

Credit and liquidity risks in euro-area sovereign yield curves*

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Abstract

In this paper, we propose a model of the joint dynamics of euro-area sovereign yield curves. The arbitrage-free valuation framework involves five factors and two regimes, one of the latter being interpreted as a crisis regime. These common factors and regimes explain most of the fluctuations in euro-area yields and spreads. The regime-switching feature of the model turns out to be particularly relevant to capture the rise in volatility experienced by fixed-income markets over the last years. In our reduced-form set up, each country is characterized by a hazard rate, specified as some linear combinations of the factors and regimes. The hazard rates incorporate both liquidity and credit components, that we aim at disentangling. The estimation suggests that a substantial share of the changes in euro-area yield differentials is liquidity-driven. Our approach is consistent with the fact that sovereign default risk is not diversifiable, which gives rise to specific risk premia that are incorporated in spreads. Once liquidity-pricing effects and risk premia are filtered out of the spreads, we obtain estimates of the actual –or real-world– default probabilities. The latter turn out to be significantly lower than their risk-neutral counterparts.

JEL codes: E43, E44, E47, G12, G24.

Keywords: default risk, liquidity risk, term structure of interest rates, regime-switching, euro-area spreads.

1 Introduction

One of the most spectacular symptoms of the crisis that began in mid-2007 is the dramatic rise in intra euro-area government-bond yield spreads. Whereas all euro-area sovereign 10-year bond yields were contained in a range of 50 bp between 2002 and 2007, the average spreads over Germany of 5 countries were higher than 100 basis points in 2009 and 2010. Since the inception of the euro in 1999 and the resulting elimination of exchange-rate risk, intra-euro-area spreads reflect the fluctuations of

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compensations demanded by investors for holding two remaining kinds of risks: credit and liquidity risks.¹ The credit risk is linked to the issuer's probability of default (PD). If investors assess that the PD of some indebted country is higher than in the past, the prices of the bonds issued by this country fall because expected loss increases. Liquidity risk arises from the potential difficulty that one may have in selling the asset before its redemption (for instance if one is required to do so in distressed market conditions, where it is difficult to find a counterpart for trade relatively quickly). The recent financial crisis illustrates why, along with credit risk, liquidity risks matter and should not be underestimated (see Brunnermeir, 2009 [14]). Disentangling credit and liquidity effects in bond prices is important in several respects. For instance, appropriate policy actions that may be needed to address a sharp rise in spreads depend on the source of the movement: if the rise in spreads reflects poor liquidity, policy actions should aim at improving market functioning. But if it is linked to credit concerns, the solvency of the debtors should be enhanced (see Codogno, Favero and Missale, 2003 [22]). Furthermore, optimal investment decisions would benefit from such a decomposition. In particular, those medium to long-term investors who buy bonds to hold them until redemption seek to buy bonds whose price is low because of poor liquidity, since it provides them with higher long-run returns than more liquid bonds with the same credit quality (see Longstaff, 2009 [59]).

In this paper, we present a no-arbitrage affine term-structure model –ATSM hereinafter– of the joint dynamics of euro-area sovereign yield curves. The framework allows for transitions between tranquil and crisis periods, which is obviously well-suited to account for the fluctuations of yields and spreads over the last three years.² In this reduced-form framework, the default probabilities are modeled directly instead of defining a stochastic process for the obligor's asset value that triggers default when the process reaches some threshold (as in Merton, 1974 [64]).³ While the focus is on default modeling, the specifications account for the pricing of some liquidity premia, as originally proposed by Duffie and Singleton (1999) [33]. The state variables, also termed with “risk factors”, follow discrete-time inter-related Gaussian processes. Exploiting the framework developed by Monfort and Renne (2011) [67], the Gaussian processes present drifts and variance-covariance matrices that are subject to regime shifts. The latter are described by a two-state Markov chain. The model is estimated using yield data covering the last twelve years. The five-factor and two-regime model accounts for more than 98% of the variances of yields driving eleven term structures

¹ Indeed, an overwhelming share of the euro-area sovereign debt is denominated in euros (see Eurostat, 2011 [37]).

² See Ang and Timmermann, 2011 [6] for a discussion about the use of regime switching in financial modeling.

³ After having developed criteria to measure the performances of credit models in terms of default discrimination and relative value analysis, Arora, Bohn and Zhu (2005) [7] compare structural (e.g. Merton's) and reduced-form models. Their results suggest that the reduced-form model outperform the others where an issuer has many bonds in the market, which is typically the case here.

of interest rates. The fact that a small set of factors is able to account for most of the fluctuations of sovereign spreads is consistent with findings by Geyer, Kossmeier and Pichler (2004) [41] and, more recently, by Longstaff et al. (2011) [57].

