bar{A}_{N+1} = \bar{A}_N + \bar{B}'_N(\mu^{Ge} - \Sigma^{Ge}\lambda_0) + \frac{1}{2}\bar{B}'_N\Sigma^{Ge}\Sigma^{Ge'}\bar{B}_N - \delta_0^{Ge} \quad (9)$$

$$\bar{B}'_{N+1} = \bar{B}'_N(\Phi^{Ge} - \Sigma^{Ge}\lambda'_1) - \delta_1^{Ge'} \quad (10)$$

with  $\bar{A}_1 = -\delta_0^{Ge}$  e  $\bar{B}'_1 = -\delta_1^{Ge'}$ .

The yield  $y_t^{(N)}$  paid in  $t$  by a bond of maturity  $N$  is given by:

$$y_t^{(N)A} = -\frac{\log(V_t^{(N+1)Ge})}{N} = A_N + B'_N X_t^{Ge} \quad (11)$$

where  $A_N = -\frac{\bar{A}_N}{N}$  and  $B_N = -\frac{\bar{B}_N}{N}$ .

This result shows, therefore, that conditional on the assumptions above, bond yields are linear functions of factors  $X_t^{Ge}$ . Once we have yields of different maturities, we estimate the parameters by imposing the no-arbitrage restrictions on cross-section estimates given by equations (9) and (10). We assume the simplifying hypothesis that the matrix  $\lambda_1$  in equation (7) is diagonal.

In short, we need to estimate the following parameters:

$$\Psi^{Ge} = (\delta_0^{Ge}, \delta_1^{Ge}, \mu^{Ge}, \phi^{Ge}, \Sigma^{Ge}, \lambda_0, \lambda_1)$$

Recall that  $\delta_0^{Ge}$  is a scalar;  $\delta_1^{Ge}$ ,  $\mu^{Ge}$  and  $\lambda_0$  are  $K$ -dimensional vectors;  $\phi^{Ge}$ ,  $\Sigma^{Ge}$  and  $\lambda_1$  are  $K \times K$  matrices. With no restrictions imposed,  $\Psi^{Ge}$  can be written as a  $(1+3K+3K^2)$ -dimensional vector of parameters.

#### 4.3. Term Structure Model for Defaultable Bonds

In this section we consider the model for pricing of securities issued by the group of countries IGS, which, according to our assumption, are defaultable securities.

Consider a defaultable bond that, at  $t$ , promises to pay  $V_{t+N}$  at maturity date  $t + N$ , and nothing before that. For any period  $s \geq t$ , let: (1)  $h_s \in [0,1]$  be the conditional probability at  $s$  of default between  $s$  and  $s + 1$ ; (2)  $\varphi_s$  be the, recovery, in units of account, in case of default; (3)  $M_s$  be the stochastic discount factor (or pricing kernel) in  $s$  of the representative buyer.

Therefore, the present value of this bond at  $t$ ,  $V_t$ , is given by:

$$V_t = h_t E_t(M_{t+1} \varphi_{t+1}) + (1 - h_t) E_t(M_{t+1} V_{t+1}) \quad (12)$$

**Hypothesis 1:** In case of default, the recovery is proportional to the face value of the bond:

$$\varphi_s = (1 - L_s) V_s \quad (13)$$

Hypothesis 1 follows Pan and Singleton (2008) and must be thought as case-specific. Substituting (13) into equation (12), we have (Bonomo and Lowenkron [2008]):

$$\begin{aligned} V_t &= h_t E_t[M_{t+1} (1 - L_{t+1}) V_{t+1}] + (1 - h_t) E_t[M_{t+1} V_{t+1}] \\ &= E_t[M_{t+1} (1 - h_t L_{t+1}) V_{t+1}] \end{aligned}$$

A common feature of this literature is allowing liquidity to affect pricing.<sup>7</sup> We make the simplistic assumption that illiquidity of the security translates into a fractional cost of rate  $l$ . Hence, the total discount rate of the security due to default and illiquidity is:

$$V_t = E_t[M_{t+1}\Theta_{t+1}V_{t+1}] \quad (14)$$

where

$$\Theta_{t+1} \equiv 1 - h_t L_{t+1} - l_{t+1}$$

is a measure of default and liquidity risk. We assume that  $\Theta_{t+1}$  is linear on factors  $\mathbf{X}_t$ , i.e.:

$$\begin{aligned} \ln\Theta_t &\equiv \theta_t = \theta_0 + \theta'_1 \mathbf{X}_t \\ &= \theta_0 + \theta'_1 \Phi \mathbf{X}_{t-1} + \theta'_1 \Sigma \varepsilon_t \end{aligned}$$

Because we are working with  $\log \Theta_t$ , there is the need of imposing an additional hypothesis:

**Hypothesis 2:** For every  $s$ ,  $\theta_s > 0$

The second hypothesis means that the discount applied on defaultable bonds as a result of the possibility of default or because of liquidity effects cannot be larger than 100%. In the absence of liquidity effects, it means that the situation in which the probability assigned to default is 100% and, in case of default, the loss rate is 100%, never occurs. We consider that this is an appropriate assumption to the real world.

Note that the stochastic discount factor used in pricing a defaultable bond is  $M_{t+1}\Theta_{t+1}$ , which is the stochastic discount factor  $M_{t+1}$  of the representative buyer of the non-defaultable bond market (of bonds issued by the German government, in our model) adjusted for the default and liquidity risk  $\Theta_{t+1}$ . Once  $\Theta_{t+1}$  carries information on the recovery rate in case of default, probability of default and liquidity effects on the price of the bond, we should think of  $\Theta_{t+1}$  as maturity-specific. So, we should rewrite equation (14) as:

$$V_t^{N+1} = E_t[M_{t+1}\Theta_{t+1}^N V_{t+1}^N] \quad (15)$$

We assume the following simplifying hypothesis on  $\ln\Theta_t^N$ :

**Hypothesis 3:**  $\ln\Theta_t^N$  has three additive components: a constant, a component linear on the maturity and a component linear on the state variables.

$$\ln\Theta_t^N \equiv \theta_t^N = \theta_0^N + \theta_1^N \mathbf{X}_t = \theta_{00} + N\theta_{01} + \theta_1^N \mathbf{X}_t \quad (16)$$

A crucial implication of this hypothesis is that  $\ln\Theta_t^N$  varies (linearly) with the maturity, but the effect of the state variables on  $\ln\Theta_t^N$  does not depend on the maturity. Note, however, that it

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<sup>7</sup> See Duffie and Singleton (1999) and Duffie, Pedersen and Singleton (2003)

does not rule out a non-linear interaction between the effect of the state variables on  $\theta_t^N$  and the maturity of the bond.

Given the model described above, the following proposition determines the prices of defaultable bonds. Thereafter, prices and other variables related to a specific country in the group of the IGS will be indexed by  $i$ .

**Proposition 1:** Assume that the short rate spread,  $s_t^{1i} \equiv y_t^{1i} - y_t^{1Ge}$ , follows an analog process of the one described in equation (4), i.e.:

$$s_t^{1i} = \delta_0^i + \delta_1^{i'} \mathbf{X}_t^i \quad (17)$$

$\mathbf{X}_t^i$  is a  $K^i$  dimensional vector of observations;  $\delta_0^i$  is a scalar and,  $\delta_1^i$ ,  $K^i$ -dimensional vector of parameters. If  $M_{t+1} \theta_{t+1}^{(N)i} V_{t+1}^{(N)i}$  follows a lognormal distribution, the restriction of no arbitrage implies that the price at  $t$  of a defaultable  $(N+1)$ -maturity bond (in the case we are evaluating, a bond issued by one of the IGS governments) can be written as:

$$V_t^{(N+1)i} = V_t^{(N+1)Ge} + \bar{D}_{N+1}^i + \bar{E}_{N+1}^{i'} \mathbf{X}_t^i$$

where  $\mathbf{X}_t^i$  represents the matrix of factors for all countries, Germany and the IGS and:

$$\bar{D}_{N+1}^i = \theta_0^{(N)i} + \bar{D}_N^i + (\theta_1^i + \bar{E}_N^i)' \mu^i + (\theta_1^i + \bar{E}_N^i)' \Sigma^i \left[ J' (\Sigma^{Ge'} \bar{B}_N - \lambda_0) + \frac{1}{2} \Sigma^{i'} (\theta_1^i + \bar{E}_N^i) \right] \quad (18)$$

$$\bar{E}_{N+1}^{i'} = (\theta_1^i + \bar{E}_N^i)' (\Phi^i - \Sigma^i J' \lambda_1^i J) \quad (19)$$

$J$  is a selection matrix such that  $\mathbf{X}_t^{Ge} = J \mathbf{X}_t^i$ .  $y_t^{(N)i}$  is given by:

$$y_t^{(N)i} = -\frac{V_t^{(N+1)i}}{N} = y_t^{(N)Ge} + D_N + E_N' \mathbf{X}_t^i$$

The spreads  $s_t^{(N)i} \equiv y_t^{(N)i} - y_t^{(N)Ge}$  can be written as

$$s_t^{(N)i} = D_N + E_N' \mathbf{X}_t^i \quad (20)$$

where  $D_N = -\frac{\bar{D}_N}{N}$  and  $E_N = -\frac{\bar{E}_N}{N}$ .

Proof in Appendix 5.

Again, we observe that the spreads are linear functions of factors. Once we have yields on bonds of different maturities, we estimate the parameters by imposing no-arbitrage restrictions on cross-section estimates given by equations (18) and (19).

For the defaultable bonds model, we need to estimate the following parameters:

$$\Psi^i = (\delta_0^i, \delta_1^i, \mu^i, \phi^i, \Sigma^i, \theta_{00}^i, \theta_{01}^i, \theta_1^i)$$

for  $i = \text{Italy (It), Spain (Sp) and Greece (Gr)}$ . Note that  $\delta_0^i$ ,  $\theta_{00}^i$  and  $\theta_{01}^i$  are scalars,  $\delta_1^i$ ,  $\mu^i$  and  $\theta_1^i$  are  $K^i$ -dimensional vectors, and  $\phi^i$  and  $\Sigma^i$  are  $K \times K$  matrices. So, with no restrictions imposed,  $\Psi^i$  can be written as a  $(3+3K^i+2K^{i2})$ -dimensional vector of parameters.<sup>8</sup>

## 5. Data

The countries we study are Germany, Italy, Greece and Spain. We use monthly data from January 1999 to March 2010 for Italy and Spain, and from January 2001 to March 2010 for Greece.<sup>9</sup> We use prices of securities with maturities of 3, 6, 12, 24, 36, 48, 60, 84 and 120 months, extracted from Bloomberg. Factors used are divided into macroeconomic variables, a risk indicator and yields-related variables. The macroeconomic variables are: Industrial production ( $IP_t = \frac{IP \text{ index}_t}{IP \text{ index}_{t-12}} - 1$ ), inflation ( $I_t = \frac{CPI_t}{CPI_{t-12}} - 1$ ), deficit/GDP ( $Def_t = \frac{\text{Fiscal Balance ex-Interest Payments}_t}{GDP_t}$ ), and debt/GDP ( $Deb_t = \frac{Debt_t}{GDP_t}$ ). The risk indicator is Moody's Baa Corporate Bond Yield ( $HY_t$ ). The yields-related variables are the 1st, 2nd and 3rd Principal Components of Yields (for Germany) or Spreads (for the IGS countries) estimated from an eigenvalue decomposition of the observed yields (spreads) covariance matrix, orthogonalized with respect to the macroeconomics and risk factors (Cochrane e Piazzesi [2008]). These yields and spreads factors represent variables other than factors considered above.

Industrial production and inflation were included in the light of the findings of Ang and Piazzesi (2003), which shows the importance of these variables, along with principal components of yields, in their term structure model applied to the US data. Deficit and debt were included to capture the fiscal effect on spreads we want to access. Moody's Baa Corporate Bond Yield was chosen to capture international finance market stress, and we expect it to reflect disposal of the international market to take risk.

Sources for the macroeconomic variables are Eurostat (all variables except  $HY$ ) and Board of Governors of the Federal Reserve System ( $HY$ ). The IMF-IFS database was used as an additional source in periods in which Eurostat was not complete. Fiscal Balance, Debt and GDP monthly data are not available. Monthly data of these variables were constructed from their quarterly observations and from monthly data of some coincident indicators, such as energy consumption, unemployment rate, imports and exports. For details, see Appendix 2.

## 6. Estimation Strategy

We use the following multi-step estimation procedure:

<sup>8</sup> Note that, if some variables included in the Germany non-defaultable model are also included in the defaultable bonds model,  $\psi^i$  and  $\psi^{Ge}$  share some of the VAR parameters of the for the country  $i$  ( $\mu^i$ ,  $\phi^i$  and  $\Sigma^i$ ).

<sup>9</sup> Greece joined the European Monetary Union in 2001.

1. Non-Defaultable Bonds – vector  $\widehat{\Psi}^{Ge}$  estimation
  - a. We estimate the short rate equation (4) and the macro dynamics (5) by OLS, obtaining  $\widehat{\delta}_0^{Ge}, \widehat{\delta}_1^{Ge}, \widehat{\mu}^{Ge}, \widehat{\Phi}^{Ge}$  and  $\widehat{\Sigma}^{Ge}$ .
  - b. We estimate the yield equation (11) also by OLS imposing restrictions (9) and (10). In this step, we obtain  $\widehat{\lambda}_0$  and  $\widehat{\lambda}_1$ . We assume the simplifying hypothesis that the matrix  $\lambda_{1t}$  in equation (7) is diagonal.
2. Defaultable Bonds – the remaining of vector  $\Psi^i$  estimation
  - a. We estimate the short rate spread equation (17) and the analog macro dynamics (5) for the country i by OLS, obtaining  $\widehat{\delta}_0^i, \widehat{\delta}_1^i, \widehat{\mu}^i, \widehat{\Phi}^i$  and  $\widehat{\Sigma}^i$ .
  - b. We estimate the yield equation (20) also by OLS using previously estimated parameters  $\widehat{\Psi}^{Ge}$  and imposing restrictions (18) and (19). In this step, we obtain  $\widehat{\theta}_{00}^i, \widehat{\theta}_{01}^i$  and  $\widehat{\theta}_1^i$ .