In addition to the yield curves of ten euro-area countries, we model the yield curve of KfW (*Kreditanstalt für Wiederaufbau*), a German agency. We identify a liquidity-related pricing factor by exploiting the term structure of the the KfW-*Bund* spreads. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than their sovereign counterparts –the *Bunds*– but are less liquid.⁴ Therefore, the KfW-*Bund* spread should be essentially liquidity-driven.⁵ The resulting liquidity-related factor contributes significantly to the dynamics of intra-euro spreads, supporting recent findings by Favero et al. (2010) [38] or Manganelli and Wolswijk (2009) [61].

We propose an efficient estimation method to bring the model to the data. The risk factors are some linear combinations of observed yields. Being observed, the estimation of the (historical) risk-factor dynamics boils down to the estimation of a Markov-switching vector-autoregression model. The regime-switching feature of the model turns out to be particularly relevant to account for the rise in volatility experienced by fixed-income markets over the last years.⁶ The fact that the factors are observed yield combinations raise internal consistency issues when it comes to estimate their risk-neutral dynamics: the model has indeed to correctly price the bond portfolios that are reflected by these yield combinations. These internal-consistency restrictions are taken into account by our estimation procedure.

Our estimation dataset is supplemented with survey-based forecasts. As evidenced by Kim and Orphanides (2005) [51], this alleviates the downward small-sample bias in the persistence of the yields obtained with conventional estimation.⁷ Such biases typically result in too stable long-horizon expectations of yields and, as a consequence, overstates the variability of term premia. Generating reliable expectations is key if one wants to use the model to recover probabilities of default from bond prices. To that respect, we propose an estimation of the term-structure of historical –or actual, or real-world– PDs implied by observed yield curves. Basically, there are two main operations to perform on the spreads to achieve this. First, one has to extract the part of the spread that is not default-related. Second, one has to remove the risk premia from the remaining part of the spread

⁴ By abuse of language, we use here the term *Bunds* for the German sovereign bonds of any maturity although this name is usually used for ten-year bonds only.

⁵ See Schwarz (2009) [72].

⁶ The pricing framework allows for risk premiums demanded by the investors to be compensated for the systematic nature of the regime shifts. Regime shifts represent a systematic risk in the sense that this risk can not be diversified away.

⁷ This way of reducing the bias is not the only one. In particular, Jardet, Monfort and Pegoraro (2009) [48] use a “near-cointegrated framework” specification of the factors (averaging a stationary and a cointegrated specification).

—these premia being defined as those parts of long-term yields that would not be present if agents were risk-neutral. Once the uncertainty regarding these two operations is taken into account, it turns out that this approach fails to produce precise estimates of the PDs, in the sense that the confidence intervals of model-based PDs often contain zero. However, the results suggest that these probabilities are often significantly lower than their risk-neutral counterparts. Yet the latter, derived from basic models like Litterman and Iben (1991) [53], are extensively used by market practitioners, who refer to them as *implied default probabilities*.⁸

Our study contributes to the term-structure modeling literature in four main directions. First, we estimate an ATSM explicitly incorporating liquidity and credit aspects on European data, in a multi-country set up.⁹ Second, we investigate the potential of the regime-switching feature in credit ATSM. Third, we propose an efficient estimation methodology, conveniently dealing with internal consistency problems and incorporating survey-based forecasts data. Fourth, we investigate the potential of credit ATSM to generate term structures of PDs. Regarding the latter point, we investigate the precision of the PDs estimates by deriving confidence intervals for these.

The remaining of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the model and details how bonds are priced in this framework. Section 4 deals with the choice and the construction of the data. Section 5 presents the estimation of the model and Section 6 examines the implication of the model in terms of liquidity and credit pricing. Section 7 summarizes the results and makes concluding remarks.

2 Related literature

There is compelling evidence that yields and spreads are affected by liquidity concerns¹⁰. However, the quantification of the liquidity premium, that is, distinguishing between the default-related and the liquidity-related components of yield spreads, remains a challenging task. In recent studies, some authors develop ATSM to breakdown different kinds of spreads into different components. These approaches are based on the assumption that there exists commonality amongst the liquidity components of prices of different bonds (see e.g. Chordia and Subrahmanyam, 2000 [20], Fontaine and Garcia, 2009 [40] or the recent paper by Dick-Nielsen, Fledhütter and Lando, 2011 [30]). For instance, Liu, Longstaff and Mandell (2006) [55] use a five-factor affine framework to jointly model

⁸ See e.g. Hull, Predescu and White (2005) [47], Berd, Mashal and Wang, 2003 [10], Caceres, Guzzo and Segoviano (2010) [16] or Cont, 2010 [24].