In the non-defaultable bonds model, the state variables  $\mathbf{X}_t^{Ge}$  are  $HY_t$  and all macroeconomic and yields-related variables from Germany, in the order:  $HY_t, IP_t, CPI_t, Def_t, Deb_t, PC1_t, PC2_t, PC3_t$ . In the defaultable bonds model, state variables  $\mathbf{X}_t^i$  are  $HY_t$  and all macroeconomic and yields-related variables from Germany and the country i.  $\mathbf{X}_t^i$  includes 1st, 2nd and 3rd principal components of spreads for country i, estimated from an eigenvalue decomposition of the observed spreads covariance matrix, orthogonalized with respect to the macroeconomics and risk factors (Cochrane e Piazzesi [2008]). These yields and spreads factors represent variables other than factors considered above. In this model, the variables are ordered the same way: First  $HY_t$ , then the German ones, and then the specific country variables (i.e.,  $K=8$  and  $K^i=15$ ). As some variables included in the Germany non-defaultable model are also included in the defaultable bonds model,  $\mu^i, \phi^i$  and  $\Sigma^i$  and  $\mu^{Ge}, \phi^{Ge}$  and  $\Sigma^{Ge}$  share some parameters. The way  $\mu^i, \phi^i$  and  $\Sigma^i$  are constructed (and the hypothesis imposed) can be seen in equations (21) to (23). The estimation of  $\widehat{\Sigma}^{Ge}$  and  $\widehat{\Sigma}^i$  demands the estimation of a SVAR. See details in Appendix 3.

$$\mu_{15 \times 1}^i = \begin{pmatrix} \mu_{8 \times 1}^{Ge} \\ \mu_{7 \times 1}^{ii} \end{pmatrix} \quad (21)$$

$$\phi_{15 \times 15}^i = \begin{pmatrix} \phi_{8 \times 8}^{Ge} & \mathbf{0}_{8 \times 7} \\ \phi_{7 \times 15}^{ii} & \end{pmatrix} \quad (22)$$

$$\Sigma_{15 \times 15}^i = \begin{pmatrix} \Sigma_{8 \times 8}^{Ge} & \mathbf{0}_{8 \times 7} \\ \Sigma_{7 \times 15}^{ii} & \end{pmatrix} \quad (23)$$

$\Psi^{Ge}$  can be written as a  $(1+3K+3K^2) = 217$ -dimensional vector of parameters and  $\Psi^i$  can be written as a  $(3+3K^i+2K^{i2}) = 498$ -dimensional vector of parameters. As 8 parameters of  $\mu^i$  come from  $\mu^{Ge}$ ,  $8 \times 8 = 64$  parameters of  $\phi^i$  come from  $\phi^{Ge}$ , and other  $8 \times 7 = 56$  are zero, and as  $8 \times 8 = 64$  parameters of  $\Sigma^i$  come from  $\Sigma^{Ge}$ , and other  $8 \times 7 = 56$  are zero, given  $\Psi^{Ge}$ ,  $\Psi^i$  has 370 unknown parameters.

## 7. Results

### 7.1. Parameter Estimates

Tables A5.1 to A5.4 in the Appendix 5 show parameter estimates for the non-defaultable (table A5.1) and defaultable bonds model (tables A5.2 for Italy, A5.3 for Spain and A5.4 for Greece). In both models, standard errors are calculated through a Bootstrap procedure (400 repetitions). The reported standard errors for the non-defaultable bonds model parameters  $\Psi^{Ge}$  take into account all the steps of estimation (steps 1.a and 1.b in section 6). The reported standard errors for the defaultable bonds model parameters  $\Psi^i$  take into account the steps of estimation of the parameters of the model (steps 2.a and 2.b in section 6), taking  $\hat{\Psi}^{Ge}$  as constant.<sup>10</sup>

Note in tables A5.2 to A5.4 that the estimated parameters  $\hat{\theta}_{00}^i$ ,  $\hat{\theta}_{01}^i$  and  $\hat{\theta}_1^i$ , related to the measure of default and liquidity risk seems to have no statistical significance. In this paper, we are going to focus our analysis in the impulse response functions and the estimated factor loadings over maturities. We believe this result is not ruling out the existence of default and liquidity premia because the recent regime (in which investors distinguish between German and IGS debts) comprises only around 20% of the total number of observations in the sample, hampering the task of estimating parameters with precision

### 7.2. What has been driving the Spreads?

From equation (20), we know that the effect of each factor on the yield curve is determined by the weights  $E^N$  that the term structure model assigns on each spread on bonds of maturity N. Figure 4 plots the 1 year bond yield spread estimated composition, i.e., the weights of factors in the 1 year bond yield spread (i.e., for N=12) multiplied by the current values of the corresponding factor along time. The factors are *HY, IP, Inf, Def, Deb*, the principal components of the spreads (summed up in *PC*) and Germany related variables (summed up in *Ge*).<sup>11</sup>

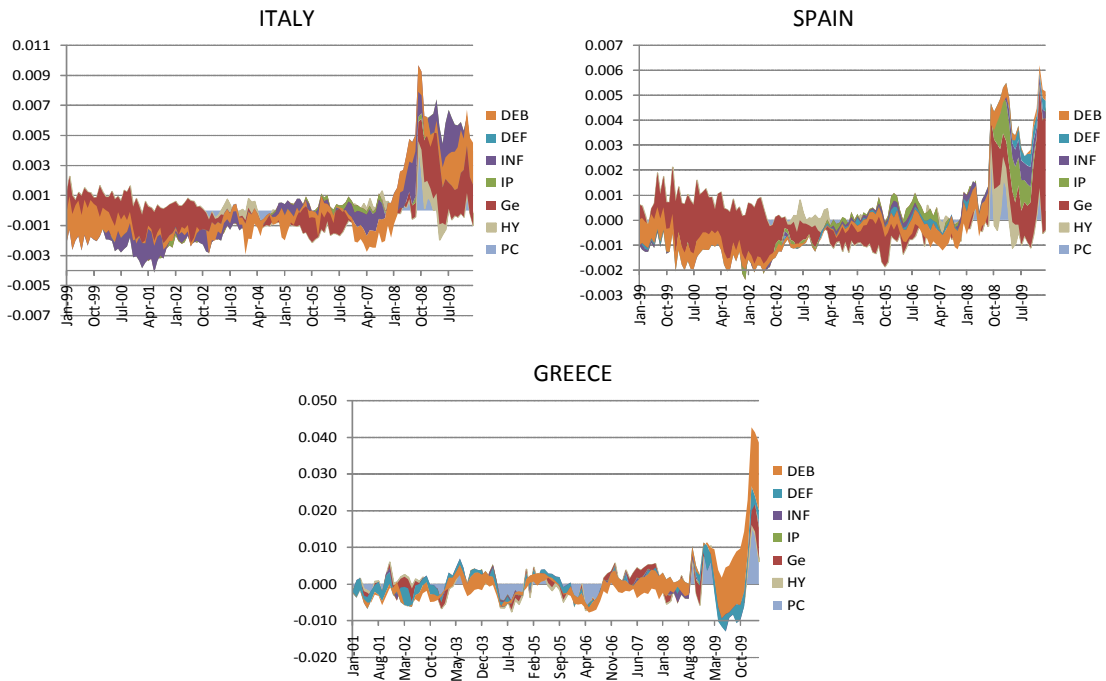
In the case of Italy, note that the recent rise in spreads has been driven mainly by Italy's debt and variables related to Germany. Inflation and market stress (*HY*) also have been playing an important role. The Germany related variables have been even more important for the widening of Spain's spreads, with smaller influence of debt, deficit, inflation and industrial production. Figure 5 plots the weights only of Germany-related factors  $E^N$  in the 1 year bond yield spread multiplied by the current values of the corresponding factor along time. It shows that, for both Italy and Spain, among the Germany related variables, the most important one for the widening of spreads is Germany debt. A possible explanation for that is a perception of markets that Germany is a lender of last resort of the largest economies in the EMU.

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<sup>10</sup> Note that step 2.b depends on previously estimated  $\hat{\Psi}^{Ge}$ . The bootstrap procedure used to estimate the reported standard errors for the defaultable bonds model parameters  $\Psi^i$  takes  $\hat{\Psi}^{Ge}$  as constant.

<sup>11</sup> The estimated parameters  $D^N$  was not taken into account in the plots. It's 0.0007 for Italy, 0.0002 for Spain and 0.0019 for Greece.

**Figure 4: 1 year bond yield spread estimated composition**



Note: Composition based on the  $E^N$  factor weights in the 1 year bond yield spread (i.e., for  $N=12$ ) multiplied by the current values of the corresponding factor along time. The factors are *HY, IP, Inf, Def, Deb*, the principal components of the spreads (summed up in *PC*) and Germany related variables (summed up in *Ge*). Y-axis scales are different among countries for a better visualization.

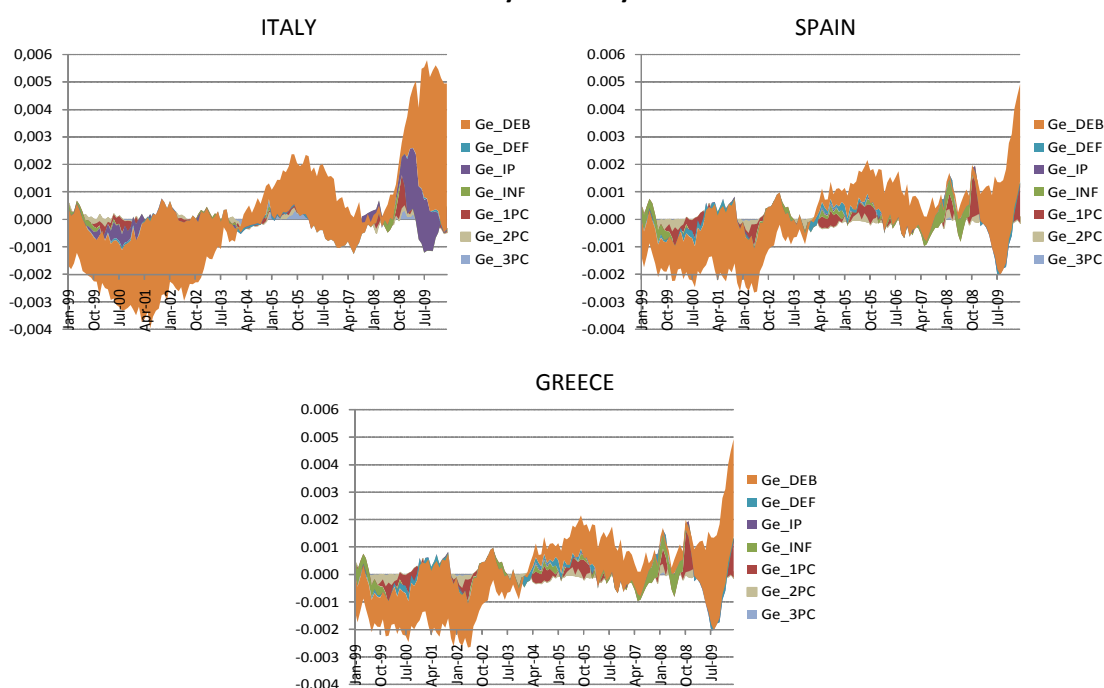
For Greece, the rise in spreads has been driven mainly by Greece's own debt (Figure 4), with smaller influence of variables other than factors considered (represented by the principal components of spreads), Greek deficit and Germany-related factors.

### 7.3. Factor loadings along the spread curve

As mentioned, the effect of each factor on the yield curve is determined by the weights  $E^N$  that the term structure model assigns on each spread of maturity  $N$ . The weights  $E^N$  also represent the initial response of yields to shocks from the various factors. Figure 6 plots the weights of *HY, Ge\_Deb, IP, Infl, Def* and *Deb* as a function of yield maturity for the three countries.

A common interesting feature is that the weight of the own country's debt is larger around the maturity of 12 months. For Italy and Spain, Germany's debt pays the most important role until the maturity of 112 and 66 months, respectively. In the case of Greece, the own country's debt is the most relevant variable in the determination of spreads. For all countries, market stress (*HY*) is the variable that has the largest weight over longer maturities (from 113 months for Italy, 69 months for Spain and 73 months for Greece). The weight on activity is always negative, suggesting that economic growth is perceived as reducing sovereign risk.