⁹ Geyer, Kossmeier and Pichler (2004) [41] have also presented a multi-country ATSM. However, their model only accounts for the spreads' dynamics (which are supposed to be driven by factors that are independent from the riskfree rates) and it does not explicitly accommodate liquidity-pricing effects.

¹⁰ See, e.g., Longstaff (2004) [58], Landschoot (2004) [52], Chen, Lesmond and Wei (2007) [19], Covitz and Downing (2007) [25] or Acharya and Pedersen (2005) [1].

Treasury, repo and swap term structures. One of their factors is related to the pricing of the Treasury-securities liquidity and another factor reflects default risk.¹¹ Feldhütter and Lando (2009) [39] develop a six-factor model for Treasury bonds, corporate bonds and swap rates that makes it possible to decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. They find that the convenience yield is by far the largest component of spreads. Longstaff, Mithal and Neis (2005) [60] use information in credit default swaps –in addition to bond prices– to obtain measures of the nondefault components in corporate spreads. They find that the nondefault component is time-varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity.

Our paper also extends the literature that considers the introduction of regime-switching in ATSM. This literature is based on strong evidences that point to the existence of regime-switching in the dynamics of the term structure of interest rates (see Hamilton, 1988 [44], Aït-Sahalia, 1996 [2], Ang and Bekaert 2002, [3] or Davies, 2004 [29] for spreads). Implied shifts in the interest-rate dynamics presents a systematic risk to investors. The pricing of such a risk has already been empirically investigated within default-free ATSM incorporating Markov-switching.¹² Building on the approaches introduced by Duffie and Singleton (1999) [33] or Duffe (1999)[31] to deal with credit risk in ATSM, Monfort and Renne (2011) [67] explore the potential of Markov-switching in credit ATSM models.¹³ The framework developed by the latter paper is exploited in the present study.

3 The model

In this section, we present the dynamics of the pricing factors and regimes. We consider three types of variables: five macroeconomic factors gathered in a vector $y_t = [y_{1,t}, y_{2,t}, y_{3,t}, y_{4,t}, y_{5,t}]'$, a regime variable z_t that can take two values: $[1, 0]'$ and $[0, 1]'$ and $d'_t = (d_{1,t}, \dots, d_{N,t})$, a set of binary variables indicating the default ($d_{n,t} = 1$) or the non-default ($d_{n,t} = 0$) states of the countries indexed by n . The next two subsections respectively describe the dynamics under the historical measure and under the risk-neutral measure. Then subsection 3.3 deals with the hazard rates and, in particular, introduces the modeling of liquidity pricing.

¹¹ As noted by Feldhütter and Lando (2009) [39], the identification of the liquidity and credit risk factors in Liu et al. relies critically on the use of the 3-month general-collateral repo rate (GC repo) as a short-term risk-free rate and of the 3-month LIBOR as a credit-risky rate. Liu et al. define the liquidity factor as the spread between the 3-month GC repo and the 3-month Treasury-bill yield (and is therefore observable). In each yield, their liquidity component is the share of the yield that is explained by this factor.

¹² See Monfort and Pegoraro, 2007 [66], Ang Bekaert and Wei, 2008 [4] and Dai, Singleton and Yang, 2007 [26].

¹³ Whereas Duffie and Singleton (1999) [33] and Duffe (1999)[31] present continuous-time models. Gouriéroux, Monfort and Polimenis, 2006 [42] study discrete-time credit ATSM, as well as Monfort and Renne (2011) [67].

3.1 Historical dynamics of factors (y_t) and regimes (z_t)

The conditional distribution of y_t given z_t is Gaussian and is given by:

$$\begin{aligned} \begin{bmatrix} y_{1,t} \\ \vdots \\ y_{p,t} \end{bmatrix} &= \begin{bmatrix} \mu_{1,1} & \mu_{1,2} \\ \vdots & \vdots \\ \mu_{p,1} & \mu_{p,2} \end{bmatrix} z_t + \Phi \begin{bmatrix} y_{1,t-1} \\ \vdots \\ y_{p,t-1} \end{bmatrix} + \begin{bmatrix} \sigma'_{11} z_t & 0 & 0 \\ \vdots & \ddots & 0 \\ \sigma'_{p1} z_t & \cdots & \sigma'_{pp} z_t \end{bmatrix} \varepsilon_t \\ &= \mu z_t + \Phi y_{t-1} + \Omega(z_t) \varepsilon_t, \end{aligned} \quad (1)$$

where the ε_t 's are independently and identically $N(0, I)$ distributed. It is a vector autoregressive model where the drift and the covariance matrix of the innovations are subject to regime shifts. The regime variable z_t follows a two-state Markov chain whose probabilities of transition are denoted with $\pi_{i,j}$. Formally:

$$P(z_t = j | z_{t-1} = i) = \pi_{i,j}. \quad (2)$$

Equation (1) implies that there is instantaneous causality between z_t and y_t , as in Ang, Bekaert and Wei (2008) [4].¹⁴ If country n has not defaulted before t , the conditional probability that country n defaults in time t is given by $1 - \exp(-\lambda_{n,t}^d)$ where the default intensity $\lambda_{n,t}^d$ is a function of (z_t, y_t) . Our framework builds on the “doubly stochastic” assumption, under which the default times of the different countries are correlated only as implied by the correlation of their default intensities. The default state is absorbing, in the sense that $d_{n,t} = 1$ implies $d_{n,t+h} = 1$ for any positive h .

The risk-free one-period rate r_{t+1} , that is the return of a one-period risk-free investment between t and $t+1$ (known in t) is a linear combination of y_t and z_t :

$$r_{t+1} = a_1 z_t + b_1 y_t.$$

3.2 The risk-neutral dynamics

Under the risk-neutral measure \mathbb{Q} , the dynamics of y_t is given by:

¹⁴ Ang et al. (2008) remark that instantaneous causality between z_t and y_t implies that the variances of the factors y_t , conditional on past values of (z_t, y_t) , embed a jump term reflecting the difference in drifts μ accross regimes. This feature is absent from the Dai, Singleton and Yang (2007) [26] setting.

$$\begin{bmatrix} y_{1,t} \\ \vdots \\ y_{p,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,1}^* & \mu_{1,2}^* \\ \vdots & \vdots \\ \mu_{p,1}^* & \mu_{p,2}^* \end{bmatrix} z_t + \Phi^* \begin{bmatrix} y_{1,t-1} \\ \vdots \\ y_{p,t-1} \end{bmatrix} + \begin{bmatrix} \sigma'_{11} z_t & 0 & 0 \\ \vdots & \ddots & 0 \\ \sigma'_{p1} z_t & \cdots & \sigma'_{pp} z_t \end{bmatrix} \varepsilon_t^* \quad (3)$$

$$= \mu^* z_t + \Phi^* y_{t-1} + \Omega(z_t) \varepsilon_t^* \quad (4)$$

where, under \mathbb{Q} , z_t is an homogenous Markov chain defined by a transition matrix $\{\pi_{ij}^*\}$, and where ε_t^* is $IIN(0, I)$.

Given the historical and the risk-neutral dynamics, it can be shown that the stochastic discount factor (s.d.f.) is exponential affine in (z_t, y_t) . More precisely, in this context, the s.d.f. $M_{t-1,t}$ between $t-1$ and t is of the form (see Monfort and Renne, 2011 [67]):

$$\begin{aligned} M_{t-1,t} = \exp & \left[-a'_1 z_{t-1} - b'_1 y_{t-1} - \frac{1}{2} \nu'(z_t, z_{t-1}, y_{t-1}) \nu(z_t, z_{t-1}, y_{t-1}) + \right. \\ & \left. + \nu'(z_t, z_{t-1}, y_{t-1}) \varepsilon_t + [\delta' z_{t-1}]' z_t \right], \end{aligned} \quad (5)$$

where δ is a 2×2 matrix whose (i, j) entry is $\ln(\pi_{ij}^*/\pi_{ij})$ and where $\Omega(z_t) \nu(z_t, y_{t-1}) = (\Phi^* - \Phi) y_{t-1} + (\mu^*(z_t) - \mu(z_t))$. The risk-sensitivity matrix δ and function ν respectively price the (standardized) innovations ε_t of y_t and the regimes z_t .

3.3 Hazard rates

As shown in Monfort and Renne (2011) [67], in such a framework, the pricing of defaultable bonds boils down to the pricing of risk-free bonds if the risk-free short rate is replaced with a short rate embedding credit and liquidity risks. The differential between the latter and the risk-free short rate is termed with hazard rate and is denoted by $\lambda_{n,t}$ (for country n). Intuitively, in the absence of liquidity pricing and with a zero recovery rate, the hazard rate would simply be the default intensity $\lambda_{n,t}^d$. Let us define a loss-adjusted credit intensity $\lambda_{n,t}^c$ that accounts for non-zero recovery rate. Building on the “recovery of market value” assumption introduced by Duffie and Singleton (1999) [33], we assume that the recovery payoff is equal to a constant fraction ζ of the bond price that would have prevailed in the absence of default. In that context, Appendix B shows that the credit intensity $\lambda_{n,t}^c$ is given by:¹⁵

$$\exp(-\lambda_{n,t}^c) = \exp(-\lambda_{n,t}^d) + \zeta [1 - \exp(-\lambda_{n,t}^d)].$$

¹⁵ Of course, when ζ is equal to zero, $\lambda_{n,t}^c = \lambda_{n,t}^d$, and when ζ is equal to one, the bond is equivalent to a risk-free bond.