**Figure 5: 1 year bond yield spread estimated composition**  
 - Only Germany related factors.



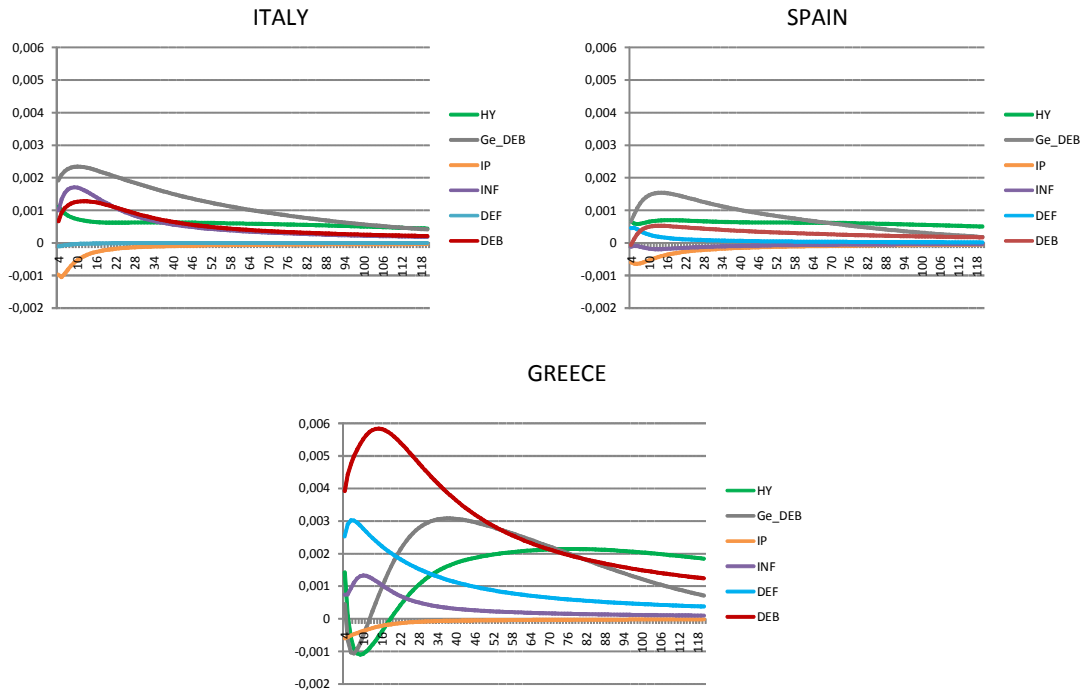
Note: Composition based on Germany related factors  $E^N$  factor weights in the 1 year bond yield spread (i.e., for  $N=12$ ) multiplied by the current values of the corresponding factor along time. The factors are  $Ge_{IP}, Ge_{Inf}, Ge_{Def}, Ge_{Deb}$ , the principal components of the Germany's yields ( $Ge_{1PC}$  to  $Ge_{3PC}$ ).

#### 7.4. Impulse Responses

Our term structure model allows us to obtain the response of the yields to shocks at all horizons, including maturities omitted in estimation. The IRs for all maturities are known analytical functions of the parameters. Figures 7 to 9 show the IR's of 6, 12 and 120 months for the three countries. These figures show the movements of the yield curve of different maturities (in rows) in response to one standard deviation shocks to  $HY, Ge_{Deb}, IP, Inf, Def$  and  $Deb$  (in columns).<sup>12</sup>

<sup>12</sup> As the reported parameter estimates, reported standard errors for the impulse responses (defaultable bonds model) takes into account the steps of estimation of the parameters of the model (steps 2.a and 2.b in section 4.1), taking  $\hat{\varphi}^{Ge}$  as constant. .

**Figure 6:  $E^N$  factor loadings of  $HY, Ge\_Deb, IP, Infl, Def$  and  $Deb$  for the three countries as a function of maturity  $N$ .**



For Italy (see Figure 7), the variable that causes the largest impact is market stress ( $HY$ ), followed by Germany's debt and the country's own debt ( $Deb$ ). A shock of one standard deviation from  $HY$ , for example, causes an initial response of the 1 year yield spreads of 35% of its standard deviation ( $0.023e-2$  of  $0.066e-2$ ).<sup>13</sup> A shock of one standard deviation from  $Ge\_Deb$  causes an initial response of the 1 year yield spreads of 34% of its standard deviation ( $0.022e-2$ ) and, from the country's debt, 27% ( $0.018e-2$ ).<sup>14</sup> The responses to shocks from  $HY$ , Germany's debt and the country's own debt get softer as the maturity increases.

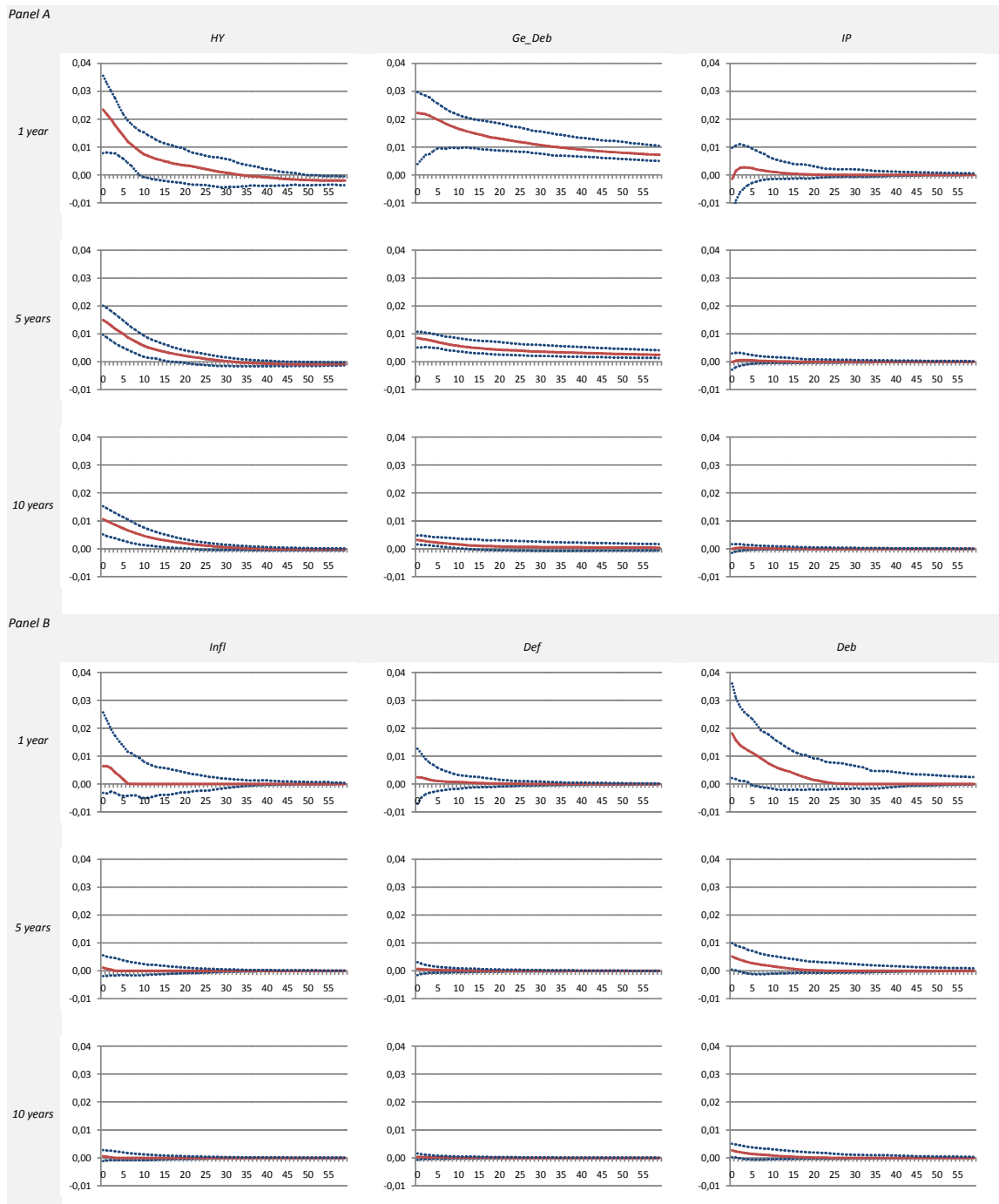
For Spain, Figure 8 shows that Germany's debt is the variable that causes the largest impact, followed by market stress ( $HY$ ) and the country's own debt. A shock of one standard deviation from Germany's debt causes an initial response of the 1 year yield spreads of 55% of its standard deviation ( $0.018e-2$  of  $0.032e-2$ ), from  $HY$ , 40% ( $0.013e-2$ ) and, from the country's own debt, 14% ( $0.004e-2$ ).

Yield spreads of Greece respond more strongly to shocks from debt and deficit. A shock of one standard deviation from  $Def$  causes an initial response of the 1 year yield spreads of 30% of its standard deviation ( $0.08e-2$  of  $0.28e-2$ ) and from  $Deb$ , 20% ( $0.06e-2$ ).

<sup>13</sup> Such small numbers are expected: Recall that  $s_t^{(N)i} \equiv y_t^{(N)i} - y_t^{(N)Ge}$ , where  $y_t^{(N)i} = \ln(1 + \frac{y_t^{(N)i}}{100})$ . If Italy's 1 year bond yield is, say, 3% per year,  $Y_t^{(12)It} = 3$ . If Germany's 1 year bond yield is, say, 2.7% per year,  $Y_t^{(12)Ge} = 2.7$  and  $s_t^{(12)It} \equiv y_t^{(12)It} - y_t^{(12)Ge} = \ln(1.030) - \ln(1.027) \approx 0.003$ .

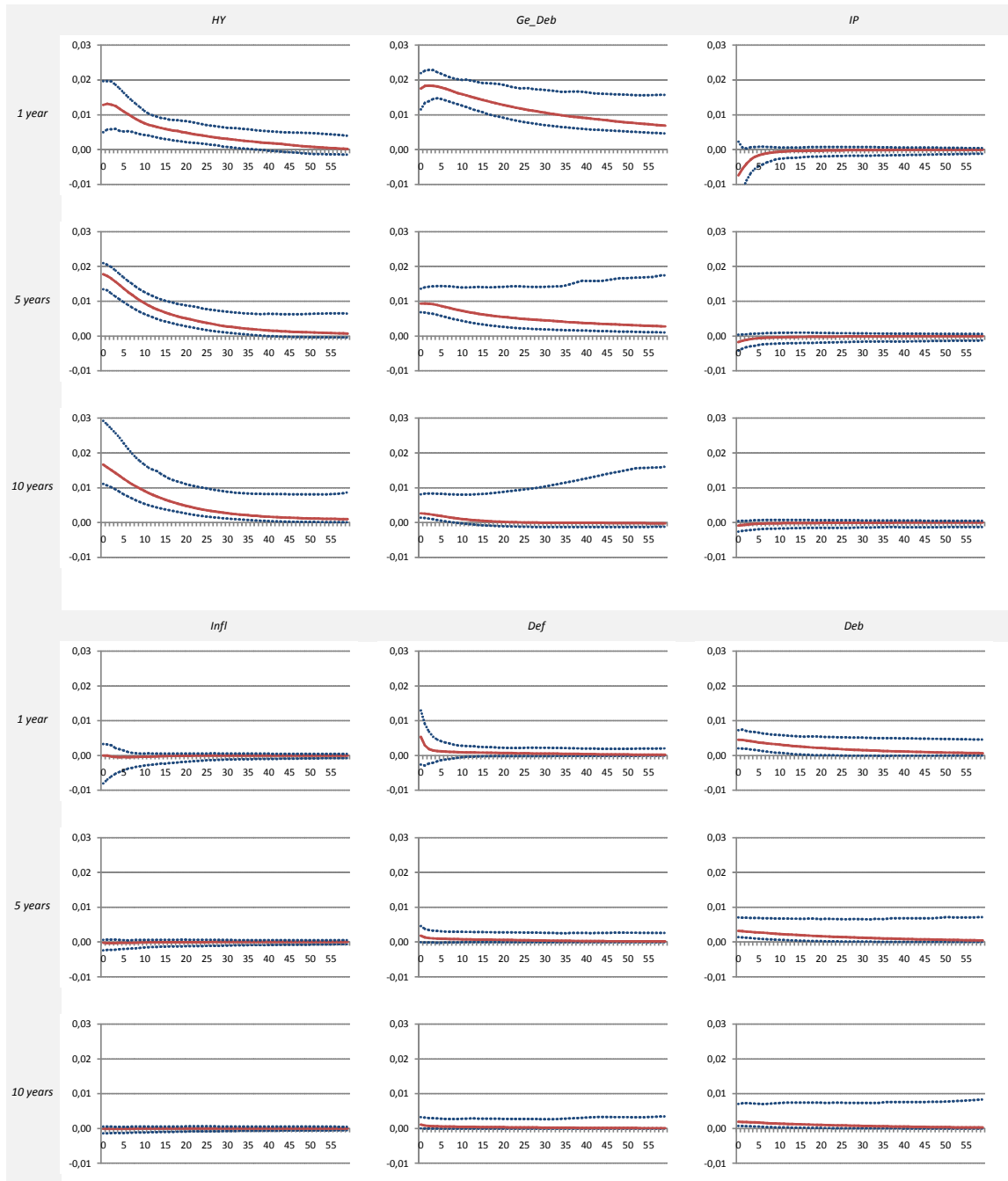
<sup>14</sup> The analysis based on standard deviation must be taken with caution because Figure 1 indicates that the spreads' standard deviation greatly changes along time.