Following e.g. Liu, Longstaff and Mandell (2006) [55], Feldhütter and Lando (2008) [39] or Fontaine and Garcia (2009) [40], liquidity-pricing effects are introduced through an illiquidity intensity denoted by $\lambda_{n,t}^\ell$.¹⁶ We assume further that credit and illiquidity intensities are affine in (z_t, y_t) . As a result, the hazard rate of the bonds issued by country n reads:

$$\lambda_{n,t} = \underbrace{(\alpha_n^c)' z_t + (\beta_n^c)' y_t}_{\text{credit-related } (\lambda_{n,t}^c)} + \underbrace{(\alpha_n^\ell)' z_t + (\beta_n^\ell)' y_t}_{\text{liquidity-related } (\lambda_{n,t}^\ell)}. \quad (6)$$

Further, we assume that the country-specific illiquidity intensities $\lambda_{n,t}^\ell$ are driven by a unique factor denoted by λ_t^ℓ , the latter being a linear combination of (z_t, y_t) . Formally, for all countries n , we have:

$$\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \times \lambda_t^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \times (\alpha_\ell' z_t + \beta_\ell' y_t). \quad (7)$$

3.4 Pricing

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see Hansen and Richard, 1987 [45] and Berholon, Monfort and Pegoraro, 2008 [12]) and that the price at t of a risk-free zero-coupon bond with residual maturity h , denoted by $B_{0,t,h}$, is given by:

$$B_{0,t,h} = E_t^{\mathbb{Q}} [\exp(-r_{t+1} - \dots - r_{t+h})], \quad (8)$$

where $r_{t+i} = a_1' z_{t+i-1} + b_1' y_{t+i-1}$, $i = 1, \dots, h$. Under our recovery assumptions, Appendix B shows that the price of a defaultable and illiquid zero-coupon bond issued by country n and with a residual maturity of h has a price at time t that is given by (if debtor n has not defaulted before time t):

$$B_{n,t,h} = E_t^{\mathbb{Q}} [\exp(-r_{t+1} - \dots - r_{t+h} - \lambda_{n,t+1} - \dots - \lambda_{n,t+h})]. \quad (9)$$

¹⁶ The affine term-structure literature is relatively silent on the interpretation or the microfoundations of the illiquidity intensity. In a theoretical paper analyzing interactions between credit and liquidity risks, He and Xiong (2011) [46] show that such an illiquidity intensity may reflect the probability of occurrence of a liquidity shock; upon the arrival of this shock, the bond investor has to exit by selling his bond at a fractional cost (i.e. the selling price is equal to a fraction of the price that would have prevailed in the absence of the liquidity shock); the fractional cost is the analogous to the fractional loss $(1 - \zeta)$ in the default case (see also Ericsson and Renault, 2006 [36] for a similar interpretation).

Since both the r_{t+i} 's and the $\lambda_{n,t+i}$'s are affine in (z_t, y_t) , and since (z_t, y_t) is compound auto-regressive of order one under \mathbb{Q} , the prices of bonds are exponential affine in (z_t, y_t) :¹⁷

$$B_{n,t,h} = \exp \left(-c'_{n,h} z_t - f'_{n,h} y_t \right) \quad (10)$$

and the associated yields are:

$$R_{n,t,h} = \frac{1}{h} \left(c'_{n,h} z_t + f'_{n,h} y_t \right), \quad (11)$$

where $(c'_{n,h}, f'_{n,h})$ are computed recursively.¹⁸

4 Data

The data are monthly and cover the period from July 1999 to March 2011. We exclude the first 6 months of 1999 so as to avoid potential effects linked to the euro introduction. The estimation involves end-of-month yields as well as survey-based yield forecasts. We consider the yield curves of ten euro-area countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal and Spain. Greece data are excluded from the analysis because appropriate euro-denominated bond yields are not available before 2001, when Greece joined the euro area. Consistently with the fact that, among sovereign euro-area bonds, the German *Bunds* are perceived to be the "safest haven" both in terms of credit quality and liquidity, we consider the German bonds as risk-free.¹⁹

Appendix C details the sources of the data and the preliminary computations performed to get end-of-month zero-coupon yields. The following subsection (4.1) introduces the KfW-*Bund* spreads that will be exploited to identify the liquidity-related latent factor λ_t^ℓ . In 4.2, we provide a preliminary analysis of euro-area yield differentials and in 4.3, we detail the computation of the factors $y_{1,t}, \dots, y_{5,t}$.