**Figure 7: Impulse response functions for Italy.**



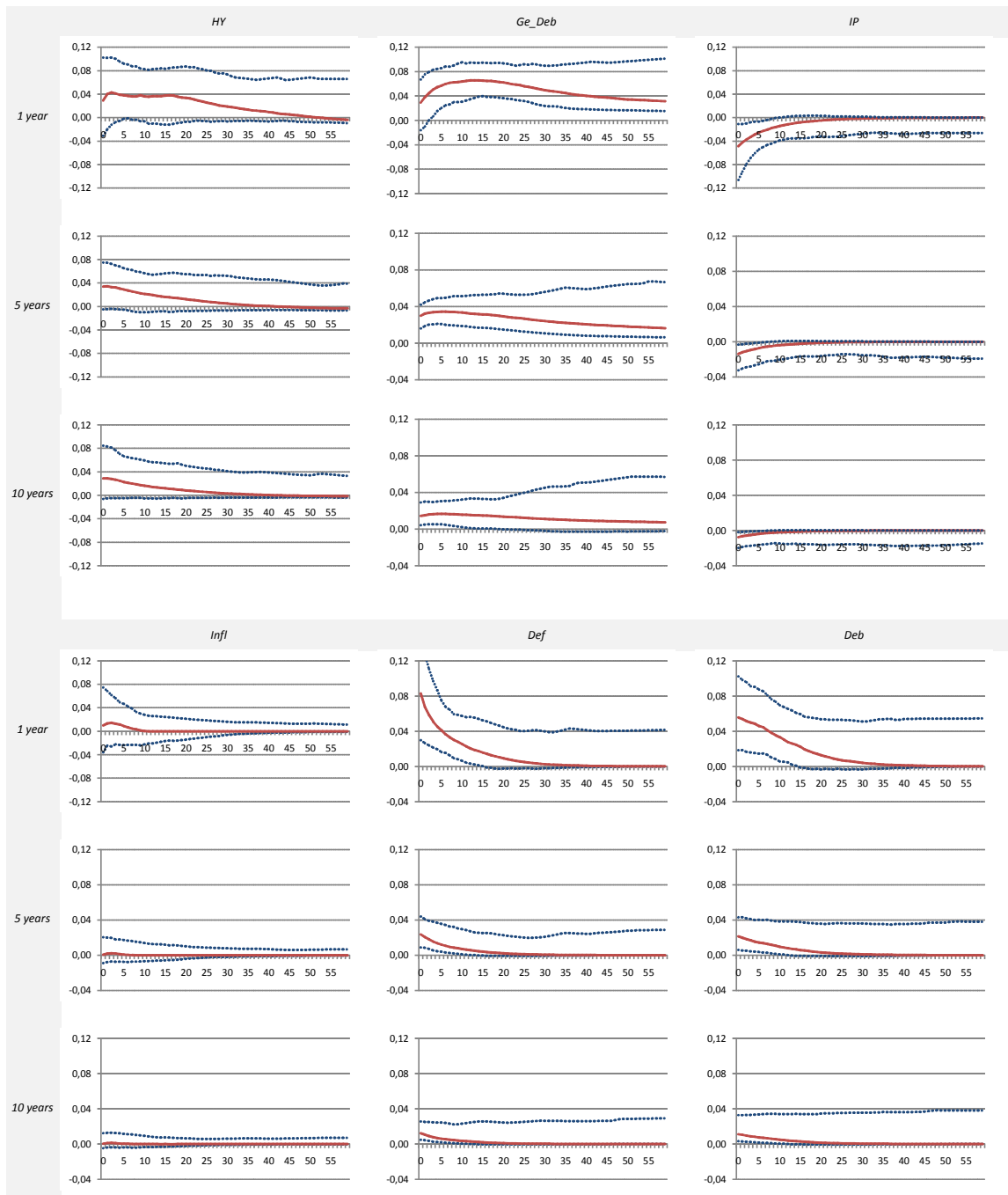
Note: All Impulse Responses (IR's) are from a one standard deviation shocks to *HY*, *Ge\_Deb* and *IP* (Panel A, in columns), *Inf*, *Def* and *Deb* (Panel B, in columns). IR's for 12 month (top row), 60 month (middle row) and 120 month bottom row) yield spreads. Confidence intervals of 95%. X-axis scales are different among countries for better visualization. Responses are multiplied by 100.

**Figure 8: Impulse response functions for Spain.**



Note: All Impulse Responses (IR's) are from a one standard deviation shocks to HY, Ge\_Deb and IP (Panel A, in columns), Inf, Defand Deb (Panel B, in columns). IR's for 12 month (top row), 60 month (middle row) and 120 month (bottom row) yield spreads. Confidence intervals of 95%. X-axis scales are different among countries for better visualization. Responses are multiplied by 100.

Figure 9: Impulse response functions for Greece.



Note: All Impulse Responses (IR's) are from a one standard deviation shocks to HY, Ge\_Deb and IP (Panel A, in columns), Inf, Defand Deb (Panel B, in columns). IR's for 12 month (top row), 60 month (middle row) and 120 month (bottom row) yield spreads. Confidence intervals of 95%. X-axis scales are different among countries for better visualization. Responses are multiplied by 100.

For all countries, the responses of spreads of different maturities to shocks to inflation have not statistical significance (at the 5% level). This result can be explained by the fact that these countries are in a monetary union, and country-specific interest rate is not a monetary policy

instrument. Only for Greece the responses of spreads to activity shocks have statistical significance, again suggesting that economic growth is perceived as reducing sovereign risk.

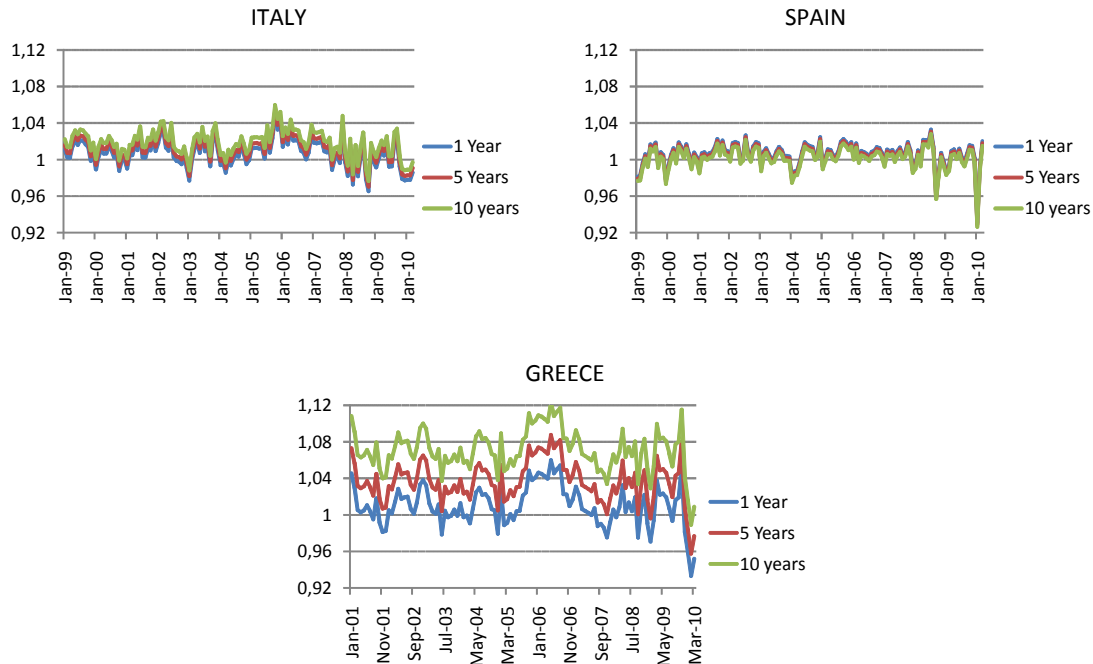
### 7.5. The Path of $\Theta_t$

From equation (17), we know that  $\Theta_t$  is a measure of default risk. Keeping  $I_t$  constant, the smaller  $\Theta_t$ , the larger  $h_t L_t$ , i.e., the larger the probability of default and/or the proportional loss in case of default. We allowed  $\Theta_t$  to change over time and across maturities. The path of  $\hat{\Theta}_t$  for maturities of 1, 5 and 10 years for Italy, Spain and Greece are shown in Figure 10.

Over the whole period of analysis (and with absolutely no restrictions on the estimated parameters)  $\hat{\Theta}_t$  is around 1 for all maturities for the three countries, indicating that the discount on the bond prices because of default and liquidity risks was always of small magnitude. Markets probably expect that the EMU would become a lender of last resort in such an extreme scenario (i.e. if one of its members experienced a severe liquidity constriction), in spite of the no-bailout clause in the EU treaty (Article 104b). Although a clear tendency in  $\Theta_t$  cannot be seen, the historical lowest levels of  $\Theta_t$  were reached after 2008 (April and October-2008 for Italy, September-2008 and January-2010 for Spain, and February and March-2010 for Greece).

An interesting common feature in the path of  $\Theta_t$  for Italy and Greece is that, the shorter the maturity, the smaller is  $\Theta_t$ . In the case of Spain, although the difference of  $\Theta_t$  across maturities is much smaller, the opposite happens: the longer the maturity, the smaller is  $\Theta_t$ . This difference may be attributed to the different roles played by liquidity and default risk across maturities and across countries.

**Figure 10: Path of  $\Theta_t^{iN}$  for maturities of 1, 5 and 10 years for Italy, Spain and Greece.**



Note:  $\ln\Theta_t^N$  follows the process  $\ln\Theta_t^{iN} \equiv \theta_t^{iN} = \theta_{00}^i + N\theta_{01}^i + \theta_1^{i'} X_t^i$ , where  $N$  is the maturity and  $X_t^i$  is Germany's and country  $i$ 's factors.

## 8. Conclusion

Following Ang and Piazzesi (2003), we use an arbitrage-free affine term structure model in which sovereign bond yield spreads are used as dependent variables in an equation that includes, among others, fiscal variables. Our main question is: in what extent these yield spreads can be attributed to economic fundamentals? In particular, we are interested in the contribution of deficit and debt in the pushing up sovereign spreads throughout the years after the onset of the economic crisis of 2007. Our idea is to distinguish the effects of fiscal shocks from the effects of shocks to other macroeconomic variables and potentially relevant indicators of risk aversion. We chose three euro-area countries for this analysis: Spain, Greece and Italy.

In the case of Italy, the recent rise in spreads has been driven mainly by Italy's debt and variables related to Germany. The Germany related variables have been even more important for the widening of Spain's spreads, with smaller influence of debt, deficit, inflation and industrial production. For both Italy and Spain, among the Germany related variables, the most important one for the widening of spreads is Germany debt. A possible explanation for that is a perception of markets that Germany is a lender of last resort of the largest economies in the EMU. For Greece, the rise in spreads has been driven mainly by Greece's own debt. A common interesting feature is that the weight of the own country's debt is larger around the maturity of 12 months. For all the countries, the weight on activity is negative, suggesting that economic growth is perceived as reducing sovereign risk.

The impulse response functions show that, for Italy, the variable that causes the largest impact is market stress, followed by Germany's debt and the country's own debt. For Spain, Germany's debt is the variable that causes the largest impact, followed by market stress and the country's own debt. Yield spreads of Greece respond more strongly to shocks from debt and deficit. A shock of one standard deviation from the country's deficit causes an initial response of the 1 year yield spreads of 30% of its standard deviation and, from its debt, 20%.

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## Appendix 1- Additional Tables

**Table A1.1: Descriptive Statistics.**

Variable	Mean	SD	Min	Max
HY	7.154	0.849	5.820	9.210
<b>A) Germany</b>				
<i>a) Yields data - per maturity <math>[\ln(1+\text{yield}/100)]*100</math></i>				
3 months	2.796	1.191	0.204	4.908
6 months	2.861	1.171	0.408	4.993
1 year	2.961	1.140	0.528	5.069
2 years	3.134	1.014	0.932	5.107
3 years	3.341	0.930	1.348	5.145
4 years	3.530	0.843	1.793	5.183
5 years	3.636	0.785	2.147	5.145
7 years	3.906	0.700	2.673	5.316
10 years	4.090	0.626	2.974	5.392
<i>b) Factors</i>				
<i>b.1) Macro variables</i>				
Industrial production	0.009	0.070	-0.247	0.097
Inflation	1.485	0.762	-0.460	3.280
Deficit	0.003	0.021	-0.047	0.051
Debt	0.645	0.040	0.578	0.737
<i>b.2) PC of Yields</i>				
1st. Principal Component	0.000	1.250	-4.645	2.390
2nd. Principal Component	0.000	0.469	-1.091	1.268
3rd. Principal Component	0.000	0.148	-0.426	0.365
<b>B) Spain</b>				
<i>a) Spreads data - per maturity <math>[\ln(1+\text{yield}^{SP}/100) - \ln(1+\text{yield}^{Ge}/100)]*100</math></i>				
3 months	0.036	0.144	-0.227	0.772
6 months	0.034	0.117	-0.106	0.738
1 year	0.032	0.115	-0.184	0.655
2 years	0.093	0.143	-0.086	0.819
3 years	0.132	0.185	-0.092	0.862
4 years	0.110	0.193	-0.104	0.936
5 years	0.156	0.213	-0.066	1.022
7 years	0.152	0.199	-0.151	0.959
10 years	0.219	0.224	-0.022	1.049
<i>b) Factors</i>				
<i>b.1) Macro variables</i>				
Industrial production	-0.006	0.064	-0.242	0.086
Inflation	2.855	1.306	-1.410	5.270
Deficit	-0.006	0.045	-0.117	0.123
Debt	0.485	0.082	0.345	0.636
<i>b.2) PC of Spreads *100</i>				
1st. Principal Component	0.000	0.206	-0.495	0.968
2nd. Principal Component	0.000	0.117	-0.237	0.605
3rd. Principal Component	0.000	0.057	-0.206	0.219

Note: For Germany, yields data are constructed the following way:  $y_t^{(N)Ge} =$

$\ln\left(1 + \frac{Y_t^{(N)Ge}}{100}\right)$ . If Germany's 1 year bond yield is, say, 2.7% per year, then  $Y_t^{(12)Ge} = 2.7$ . Spreads are calculated as:  $s_t^{(N)i} \equiv y_t^{(N)i} - y_t^{(N)Ge}$ . Yields and spreads are multiplied by 100

Table A1.1- continuation

Variable	Mean	SD	Min	Max
<b>C) Italy</b>				
<i>a) Spreads data - per maturity <math>[\ln(1+\text{yield}^{It}/100) - \ln(1+\text{yield}^{Ge}/100)]*100</math></i>				
3 months	0.067	0.148	-0.153	0.924
6 months	0.063	0.146	-0.125	0.926
1 year	0.066	0.143	-0.077	0.668
2 years	0.158	0.218	-0.014	1.184
3 years	0.188	0.273	-0.009	1.488
4 years	0.197	0.260	0.002	1.359
5 years	0.223	0.263	-0.026	1.287
7 years	0.201	0.205	0.007	1.075
10 years	0.312	0.266	0.052	1.318
<i>b) Factors</i>				
<i>b.1) Macro variables</i>				
Industrial production	-0.013	0.066	-0.256	0.083
Inflation	2.194	0.719	0.000	4.080
Deficit	0.003	0.020	-0.038	0.064
Debt	1.093	0.037	1.028	1.173
<i>b.2) PC of Spreads *100</i>				
1st. Principal Component	0.000	0.227	-0.742	0.811
2nd. Principal Component	0.000	0.115	-0.392	0.597
3rd. Principal Component	0.000	0.051	-0.129	0.139
<b>D) Greece</b>				
<i>a) Spreads data - per maturity <math>[\ln(1+\text{yield}^{Gr}/100) - \ln(1+\text{yield}^{Ge}/100)]*100</math></i>				
3 months	0.254	0.685	-0.334	3.928
6 months	0.246	0.708	-0.263	4.055
1 year	0.281	0.748	-0.196	4.344
2 years	0.377	0.820	-0.170	4.613
3 years	0.454	0.866	0.025	4.674
4 years	0.460	0.851	-0.013	4.458
5 years	0.519	0.826	0.078	4.099
7 years	0.492	0.736	0.062	3.751
10 years	0.566	0.693	0.081	3.649
<i>b) Factors</i>				
<i>b.1) Macro variables</i>				
Industrial production	-0.011	0.042	-0.127	0.070
Inflation	3.157	0.916	0.490	4.910
Deficit	0.006	0.042	-0.079	0.122
Debt	1.001	0.050	0.943	1.168
<i>b.2) PC of Spreads *100</i>				
1st. Principal Component	0.000	1.122	-1.997	3.965
2nd. Principal Component	0.000	0.227	-0.671	0.777
3rd. Principal Component	0.000	0.083	-0.328	0.176

**Table A1.2: Debt and Deficit Evolution**

	Deficit	Debt
<b>Spain</b>	Critical Recent Evolution: After three years of positive fiscal balances (from 2005 to 2007), Spain incurred in high fiscal deficits in 2008 and 2009. In this last year, the deficit/GDP ratio reached 11.1%.	In the end of 2009, Spain had the 5th (out of 16) lowest Debt/GDP ratio in the euro-area (53.2%). It was the lowest Debt/GDP ratio amongst the 8 biggest economies of the euro-area.
<b>Italy</b>	Positive annual deficit in the whole sample (that begins in 1999). Annual Deficit/GDP increased from 2007 to 2009 (the ratio was 1.5% in 2007, 2.7% in 2008 and 5.7% in 2009), but in terms of p.p. of GDP, the increase was relatively modest (see Figure 2).	In the end of 2009, Italy had the highest Debt/GDP ratio in the euro-area (116.0%).
<b>Greece</b>	Critical increase in the deficit/GDP ratio from 2006 to 2009.	In the end of 2009, Greece had the second highest Debt/GDP ratio in the euro-area (115,1%).

Obs.: Data are subject to revisions. Data source: Eurostat. Accessed October 2010

## Appendix 2 - Data: Quarterly to Monthly Frequency

Fiscal Balance, Debt and GDP monthly data are not available. Monthly data of these variables were constructed from their quarterly observations and from monthly data of some coincident indicators, such as energy consumption, unemployment rate, imports and exports.

- Quarterly data
  - General Government Revenues
  - General Government Expenditures
  - Net Lending/Borrowing
  - GDP
- Monthly data (activity):
  - Unemployment rate (UN)
  - Industrial Consumption (IP)
  - Consumer Price Index (CPI)
  - Imports and Exports (M and X)
  - Energy consumption - GWh (EN)
  - Car Registrations (CAR)

Transformation method:

- 1st step: quarterly data regressions

$$Z_t^q = \alpha + \beta_{IP} IP_t^q + \beta_{UN} UN_t^q + \beta_{CPI} CPI_t^q + \beta_M M_t^q + \beta_X X_t^q + \beta_{EN} EN_t^q + \beta_{CAR} CAR_t^q + \varphi_q + \varepsilon_t$$

where Z= general government net lending/borrowing, total revenues, total expenditure or GDP. The superscript q means quarterly observation data.

- 2nd step: quarterly to monthly frequency

$$\hat{Z}_t^m = \hat{\alpha} + \hat{\beta}_{IP} IP_t^m + \hat{\beta}_{UN} UN_t^m + \hat{\beta}_{CPI} CPI_t^m + \hat{\beta}_M M_t^m + \hat{\beta}_X X_t^m + \hat{\beta}_{EN} EN_t^m + \hat{\beta}_{CAR} CAR_t^m + \hat{\varphi}_q$$

$\hat{Z}_t^m$  is the variable used in our estimations.

### Appendix 3: Identification Hypothesis

We can rewrite equation (5) in the following structural version:

$$AX_t = \alpha_0 + \alpha_1 X_{t-1} + Bu_t \quad (\text{A3.1})$$

where  $A, \alpha_1$  and  $B$  are  $K \times K$  matrices and  $\alpha_0$  is a  $K$ -dimensional vector. If  $A$  is an invertible matrix, (A3.1) can be rewritten as:

$$X_t = A^{-1}\alpha_0 + A^{-1}\alpha_1 X_{t-1} + A^{-1}Bu_t$$

where, in an analogy to equation (6),  $A^{-1}\alpha_0 = \mu$ ,  $A^{-1}\alpha_1 = \Phi$  and  $A^{-1}B = \Sigma$ . In the estimation of the dynamic of the state variable of Germany, we impose the following restrictions on  $A$  and  $B$  in the estimation of Germany's model:

$$A^{Ge} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}^{Ge} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31}^{Ge} & a_{32}^{Ge} & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{41}^{Ge} & a_{42}^{Ge} & a_{43}^{Ge} & 1 & 0 & 0 & 0 & 0 \\ a_{51}^{Ge} & a_{52}^{Ge} & a_{53}^{Ge} & a_{54}^{Ge} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{65}^{Ge} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{75}^{Ge} & a_{76}^{Ge} & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{85}^{Ge} & a_{86}^{Ge} & a_{87}^{Ge} & 1 \end{pmatrix}_{8 \times 8}$$

$$B^{Ge} = \begin{pmatrix} b_{11}^{Ge} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{22}^{Ge} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{33}^{Ge} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{44}^{Ge} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{55}^{Ge} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66}^{Ge} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{77}^{Ge} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{88}^{Ge} \end{pmatrix}_{8 \times 8}$$

We impose the following restrictions on  $A$  and  $B$  in ten estimation of country  $i$ 's model:

$$A^i = \begin{pmatrix} A^{Ge} & \mathbf{0}_{8 \times 7} \\ Asp^i & \end{pmatrix} \quad B^i = \begin{pmatrix} B^{Ge} & \mathbf{0}_{8 \times 7} \\ \mathbf{0}_{7 \times 8} & B^i \end{pmatrix}$$

where

$$Asp^i = \begin{pmatrix} a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & 1 & 0 & 0 & 0 & 0 \\ a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & 1 & 0 & 0 & 0 \\ a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & a_{11}^i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11}^i & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11}^i & a_{11}^i & 1 \end{pmatrix}_{7 \times 15}$$

and

$$B^i = \begin{pmatrix} b_{11}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{11}^i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{11}^i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{11}^i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{11}^i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{11}^i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{11}^i \end{pmatrix}_{7 \times 7}$$

The over identification restrictions on matrix A is due to the method we used to find the principal components of yields and spreads: they were estimated from an eigenvalue decomposition of the observed yields (spreads) covariance matrix, orthogonalized with respect to the macroeconomics and risk factors.

#### Appendix 4: Proof of Proposition 1

The proof of Proposition 1 has two steps: In the first one, we prove that, given our assumptions, the short rate spread  $s_t^1 \equiv y_t^1 - y_t^{1Ge}$  is linear in  $X_t$ ; in the second step, we show that, given that the short rate spread is linear in  $X_t$  and the dynamics of Germany yield curve given by Result 1, spreads of other maturities are given by equations (18) to (20).

First step: the short rate spread  $s_t^1 \equiv y_t^1 - y_t^{1Ge}$  is linear in  $X_t$ .

Consider  $V_t^N$  the price at  $t$  of a bond that pays no coupon and makes a payment of \$ 1 at time  $t + N$ . For  $N=0$ ,  $V_t^N = 1$ . So equation (15) can be written as

$$V_t^1 = E_t[M_{t+1}\theta_{t+1}^0] \quad (A4.1)$$

If  $M_{t+1}\theta_{t+1}^0 V_{t+1}^N$  follows a lognormal distribution, equation (A4.1) gives:

$$v_t^1 = E_t[m_{t+1} + \theta_{t+1}^0] + \frac{1}{2} Var_t[m_{t+1} + \theta_{t+1}^0]$$

where lowercase letters denote variables in *log*. Substituting equations (5), (6) and (16), we have:

$$\begin{aligned} v_t^1 = E_t \left[ -\frac{1}{2} \lambda_t' \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} - \lambda_t' \varepsilon_{t+1}^{Ge} + \theta_{00} + 0 * \theta_{01} + \theta_1' (\mu + \Phi X_t + \Sigma \varepsilon_{t+1}) \right] \\ + \frac{1}{2} Var_t \left[ -\frac{1}{2} \lambda_t' \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} - \lambda_t' \varepsilon_{t+1}^{Ge} + \theta_{00} + 0 * \theta_{01} \right. \\ \left. + \theta_1' (\mu + \Phi X_t + \Sigma \varepsilon_{t+1}) \right] = \end{aligned}$$

$$\begin{aligned}
&= E_t \left[ -\frac{1}{2} \lambda_t' \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + \theta_1' (\mu + \Phi X_t) \right] + \frac{1}{2} Var_t [-\lambda_t' \varepsilon_{t+1}^{Ge} + \theta_1' \Sigma \varepsilon_{t+1}] \\
&= -\frac{1}{2} \lambda_t' \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + \theta_1' (\mu + \Phi X_t) + \frac{1}{2} Var_t [(-\lambda_t' J + \theta_1' \Sigma) \varepsilon_{t+1}]
\end{aligned}$$

where  $J$  is a selection  $(K^{Ge} \times K)$ -dimensional matrix such that  $\varepsilon_t^{Ge} = J \varepsilon_t$ . Note that  $JJ' = I_{K^{Ge}}$ . So

$$v_t^1 = -\delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + \theta_1' (\mu + \Phi X_t) - \theta_1' \Sigma J' \lambda_t + \frac{1}{2} \theta_1' \Sigma \Sigma' \theta_1$$

Substituting equation (7),

$$v_t^1 = -\delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + \theta_1' (\mu + \Phi X_t) - \theta_1' \Sigma J' (\lambda_0 + \lambda_1' X_t^{Ge}) + \frac{1}{2} \theta_1' \Sigma \Sigma' \theta_1$$

We know from equation (4) that  $v_t^{1Ge} = -r_t^{Ge} = -\delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge}$ . So

$$v_t^1 = v_t^{1Ge} + \left[ \theta_{00} + \theta_1' \mu - \theta_1' \Sigma J' \lambda_t - \theta_1' \Sigma J' \lambda_0 + \frac{1}{2} \theta_1' \Sigma \Sigma' \theta_1 \right] + [\theta_1' \Phi - \theta_1' \Sigma J' \lambda_1'] X_t$$

This means that:

$$v_t^1 = v_t^{1Ge} + \bar{D}_1 + \bar{E}_1' X_t \tag{A4.2}$$

where

$$\bar{D}_1 = \left[ \theta_{00} + \theta_1' \mu - \theta_1' \Sigma J' \lambda_0 + \frac{1}{2} \theta_1' \Sigma \Sigma' \theta_1 \right]$$

and

$$\bar{E}_1' = [\theta_1' \Phi - \theta_1' \Sigma J' \lambda_1'].$$

Equation (A4.2) can be written the following way:

$$s_t^1 \equiv y_t^1 - y_t^{1Ge} = D_1 + E_1' X_t$$

where  $\bar{D}_1 = -D_1$  and  $\bar{E}_1' = -E_1'$

Second step: given that the short rate spread is linear in  $X_t$  and the dynamics of Germany yield curve given by Result 1, spreads of other maturities are given by equations (18) to (20).