4.1 The KfW-*Bund* spread

Our identification of a liquidity-related latent factor is partly based on the yield spread between German federal bonds and KfW agency bonds. The latter are less liquid than the sovereign counter-

¹⁷ Appendix A.1 derives the Laplace transform of (z_t, y_t) and shows that (z_t, y_t) is Compound auto-regressive of order one. Appendix A.2 shows how to compute the multi-horizon Laplace transform of compound auto-regressive processes. (See Darolles, Gourieroux and Jasiak, 2006 [27] or Bertholon, Monfort and Pegoraro, 2008 [12] for in-depth presentations of compound auto-regressive –or Car– processes.)

¹⁸ The general recursive formulas are presented in Appendix A.2. To apply these in the current case, one has (a) to use the Laplace transform of (z_t, y_t) presented in Appendix A.1 and (b) take a sequence ω_h , $h = 1, \dots, H$ defined by $\omega_H = (-\alpha'_n, -\beta'_n)$ and $\omega_h = (-\alpha'_n - a'_1, -\beta'_n - b'_1, -\gamma'_n)$ for $h = 1, \dots, H - 1$, with $c_{n,0} = a_1$ and $f_{n,0} = b_1$.

¹⁹ In particular, the German bond market is the only one in Europe that has a liquid futures market, which boosts demand for the German Bund compared to other euro area debt and bolsters its liquidity (see e.g. Pagano and von Thadden, 2004 [71], Ejlsing and Sihoven, 2009 [35] or Barrios et al., 2009 [8]).

parts, but are explicitly and fully guaranteed against default by the German federal government.²⁰ Consequently, the spread between these two kinds of bonds can be seen as a measure of the German government bond-market liquidity premium demanded by investors. In the same spirit, Longstaff (2004) [58] computes liquidity premia based on the spread between U.S. Treasuries and bonds issued by Refcorp, that are guaranteed by the Treasury.

In order to check that this liquidity-pricing measure is not purely specific to Germany, we look at comparable spreads in alternative countries.²¹ Let us first consider two debtors whose issuances are guaranteed by the French government, namely the CADES (Caisse d’amortissement de la dette sociale) and the SFEF (Société de financement de l’économie française).²² The right plot in Figure 1 shows that, over the recent period –when the French spreads are available–, the KfW-*Bund* spread shares most of its fluctuations with the spread between SFEF bonds and French Treasury bonds (OATs), as well as with the CADES-OAT spread. The same plot displays the spreads of government-guaranteed bank bonds –issued by the Dutch NIBC bank and the Austrian Raiffeisen Zentralbank– over their respective sovereign counterparts. These spreads also show strong correlations with the KfW-*Bund* spread.

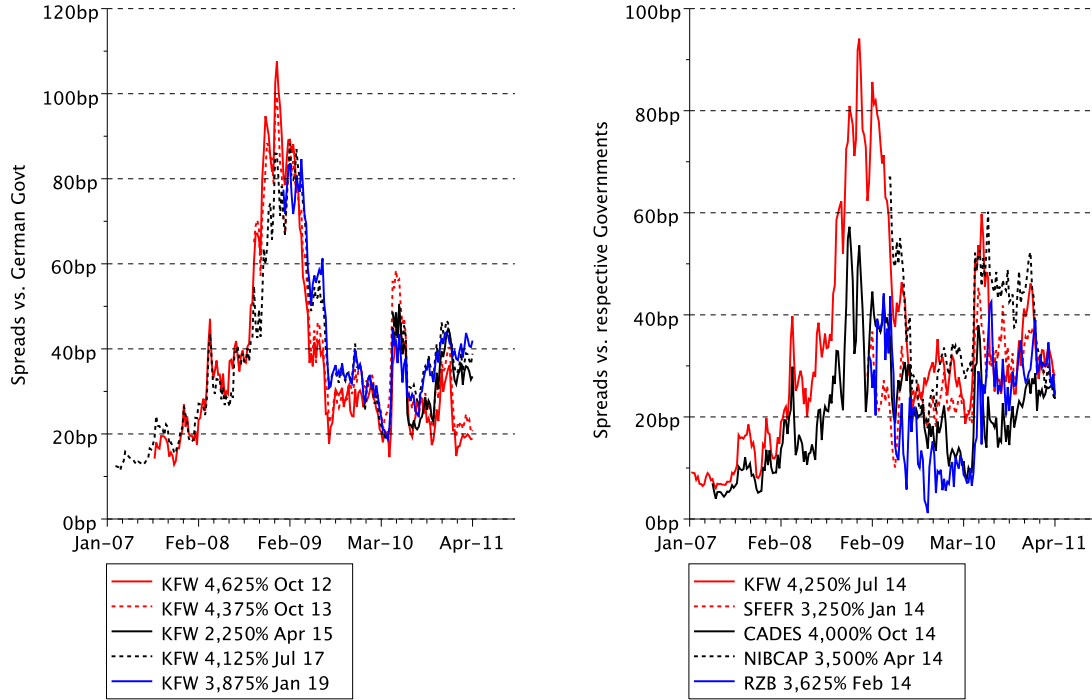
²⁰ An understanding between the European Commission and the German Federal Ministry of Finance (1 March 2002) stated that the guarantee of the Federal Republic of Germany will continue to be available to KfW. The three main rating agencies –Fitch, Standard and Poor’s and Moody’s– have assigned a triple-A rating to KfW (see KfW website www.kfw.de/EN_Home/Investor_Relations/Rating.jsp). In addition, as the German federal bonds, KfW’s bonds are zero-weighted under the Basle capital rules. The relevance of the KfW-*Bund* spread as a liquidity proxy is also pointed out by McCauley (1999) [63], the ECB, 2009 [34] and is exploited by Schwarz (2009) [72].