From equation (15), and assuming the hypothesis that  $M_{t+1} \Theta_{t+1}^N V_{t+1}^N$  follows a lognormal distribution, we have:

$$v_t^{N+1} = E_t [m_{t+1} + \theta_{t+1}^N + v_{t+1}^N] + \frac{1}{2} Var_t [m_{t+1} + \theta_{t+1}^N + v_{t+1}^N]$$

If  $y_t^N - y_t^{NGe} = D_N + E'_N X_t$  for  $N = 1$ , substituting equations (5), (6), (16), (A4.2) and (8) to (10) we have:

$$\begin{aligned}
v_t^{N+1} &= E_t \left[ -\frac{1}{2} \lambda'_t \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} - \lambda'_t \varepsilon_{t+1}^{Ge} + \theta_{00} + N * \theta_{01} + \theta'_1 (\mu + \Phi X_t + \Sigma \varepsilon_{t+1}) + \bar{A}_N \right. \\
&\quad \left. + \bar{B}'_N X_{t+1}^{Ge} + \bar{D}_N + \bar{E}'_N X_{t+1} \right] \\
&\quad + \frac{1}{2} Var_t \left[ -\frac{1}{2} \lambda'_t \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_{t+1}^{Ge} - \lambda'_t \varepsilon_{t+1}^{Ge} + \theta_{00} + N * \theta_{01} \right. \\
&\quad \left. + \theta'_1 (\mu + \Phi X_t + \Sigma \varepsilon_{t+1}) + \bar{A}_N + \bar{B}'_N X_t^{Ge} + \bar{D}_N + \bar{E}'_N X_{t+1} \right] \\
&= -\frac{1}{2} \lambda'_t \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + N * \theta_{01} + \theta'_1 (\mu + \Phi X_t) + \bar{A}_N + \bar{B}'_N (\mu^{Ge} + \Phi^{Ge} X_t^{Ge}) + \bar{D}_N + \\
&\quad \bar{E}'_N (\mu + \Phi X_t) + \frac{1}{2} Var_t [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) \varepsilon_{t+1}^{Ge} + (\theta'_1 + \bar{E}'_N) \Sigma \varepsilon_{t+1}] \tag{A4.3}
\end{aligned}$$

Opening only  $Var_t [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) \varepsilon_{t+1}^{Ge} + (\theta'_1 + \bar{E}'_N) \Sigma \varepsilon_{t+1}]$ , we have:

$$\begin{aligned}
Var_t [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) \varepsilon_{t+1}^{Ge} + (\theta'_1 + \bar{E}'_N) \Sigma \varepsilon_{t+1}] &= Var_t \{ [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) J + (\theta'_1 + \bar{E}'_N) \Sigma] \varepsilon_{t+1} \} \\
&= [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) J + (\theta'_1 + \bar{E}'_N) \Sigma] * [(\bar{B}'_N \Sigma^{Ge} - \lambda'_t) J + (\theta'_1 + \bar{E}'_N) \Sigma]' \\
&= (\bar{B}'_N \Sigma^{Ge} - \lambda'_t) J J' (\bar{B}'_N \Sigma^{Ge} - \lambda'_t)' + 2(\theta'_1 + \bar{E}'_N) \Sigma J' (\bar{B}'_N \Sigma^{Ge} - \lambda'_t)' + (\theta'_1 + \bar{E}'_N) \Sigma \Sigma' (\theta'_1 + \bar{E}'_N)' \\
&= \lambda'_t \lambda_t - 2\bar{B}'_N \Sigma^{Ge} \lambda_t + \bar{B}'_N \Sigma^{Ge} \Sigma^{Ge'} \bar{B}_N - 2\theta'_1 \Sigma J' \lambda_t + 2\theta'_1 \Sigma J' \Sigma^{Ge'} \bar{B}_N - 2\bar{E}'_N \Sigma J' \lambda_t + 2\bar{E}'_N \Sigma J' \Sigma^{Ge'} \bar{B}_N \\
&\quad + (\theta'_1 + \bar{E}'_N) \Sigma \Sigma' (\theta'_1 + \bar{E}'_N)'
\end{aligned}$$

Substituting into equation (A4.3),

$$\begin{aligned}
v_t^{N+1} &= -\frac{1}{2} \lambda'_t \lambda_t - \delta_0^{Ge} - \delta_1^{Ge'} X_t^{Ge} + \theta_{00} + N * \theta_{01} + \theta'_1 (\mu + \Phi X_t) + \bar{A}_N + \bar{B}'_N (\mu^{Ge} + \Phi^{Ge} X_t^{Ge}) \\
&\quad + \bar{D}_N + \bar{E}'_N (\mu + \Phi X_t) + \frac{1}{2} [\lambda'_t \lambda_t - 2\bar{B}'_N \Sigma^{Ge} \lambda_t + \bar{B}'_N \Sigma^{Ge} \Sigma^{Ge'} \bar{B}_N - 2\theta'_1 \Sigma J' \lambda_t \\
&\quad + 2\theta'_1 \Sigma J' \Sigma^{Ge'} \bar{B}_N - 2\bar{E}'_N \Sigma J' \lambda_t + 2\bar{E}'_N \Sigma J' \Sigma^{Ge'} \bar{B}_N + (\theta'_1 + \bar{E}'_N) \Sigma \Sigma' (\theta'_1 + \bar{E}'_N)']
\end{aligned}$$

Substituting equations (7) to (10) and rearranging the equation above, gives:

$$v_t^{N+1} = v_t^{N+1, Ge} + \bar{D}_{N+1} + \bar{E}'_{N+1} X_t$$

where

$$\begin{aligned}
\bar{D}_{N+1} &= \theta_0^{(N)} + \bar{D}_N^i + (\theta_1^i + \bar{E}_N^i)' \mu^i + (\theta_1^i + \bar{E}_N^i)' \Sigma^i \left[ J' (\Sigma^{Ge'} \bar{B}_N - \lambda_0) + \frac{1}{2} \Sigma^i (\theta_1^i + \bar{E}_N^i) \right] \\
\bar{E}'_{N+1} &= (\theta_1^i + \bar{E}_N^i)' (\Phi^i - \Sigma^i J' \lambda_1^i J)
\end{aligned}$$

Appendix 5: Parameter Estimates

Table A5.1: Parameter Estimates – Germany<sup>1</sup>

Alemanha									
VAR dos Fatores									
$X_t^{Ge} = \mu^{Ge} + \Phi^{Ge} X_{t-1}^{Ge} + \Sigma^{Ge} \epsilon_t^{Ge}$									
	$\mu^{Ge}$	Matriz $\Phi^{Ge}$							
		$HY_{t-1}$	$IP_{t-1}^{Ge}$	$CPI_{t-1}^{Ge}$	$Def_{t-1}^{Ge}$	$Deb_{t-1}^{Ge}$	Princ. Comp1 $_{t-1}^{Ge}$	Princ. Comp2 $_{t-1}^{Ge}$	Princ. Comp3 $_{t-1}^{Ge}$
$HY_t$	-0.010 (0.023)	0.923*** (0.043)	-0.016 (0.035)	0.059** (0.029)	-0.022 (0.032)	-0.059 (0.037)	0.004 (0.023)	-0.025 (0.023)	0.04* (0.024)
$IP_t^{Ge}$	0.007 (0.026)	-0.056 (0.049)	0.954*** (0.038)	-0.033 (0.031)	0.025 (0.034)	0.021 (0.044)	0.091*** (0.029)	-0.076*** (0.025)	-0.05* (0.026)
$CPI_t^{Ge}$	0.009 (0.031)	-0.064 (0.058)	0.12** (0.048)	0.838*** (0.041)	0.014 (0.042)	-0.036 (0.052)	0.109*** (0.032)	0.058* (0.031)	-0.072** (0.030)
$Def_t^{Ge}$	-0.008 (0.056)	-0.561*** (0.109)	-0.351*** (0.087)	-0.233*** (0.068)	0.268*** (0.074)	-0.399*** (0.099)	-0.199*** (0.053)	-0.257*** (0.056)	0.012 (0.058)
$Deb_t^{Ge}$	0.023** (0.010)	-0.011 (0.016)	-0.051*** (0.013)	-0.005 (0.012)	0.010 (0.012)	0.989*** (0.016)	-0.021** (0.010)	-0.006 (0.010)	0.016* (0.009)
Princ. Comp1 $_t^{Ge}$	-0.024 (0.050)	-0.192** (0.091)	-0.108 (0.070)	-0.174*** (0.059)	-0.328*** (0.066)	-0.188** (0.084)	0.804*** (0.049)	-0.059 (0.050)	0.155*** (0.048)
Princ. Comp2 $_t^{Ge}$	-0.028 (0.058)	-0.176 (0.108)	-0.113 (0.084)	0.004 (0.072)	-0.387*** (0.074)	-0.088 (0.102)	-0.1* (0.056)	0.62*** (0.058)	0.048 (0.058)
Princ. Comp3 $_t^{Ge}$	-0.023 (0.059)	-0.009 (0.107)	0.039 (0.088)	0.019 (0.065)	-0.010 (0.070)	0.005 (0.097)	-0.135** (0.061)	0.031 (0.055)	0.744*** (0.056)
		Matriz $\Sigma^{Ge}$							
		$HY_t$	$IP_t^{Ge}$	$CPI_t^{Ge}$	$Def_t^{Ge}$	$Deb_t^{Ge}$	Princ. Comp1 $_t^{Ge}$	Princ. Comp2 $_t^{Ge}$	Princ. Comp3 $_t^{Ge}$
$HY_t$		0.27*** (0.041)	-	-	-	-	-	-	-
$IP_t^{Ge}$		0.008 (0.024)	0.309*** (0.026)	-	-	-	-	-	-
$CPI_t^{Ge}$		0.021 (0.031)	-0.078** (0.033)	0.356*** (0.022)	-	-	-	-	-
$Def_t^{Ge}$		-0.191*** (0.042)	-0.029 (0.066)	0.037 (0.067)	0.634*** (0.043)	-	-	-	-
$Deb_t^{Ge}$		-0.014* (0.007)	-0.014* (0.008)	0.001 (0.009)	0.021** (0.009)	0.104*** (0.013)	-	-	-
Princ. Comp1 $_t^{Ge}$		-	-	-	-	-	0.583*** (0.049)	-	-
Princ. Comp2 $_t^{Ge}$		-	-	-	-	-	0.103* (0.055)	0.67*** (0.035)	-
Princ. Comp3 $_t^{Ge}$		-	-	-	-	-	0.037 (0.074)	-0.166** (0.064)	0.633*** (0.054)
Regressão da taxa curta									
$r_t^{Ge} = \delta_0^{Ge} + \delta_1^{Ge} X_t^{Ge} + u_t^{Ge}$									
	$\delta_0^{Ge}$	Vetor $\delta_1^{Ge}$							
		$HY_t$	$IP_t^{Ge}$	$CPI_t^{Ge}$	$Def_t^{Ge}$	$Deb_t^{Ge}$	Princ. Comp1 $_t^{Ge}$	Princ. Comp2 $_t^{Ge}$	Princ. Comp3 $_t^{Ge}$
	0.028*** (0.000)	0.001*** (0.000)	0.004*** (0.000)	0.003*** (0.000)	-0.003*** (0.000)	-0.005*** (0.000)	0.004*** (0.000)	0.002*** (0.000)	-0.001*** (0.000)
Prices of Risk $\lambda_0$ and $\lambda_1$									
$\lambda_t^{Ge} = \lambda_0^{Ge} + \lambda_1^{Ge} X_t^{Ge}$									
	$\lambda_0^{Ge}$	Matriz $\lambda_1^{Ge}$							
		$HY_t$	$IP_t^{Ge}$	$CPI_t^{Ge}$	$Def_t^{Ge}$	$Deb_t^{Ge}$	Princ. Comp1 $_t^{Ge}$	Princ. Comp2 $_t^{Ge}$	Princ. Comp3 $_t^{Ge}$
$HY_t$	4.345 (6.318)	-0.109 (0.254)	-	-	-	-	-	-	-
$IP_t^{Ge}$	-11.419 (13.737)	-	0.195 (0.191)	-	-	-	-	-	-
$CPI_t^{Ge}$	-18.672 (18.390)	-	-	0.505 (0.483)	-	-	-	-	-
$Def_t^{Ge}$	-7.576 (12.306)	-	-	-	-0.135 (0.160)	-	-	-	-
$Deb_t^{Ge}$	-0.559 (1.804)	-	-	-	-	0.384 (0.511)	-	-	-
Princ. Comp1 $_t^{Ge}$	2.281 (5.953)	-	-	-	-	-	0.075 (0.075)	-	-
Princ. Comp2 $_t^{Ge}$	0.337 (0.888)	-	-	-	-	-	-	0.060 (0.435)	-
Princ. Comp3 $_t^{Ge}$	-19.197 (30.580)	-	-	-	-	-	-	-	1.545 (1.005)

<sup>1</sup> Estimated Standard Errors in Parentheses (\*\*\*1% level; \*\*5% level; \*10% level)