²¹ Note that such alternative (term structures of) spreads are not available on our whole estimation period (1999–2011).

²² Note that contrary to the ones issued by the SFEF, those issued by the CADES do not benefit from the explicit –but only implicit– guarantee from the French government. However, both issuer are triple-A rated, as the French government.

Fig. 1: Differentials between government and government-guaranteed bonds

Notes: The left plot shows the spreads between KfW bond yields and their sovereign counterparts. In the right plot, the spread between a KfW bond maturing in 2014 and its sovereign counterpart is compared with various other spreads between Government-guaranteed European bonds and their respective sovereign counterparts: SFEFR and CADES bonds are guaranteed by the French government (implicitly in the CADES case), the NIBCAP and RZB bonds are respectively guaranteed by the Dutch and Austrian governments. The yields come from Barclays Capital.



4.2 Euro-area government yields

Table 1 suggests that euro-area government yields are highly correlated across countries and across maturities (see also Favero, Pagano and von Thadden, 2010 [38]). Table 2 reports the correlations between the spreads vs. Germany for different countries over the sample periods and presents a principal-component analysis of these spreads across countries. The correlations suggest that spreads largely comove across countries. The principal-component analysis (see lower part of Table 2) indicates that, for different maturities (2, 5 and 10 years), the first two principal components roughly explain 90% of the spread variances across countries. This suggests that a model with a limited number of common factors may be able to explain the bulk of euro-area yield-differential fluctuations. The estimation is based on four benchmark maturities per country: 1, 2, 5 and 10 years. The short end of the risk-free yield curve is augmented by the 1-month EONIA swap.²³

²³ Data providers such as Bloomberg do not propose 1-month sovereign German yields. We decide to replace it with the 1-month EONIA swap rates as swap yields are often considered as risk-free yields, see e.g. Collin-Dufresne, Goldstein and Martin (2001) [23].

Tab. 1: **Descriptive statistics of selected yields**

Notes: The table reports summary statistics for selected yields. The data are monthly and cover the period from July 1999 to March 2011. Two auto-correlations are shown (the 1-month and the 1-year auto-correlations). The yields are continuously compounded and are in percentage annual terms (see Appendix C for details about their construction). The lower panel of the table presents the covariances and the correlations (in italics) of the yields. The 1-month rate is the 1-month EONIA swap.

	German yds			Italian yds		Portuguese yds		Irish yds	
	1-mth	2-year	10-year	2-year	10-year	2-year	10-year	2-year	10-year
Mean	2.761	2.961	4.086	3.288	4.517	3.428	4.609	3.537	4.672
Median	2.832	3.091	4.084	3.32	4.459	3.474	4.45	3.51	4.568
Standard dev.	1.371	1.18	0.718	1.046	0.581	1.046	0.79	1.125	1.048
Skewness	-0.243	-0.303	-0.076	0.175	0.17	0.488	0.952	1.285	1.974
Kurtosis	2.09	2.131	2.323	2.059	2.189	3.664	4.967	6.398	9.232
Auto-cor. (lag 1)	0.998	0.988	0.973	0.98	0.962	0.896	0.962	0.936	0.963
Auto-cor. (lag 12)	0.475	0.53	0.586	0.491	0.571	0.132	0.29	0.037	0.279
Correlations \ Covariances									
1-mth EONIA swap	1.867	1.521	0.73	1.306	0.432	0.835	0.114	0.636	-0.276
German 2-yr yd	<i>0.946</i>	1.385	0.713	1.149	0.423	0.728	0.144	0.577	-0.18
German 10-yr yd	<i>0.744</i>	<i>0.843</i>	0.516	0.588	0.345	0.349	0.201	0.283	0.037
Italian 2-yr yd	<i>0.917</i>	<i>0.937</i>	<i>0.785</i>	1.086	0.451	0.84	0.316	0.747	0.12
Italian 10-yr yd	<i>0.545</i>	<i>0.619</i>	<i>0.827</i>	<i>0.746</i>	0.337	0.406	0.343	0.415	0.325
Portug. 2-yr yd	<i>0.586</i>	<i>0.593</i>	<i>0.466</i>	<i>0.773</i>	<i>0.671</i>	1.086	0.605	1.095	0.61
Portug. 10-yr yd	<i>0.106</i>	<i>0.155</i>	<i>0.355</i>	<i>0.384</i>	<i>0.75</i>	<i>0.737</i>	0.622	0.721	0.753
Irish 2-yr yd	<i>0.415</i>	<i>0.438</i>	<i>0.351</i>	<i>0.639</i>	<i>0.638</i>	<i>0.938</i>	<i>0.815</i>	1.256	0.855
Irish 10-yr yd	<i>-0.193</i>	<i>-0.146</i>	<i>0.049</i>	<i>0.11</i>	<i>0.536</i>	<i>0.561</i>	<i>0.913</i>	<i>0.73</i>	1.092