Table A5.2: Parameter Estimates – Italy<sup>1</sup>

Italy																
VAR of Factors $X_t$																
$X_t^i = \mu^i + \Phi^i X_{t-1}^i + \Sigma^i \epsilon_t^i$																
	$\mu^{i,IT}$	Matriz $\Phi^{i,IT}$														
		HY <sub>t-1</sub>	IP <sup>Ge</sup> <sub>t-1</sub>	CPI <sup>Ge</sup> <sub>t-1</sub>	Def <sup>Ge</sup> <sub>t-1</sub>	Deb <sup>Ge</sup> <sub>t-1</sub>	Princ. Comp1 <sup>Ge</sup> <sub>t-1</sub>	Princ. Comp2 <sup>Ge</sup> <sub>t-1</sub>	Princ. Comp3 <sup>Ge</sup> <sub>t-1</sub>	IP <sup>It</sup> <sub>t-1</sub>	CPI <sup>It</sup> <sub>t-1</sub>	Def <sup>It</sup> <sub>t-1</sub>	Deb <sup>It</sup> <sub>t-1</sub>	Princ. Comp1 <sup>It</sup> <sub>t-1</sub>	Princ. Comp2 <sup>It</sup> <sub>t-1</sub>	Princ. Comp3 <sup>It</sup> <sub>t-1</sub>
IP <sup>It</sup> <sub>t</sub>	0.006 (0.029)	-0.067 (0.062)	0.833*** (0.094)	-0.106** (0.050)	0.043 (0.045)	-0.052 (0.061)	0.099*** (0.032)	-0.101*** (0.030)	-0.037 (0.029)	0.121 (0.093)	0.079 (0.056)	0.000 (0.053)	0.179*** (0.053)	-0.071** (0.027)	0.001 (0.028)	0.064 (0.026)
CPI <sup>It</sup> <sub>t</sub>	-0.003 (0.019)	-0.089** (0.042)	-0.015 (0.065)	0.106*** (0.037)	0.044 (0.020)	-0.063 (0.041)	0.044 (0.022)	-0.001 (0.019)	-0.032 (0.020)	0.058 (0.062)	0.911*** (0.041)	-0.033 (0.038)	0.086 (0.036)	0.003 (0.020)	0.010 (0.019)	0.003 (0.019)
Def <sup>It</sup> <sub>t</sub>	0.000 (0.046)	-0.126 (0.105)	-0.439*** (0.166)	-0.102 (0.084)	0.053 (0.074)	0.375*** (0.094)	-0.146*** (0.049)	-0.109** (0.047)	0.106 (0.049)	-0.058 (0.150)	0.002 (0.093)	0.015 (0.091)	-0.382*** (0.085)	0.049 (0.051)	-0.17*** (0.046)	0.088 (0.048)
Deb <sup>It</sup> <sub>t</sub>	0.002 (0.024)	0.083 (0.051)	-0.082 (0.088)	-0.035 (0.050)	-0.003 (0.039)	0.023 (0.050)	-0.031 (0.028)	-0.010 (0.026)	0.007 (0.027)	0.052 (0.084)	-0.081 (0.054)	0.023 (0.052)	0.85*** (0.045)	0.027 (0.026)	-0.009 (0.024)	-0.015 (0.025)
Princ. Comp1 <sup>It</sup> <sub>t</sub>	-0.010 (0.073)	0.011 (0.141)	0.086 (0.249)	-0.194 (0.130)	-0.232** (0.107)	0.028 (0.141)	0.038 (0.080)	-0.017 (0.067)	-0.125* (0.072)	-0.008 (0.235)	0.114 (0.139)	0.143 (0.133)	-0.036 (0.133)	0.54*** (0.072)	-0.011 (0.068)	0.065 (0.067)
Princ. Comp2 <sup>It</sup> <sub>t</sub>	-0.014 (0.071)	-0.547*** (0.268)	0.405 (0.147)	-0.083 (0.124)	-0.103 (0.112)	-0.221 (0.139)	0.204*** (0.075)	0.024 (0.076)	0.069 (0.075)	-0.806*** (0.241)	0.291 (0.147)	-0.222 (0.143)	0.218 (0.129)	0.064 (0.072)	0.455*** (0.068)	-0.019 (0.072)
Princ. Comp3 <sup>It</sup> <sub>t</sub>	0.013 (0.079)	0.301 (0.169)	0.302 (0.277)	0.027 (0.158)	0.111 (0.135)	0.149 (0.168)	-0.101 (0.093)	-0.038 (0.084)	-0.039 (0.086)	-0.136 (0.262)	-0.136 (0.162)	0.013 (0.170)	-0.046 (0.155)	-0.011 (0.082)	0.016 (0.085)	0.285*** (0.078)
Matriz $\Sigma^{i,IT}$																
	HY	IP <sup>Ge</sup>	CPI <sup>Ge</sup>	Def <sup>Ge</sup>	Deb <sup>Ge</sup>	Princ. Comp1 <sup>Ge</sup>	Princ. Comp2 <sup>Ge</sup>	Princ. Comp3 <sup>Ge</sup>	IP <sup>It</sup>	CPI <sup>It</sup>	Def <sup>It</sup>	Deb <sup>It</sup>	Princ. Comp1 <sup>It</sup>	Princ. Comp2 <sup>It</sup>	Princ. Comp3 <sup>It</sup>	
IP <sup>It</sup>	0.000 (0.024)	-0.001 (0.027)	-0.002 (0.032)	-0.002 (0.029)	-0.001 (0.055)	-0.001 (0.026)	0.001 (0.031)	-0.001 (0.030)	0.31*** (0.038)	-	-	-	-	-	-	
CPI <sup>It</sup>	0.000 (0.018)	-0.002 (0.019)	-0.002 (0.022)	-0.001 (0.021)	-0.058 (0.087)	0.000 (0.017)	0.000 (0.022)	0.000 (0.021)	0.010 (0.036)	0.146 (0.102)	-	-	-	-	-	
Def <sup>It</sup>	0.001 (0.039)	-0.001 (0.041)	-0.004 (0.053)	-0.001 (0.050)	-0.023 (0.047)	0.001 (0.043)	-0.005 (0.051)	0.000 (0.052)	-0.042 (0.050)	-0.003 (0.046)	0.515*** (0.056)	-	-	-	-	
Deb <sup>It</sup>	0.001 (0.022)	0.001 (0.024)	-0.001 (0.026)	-0.001 (0.027)	-0.005 (0.023)	-0.001 (0.024)	-0.001 (0.026)	-0.001 (0.028)	0.024 (0.027)	0.005 (0.019)	0.021 (0.025)	0.275*** (0.030)	-	-	-	
Princ. Comp1 <sup>It</sup>	-	-	-	-	-	-	-	-	-	-	-	0.801*** (0.095)	-	-	-	
Princ. Comp2 <sup>It</sup>	-	-	-	-	-	-	-	-	-	-	-	-	-0.036 (0.157)	0.787*** (0.093)	-	
Princ. Comp3 <sup>It</sup>	-	-	-	-	-	-	-	-	-	-	-	-	-0.026 (0.126)	0.049 (0.117)	0.925*** (0.064)	
Short Rate Regression																
$\text{spread}_t^i = \delta_0^i + \delta_1^i X_t^i + u_t^i$																
	Vetor $100 \cdot \delta_1$															
	HY <sub>t</sub>	IP <sup>Ge</sup> <sub>t</sub>	CPI <sup>Ge</sup> <sub>t</sub>	Def <sup>Ge</sup> <sub>t</sub>	Deb <sup>Ge</sup> <sub>t</sub>	Princ. Comp1 <sup>Ge</sup> <sub>t</sub>	Princ. Comp2 <sup>Ge</sup> <sub>t</sub>	Princ. Comp3 <sup>Ge</sup> <sub>t</sub>	IP <sup>It</sup> <sub>t</sub>	CPI <sup>It</sup> <sub>t</sub>	Def <sup>It</sup> <sub>t</sub>	Deb <sup>It</sup> <sub>t</sub>	Princ. Comp1 <sup>It</sup> <sub>t</sub>	Princ. Comp2 <sup>It</sup> <sub>t</sub>	Princ. Comp3 <sup>It</sup> <sub>t</sub>	
	0.067*** (0.002)	0.084*** (0.004)	-0.001 (0.007)	0.001 (0.004)	-0.005 (0.003)	0.14*** (0.004)	-0.029*** (0.002)	0.004 (0.002)	0.032*** (0.002)	-0.019*** (0.007)	0.038*** (0.004)	-0.02*** (0.004)	0.003 (0.004)	0.044*** (0.002)	0.083*** (0.002)	-0.026*** (0.002)
Recovery Intensity Coefficients $\theta_0$ and $\theta_1$																
$\ln(\theta^i) = \theta_0^i + \theta_1^i X_t^i$																
	Vetor $\theta_1$															
	HY <sub>t</sub>	IP <sup>Ge</sup> <sub>t</sub>	CPI <sup>Ge</sup> <sub>t</sub>	Def <sup>Ge</sup> <sub>t</sub>	Deb <sup>Ge</sup> <sub>t</sub>	Princ. Comp1 <sup>Ge</sup> <sub>t</sub>	Princ. Comp2 <sup>Ge</sup> <sub>t</sub>	Princ. Comp3 <sup>Ge</sup> <sub>t</sub>	IP <sup>It</sup> <sub>t</sub>	CPI <sup>It</sup> <sub>t</sub>	Def <sup>It</sup> <sub>t</sub>	Deb <sup>It</sup> <sub>t</sub>	Princ. Comp1 <sup>It</sup> <sub>t</sub>	Princ. Comp2 <sup>It</sup> <sub>t</sub>	Princ. Comp3 <sup>It</sup> <sub>t</sub>	
	0.000 (0.000)	0.081 (0.096)	-0.014 (0.05)	-0.006 (0.06)	-0.006 (0.02)	-0.015 (0.04)	-0.014 (0.02)	0.000 (0.02)	-0.006 (0.02)	0.006 (0.03)	0.001 (0.03)	0.000 (0.01)	0.005 (0.02)	0.001 (0.01)	-0.005 (0.01)	-0.002 (0.01)

<sup>1</sup> Estimated Standard Errors in Parentheses (\*\*\*1% level; \*\*5% level; \*10% level)

Table A5.3: Parameter Estimates – Spain<sup>1</sup>

Spain																
VAR of Factors $X_t$																
$X_t^{Sp} = \mu^{Sp} + \Phi^{Sp} X_{t-1}^{Sp} + \Sigma^{Sp} \varepsilon_t^{Sp}$																
	$\mu^{Sp,Sp}$	Matrix $\Phi^{Sp,Sp}$														
		HY <sub>t-1</sub>	IP <sub>t-1</sub> <sup>Sp</sup>	CPI <sub>t-1</sub> <sup>Sp</sup>	Def <sub>t-1</sub> <sup>Sp</sup>	Deb <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp1 <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp2 <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp3 <sub>t-1</sub> <sup>Sp</sup>	IP <sub>t-1</sub> <sup>Sp</sup>	CPI <sub>t-1</sub> <sup>Sp</sup>	Def <sub>t-1</sub> <sup>Sp</sup>	Deb <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp1 <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp2 <sub>t-1</sub> <sup>Sp</sup>	Princ. Comp3 <sub>t-1</sub> <sup>Sp</sup>
IP <sub>t</sub> <sup>Sp</sup>	-0.002 (0.028)	-0.214*** (0.057)	0.372*** (0.088)	-0.134* (0.068)	-0.039 (0.038)	0.029 (0.066)	0.106*** (0.035)	-0.13*** (0.032)	-0.027 (0.033)	0.344*** (0.087)	0.078 (0.073)	-0.123** (0.057)	0.279*** (0.069)	-0.113*** (0.029)	-0.064** (0.027)	0.052* (0.029)
CPI <sub>t</sub> <sup>Sp</sup>	0.002 (0.020)	-0.066 (0.040)	0.162*** (0.060)	0.16*** (0.046)	0.078*** (0.029)	-0.064 (0.046)	0.058** (0.023)	0.042** (0.021)	0.016 (0.058)	0.054 (0.053)	0.766*** (0.039)	0.114*** (0.048)	0.088* (0.048)	-0.032 (0.020)	-0.032 (0.020)	-0.017 (0.020)
Def <sub>t</sub> <sup>Sp</sup>	0.022 (0.050)	0.214** (0.100)	-0.136 (0.141)	0.088 (0.117)	0.087 (0.070)	0.476*** (0.123)	-0.029 (0.056)	0.076 (0.056)	-0.148*** (0.053)	-0.256* (0.142)	0.002 (0.125)	0.254*** (0.094)	0.222* (0.124)	0.081 (0.049)	0.022 (0.046)	0.007 (0.051)
Deb <sub>t</sub> <sup>Sp</sup>	-0.009 (0.007)	0.019 (0.015)	-0.015 (0.020)	-0.012 (0.016)	-0.009 (0.010)	0.010 (0.016)	-0.008 (0.008)	-0.001 (0.008)	-0.009 (0.008)	0.000 (0.021)	-0.007 (0.019)	0.018 (0.014)	0.972*** (0.017)	0.010 (0.007)	0.005 (0.007)	-0.002 (0.007)
Princ. Comp1 <sub>t</sub> <sup>Sp</sup>	0.009 (0.075)	0.096 (0.153)	0.274 (0.228)	0.179 (0.181)	-0.001 (0.103)	0.215 (0.186)	-0.064 (0.085)	0.062 (0.081)	-0.034 (0.193)	-0.197 (0.223)	-0.130 (0.144)	-0.034 (0.192)	0.222 (0.074)	0.419*** (0.073)	-0.208*** (0.073)	-0.115 (0.071)
Princ. Comp2 <sub>t</sub> <sup>Sp</sup>	-0.019 (0.073)	-0.308** (0.146)	-0.032 (0.201)	-0.122 (0.159)	-0.155 (0.099)	-0.112 (0.183)	0.130 (0.080)	0.064 (0.081)	-0.14* (0.192)	-0.142 (0.196)	0.228 (0.132)	0.116 (0.178)	0.100 (0.069)	-0.161** (0.071)	0.473*** (0.071)	-0.087 (0.066)
Princ. Comp3 <sub>t</sub> <sup>Sp</sup>	-0.017 (0.071)	0.123 (0.140)	-0.491** (0.214)	0.112 (0.171)	0.100 (0.105)	-0.059 (0.175)	0.124 (0.086)	0.032 (0.083)	0.083 (0.082)	0.692*** (0.216)	0.019 (0.205)	0.229 (0.138)	-0.269 (0.166)	0.091 (0.071)	0.027 (0.076)	0.463*** (0.073)
Matrix $\Sigma^{Sp,Sp}$																
	HY	IP <sup>Sp</sup>	CPI <sup>Sp</sup>	Def <sup>Sp</sup>	Deb <sup>Sp</sup>	Princ. Comp1 <sup>Sp</sup>	Princ. Comp2 <sup>Sp</sup>	Princ. Comp3 <sup>Sp</sup>	IP <sup>Sp</sup>	CPI <sup>Sp</sup>	Def <sup>Sp</sup>	Deb <sup>Sp</sup>	Princ. Comp1 <sup>Sp</sup>	Princ. Comp2 <sup>Sp</sup>	Princ. Comp3 <sup>Sp</sup>	
IP <sup>Sp</sup>	0.001 (0.024)	-0.002 (0.027)	-0.003 (0.032)	-0.003 (0.031)	-0.002 (0.043)	-0.001 (0.027)	0.001 (0.030)	-0.004 (0.029)	0.312*** (0.027)	-	-	-	-	-	-	
CPI <sup>Sp</sup>	-0.001 (0.018)	0.000 (0.020)	0.000 (0.021)	0.000 (0.022)	-0.054 (0.086)	0.000 (0.019)	0.001 (0.022)	0.000 (0.022)	0.040 (0.035)	0.149 (0.100)	-	-	-	-	-	
Def <sup>Sp</sup>	0.002 (0.041)	0.000 (0.045)	0.004 (0.055)	0.006 (0.048)	-0.022 (0.049)	0.003 (0.046)	-0.001 (0.052)	0.002 (0.051)	-0.133** (0.051)	0.012 (0.039)	0.509*** (0.048)	-	-	-	-	
Deb <sup>Sp</sup>	0.000 (0.006)	0.000 (0.006)	0.000 (0.007)	0.000 (0.008)	-0.001 (0.007)	0.000 (0.007)	0.000 (0.007)	0.000 (0.008)	0.006 (0.007)	0.000 (0.006)	0.018*** (0.006)	0.074*** (0.005)	-	-	-	
Princ. Comp1 <sup>Sp</sup>	-	-	-	-	-	-	-	-	-	-	-	-	0.857*** (0.102)	-	-	
Princ. Comp2 <sup>Sp</sup>	-	-	-	-	-	-	-	-	-	-	-	-	0.165 (0.099)	0.78*** (0.059)	-	
Princ. Comp3 <sup>Sp</sup>	-	-	-	-	-	-	-	-	-	-	-	-	0.039 (0.150)	0.036 (0.113)	0.799*** (0.065)	
Short Rate Regression																
$\text{spread}_t^{Sp} = \delta^{Sp}_0 + \delta^{Sp}_1 X_t^{Sp} + u_t^{Sp}$																
	Vector $100 \cdot \delta_1$															
	0.036*** (0.003)	0.062*** (0.005)	0.045*** (0.008)	0.02*** (0.006)	-0.009** (0.004)	0.038*** (0.006)	-0.021*** (0.003)	-0.007** (0.003)	0.028*** (0.003)	-0.049*** (0.008)	-0.022*** (0.007)	0.031*** (0.005)	-0.024*** (0.007)	0.045*** (0.003)	0.089*** (0.003)	-0.009*** (0.003)
Recovery Intensity Coefficients $\theta_0$ and $\theta_1$																
$\ln(\theta^{Sp}) = \theta^{Sp}_0 + \theta^{Sp}_1 X_t^{Sp}$																
	Vector $\theta_1$															
	0.000 (0.000)	0.003 (0.132)	0.003 (0.014)	-0.007 (0.008)	0.004 (0.006)	0.003 (0.014)	-0.003 (0.008)	0.002 (0.004)	0.003 (0.006)	0.001 (0.006)	0.014 (0.015)	0.000 (0.005)	0.008 (0.010)	-0.006 (0.010)	-0.004 (0.005)	-0.005* (0.005)

<sup>1</sup> Estimated Standard Errors in Parentheses (\*\*\*1% level; \*\*5% level; \*10% level)

Table A5.3: Parameter Estimates – Greece<sup>1</sup>

Greece																	
VAR of Factors $X_t$																	
$X_t^{Gr} = \mu^{Gr} + \Phi^{Gr} X_{t-1}^{Gr} + \varepsilon_t^{Gr}$																	
	$\mu^{Gr}$	Matrix $\Phi^{Gr, Gr}$															
		HY <sub>t-1</sub>	IP <sub>t-1</sub> <sup>Gr</sup>	CPI <sub>t-1</sub> <sup>Gr</sup>	Def <sub>t-1</sub> <sup>Gr</sup>	Deb <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t-1</sub> <sup>Gr</sup>	IP <sub>t-1</sub> <sup>Gr</sup>	CPI <sub>t-1</sub> <sup>Gr</sup>	Def <sub>t-1</sub> <sup>Gr</sup>	Deb <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t-1</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t-1</sub> <sup>Gr</sup>	
IP <sub>t</sub> <sup>Gr</sup>	0.026 (0.063)	-0.145 (0.124)	0.296*** (0.100)	-0.103 (0.094)	0.002 (0.079)	-0.132 (0.143)	0.204*** (0.052)	-0.252*** (0.062)	0.031 (0.061)	-0.016 (0.097)	0.209** (0.093)	0.005 (0.115)	-0.342*** (0.053)	-0.059 (0.053)	-0.091* (0.053)	0.105** (0.053)	
CPI <sub>t</sub> <sup>Gr</sup>	-0.018 (0.038)	-0.232*** (0.084)	0.134** (0.065)	0.147** (0.058)	0.036 (0.050)	-0.252** (0.096)	0.026 (0.034)	0.046 (0.038)	-0.038 (0.038)	-0.057 (0.059)	0.754*** (0.062)	0.027 (0.061)	0.219*** (0.073)	0.050 (0.032)	0.023 (0.035)	0.053 (0.035)	
Def <sub>t</sub> <sup>Gr</sup>	-0.106 (0.067)	0.114 (0.129)	-0.33*** (0.099)	0.045 (0.095)	-0.123 (0.081)	0.646*** (0.159)	-0.104* (0.059)	-0.042 (0.066)	-0.089 (0.097)	0.095 (0.097)	0.048 (0.097)	0.261*** (0.098)	-0.193 (0.122)	0.15** (0.058)	0.032 (0.057)	-0.040 (0.056)	
Deb <sub>t</sub> <sup>Gr</sup>	0.013 (0.019)	0.035 (0.038)	-0.076** (0.033)	0.013 (0.029)	0.010 (0.023)	0.057 (0.046)	-0.051*** (0.016)	0.005 (0.020)	-0.031 (0.019)	0.036 (0.030)	-0.058* (0.029)	0.021 (0.030)	0.92*** (0.036)	0.05*** (0.018)	-0.036** (0.016)	-0.038** (0.016)	
Princ. Comp1 <sub>t</sub> <sup>Gr</sup>	0.157** (0.069)	0.064 (0.139)	0.372*** (0.114)	-0.201** (0.099)	-0.080 (0.081)	-0.262 (0.164)	-0.006 (0.062)	0.021 (0.069)	-0.110 (0.068)	0.076 (0.103)	0.336*** (0.103)	0.224 (0.138)	0.687*** (0.062)	-0.035 (0.056)	0.001 (0.059)		
Princ. Comp2 <sub>t</sub> <sup>Gr</sup>	0.053 (0.080)	0.026 (0.155)	0.066 (0.132)	-0.095 (0.114)	-0.132 (0.091)	0.034 (0.177)	0.038 (0.072)	-0.112 (0.082)	-0.13* (0.076)	0.126 (0.128)	0.035 (0.115)	0.022 (0.148)	0.012 (0.064)	0.587*** (0.063)	-0.16** (0.065)		
Princ. Comp3 <sub>t</sub> <sup>Gr</sup>	0.128 (0.084)	0.413** (0.166)	0.558*** (0.137)	0.098 (0.128)	0.194* (0.104)	0.033 (0.198)	-0.202** (0.083)	0.014 (0.081)	0.257*** (0.082)	0.044 (0.137)	-0.294** (0.122)	0.023 (0.133)	0.3* (0.160)	-0.062 (0.070)	0.365*** (0.071)	0.074	
Matrix $\varepsilon^{Gr, Gr}$																	
	HY	IP <sup>Gr</sup>	CPI <sup>Gr</sup>	Def <sup>Gr</sup>	Deb <sup>Gr</sup>	Princ. Comp1 <sup>Gr</sup>	Princ. Comp2 <sup>Gr</sup>	Princ. Comp3 <sup>Gr</sup>	IP <sup>Gr</sup>	CPI <sup>Gr</sup>	Def <sup>Gr</sup>	Deb <sup>Gr</sup>	Princ. Comp1 <sup>Gr</sup>	Princ. Comp2 <sup>Gr</sup>	Princ. Comp3 <sup>Gr</sup>		
IP <sup>Gr</sup>	-0.002 (0.039)	0.000 (0.047)	-0.002 (0.043)	-0.003 (0.044)	-0.009 (0.057)	0.001 (0.042)	0.000 (0.043)	-0.003 (0.042)	0.464*** (0.041)	-	-	-	-	-	-		
CPI <sup>Gr</sup>	-0.003 (0.025)	0.001 (0.028)	0.000 (0.031)	0.001 (0.028)	-0.031 (0.088)	0.000 (0.027)	0.001 (0.030)	-0.001 (0.028)	-0.030 (0.039)	0.259** (0.100)	-	-	-	-	-		
Def <sup>Gr</sup>	-0.001 (0.038)	0.002 (0.044)	-0.002 (0.047)	0.002 (0.049)	-0.013 (0.054)	0.002 (0.043)	0.000 (0.050)	0.003 (0.049)	-0.202*** (0.053)	0.045 (0.061)	0.433*** (0.054)	-	-	-	-		
Deb <sup>Gr</sup>	0.000 (0.013)	0.000 (0.014)	0.001 (0.015)	0.002 (0.016)	-0.002 (0.015)	0.001 (0.013)	0.000 (0.015)	0.000 (0.014)	-0.033* (0.018)	0.002 (0.016)	0.023* (0.014)	0.14*** (0.014)	-	-	-		
Princ. Comp1 <sup>Gr</sup>	-	-	-	-	-	-	-	-	-	-	-	-	0.544*** (0.065)	-	-		
Princ. Comp2 <sup>Gr</sup>	-	-	-	-	-	-	-	-	-	-	-	-	0.024 (0.079)	0.609*** (0.088)	-		
Princ. Comp3 <sup>Gr</sup>	-	-	-	-	-	-	-	-	-	-	-	-	-0.108 (0.092)	0.126 (0.155)	0.609*** (0.079)		
Short Rate Regression																	
$\text{spread}_t^{Gr} = \delta_0^{Gr} + \delta_1^{Gr} X_t^{Gr} + u_t^{Gr}$																	
	$100\delta_0$	Vector $100\delta_1$															
	0.301*** (0.003)	HY <sub>t</sub>	IP <sub>t</sub> <sup>Gr</sup>	CPI <sub>t</sub> <sup>Gr</sup>	Def <sub>t</sub> <sup>Gr</sup>	Deb <sub>t</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t</sub> <sup>Gr</sup>	IP <sub>t</sub> <sup>Gr</sup>	CPI <sub>t</sub> <sup>Gr</sup>	Def <sub>t</sub> <sup>Gr</sup>	Deb <sub>t</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t</sub> <sup>Gr</sup>	
		0.382*** (0.006)	0.252*** (0.004)	-0.054*** (0.004)	0.03*** (0.003)	0.295*** (0.007)	-0.206*** (0.003)	-0.057*** (0.003)	-0.031*** (0.003)	0.002 (0.004)	0.122*** (0.004)	0.168*** (0.004)	0.318*** (0.005)	0.351*** (0.002)	0.137*** (0.002)	-0.029*** (0.002)	
Recovery Intensity Coefficients $\theta_0$ and $\theta_1$																	
$\ln(\theta_t^{Gr}) = \theta_0^{Gr} + \theta_1^{Gr} X_t^{Gr}$																	
	$\theta_0$	$\theta_1$	HY <sub>t</sub>	IP <sub>t</sub> <sup>Gr</sup>	CPI <sub>t</sub> <sup>Gr</sup>	Def <sub>t</sub> <sup>Gr</sup>	Deb <sub>t</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t</sub> <sup>Gr</sup>	IP <sub>t</sub> <sup>Gr</sup>	CPI <sub>t</sub> <sup>Gr</sup>	Def <sub>t</sub> <sup>Gr</sup>	Deb <sub>t</sub> <sup>Gr</sup>	Princ. Comp1 <sub>t</sub> <sup>Gr</sup>	Princ. Comp2 <sub>t</sub> <sup>Gr</sup>	Princ. Comp3 <sub>t</sub> <sup>Gr</sup>
	0.000 (0.001)	0.030 (3.273)	0.016 (0.192)	0.012 (0.113)	-0.004 (0.125)	0.006 (0.242)	0.005 (0.090)	0.004 (0.048)	0.006 (0.143)	-0.003 (0.060)	0.010 (0.173)	0.001 (0.069)	0.004 (0.084)	0.005 (0.110)	-0.013 (0.020)	-0.007 (0.026)	-0.007 (0.037)

<sup>1</sup> Estimated Standard Errors in Parentheses (\*\*\*1% level; \*\*5% level; \*10% level)