4.3 Construction of the factors y_t

As explained in Section 3.4, our framework implies that (modeled) bond yields end up being some linear combinations of the regime variables z_t and of the factors y_t . Therefore, appropriate factors have to capture a large share of the common fluctuations of yields. Natural candidates for the y_t 's are the principal components of the set of yields time series. However, since we do not have survey-based forecasts of all the yields that we consider in the estimation –there are 40 of them–, doing so would deprive us of survey-based forecasts of the factors. If, as in Kim and Orphanides (2005) [51], we want to incorporate such data in the estimation of the historical dynamics of the factors, these need to be based on variables for which some forecasts are available. To that respect, the *Consensus Forecasts* provide us with 3-month-ahead and 12-month-ahead forecasts of the 10-year sovereign yields of 5 countries: France, Germany, Italy, the Netherlands and Spain. As a consequence, if we construct some factors that are given by combinations of these yields, 3-month and 12-month ahead survey-based forecasts of these factors can be included in the estimation procedure. (The advantages of using survey forecasts in the estimation of the historical dynamics of the factor are outlined in Section 1.)

Tab. 2: **Correlations and preliminary analysis of euro-area yield differentials**

Notes: Panel A reports the covariances and correlations (in italics) of 10-year spreads (vs. Germany) across nine euro-area countries. Panel B presents results of principal-component analyses carried out on the spreads. There are three analyses that correspond respectively to three maturities: 2 years, 5 years and 10 years. For each analysis, Panel B reports the eigenvalues of the covariance matrices and the proportions of variance explained by the corresponding component (denoted by "Prop. of var." in Panel B).

Panel A: Covariance and <i>correlations</i> of 10-year spreads vs. Germany									
	Fr.	It.	Sp.	Au.	Be.	Fi.	Po.	Ne.	Ir.
France	0.015	0.045	0.052	0.023	0.027	0.012	0.077	0.013	0.117
Italy	<i>0.915</i>	0.163	0.202	0.062	0.088	0.023	0.313	0.032	0.459
Spain	<i>0.818</i>	<i>0.951</i>	0.277	0.069	0.113	0.025	0.434	0.032	0.623
Austria	<i>0.867</i>	<i>0.684</i>	<i>0.585</i>	0.05	0.047	0.028	0.093	0.024	0.15
Belgium	<i>0.922</i>	<i>0.887</i>	<i>0.871</i>	<i>0.854</i>	0.061	0.024	0.169	0.023	0.253
Finland	<i>0.599</i>	<i>0.358</i>	<i>0.293</i>	<i>0.767</i>	<i>0.615</i>	0.026	0.028	0.015	0.053
Portugal	<i>0.738</i>	<i>0.904</i>	<i>0.962</i>	<i>0.486</i>	<i>0.8</i>	<i>0.204</i>	0.736	0.042	1.03
Netherlands	<i>0.878</i>	<i>0.67</i>	<i>0.514</i>	<i>0.911</i>	<i>0.794</i>	<i>0.785</i>	<i>0.41</i>	0.014	0.074
Ireland	<i>0.783</i>	<i>0.918</i>	<i>0.956</i>	<i>0.545</i>	<i>0.83</i>	<i>0.263</i>	<i>0.97</i>	<i>0.497</i>	1.534
Panel B: Principal components									
Component	1	2	3	4	5	6	7	8	9
<i>2-year spread</i>									
Eigenvalue	6.07	1.46	0.71	0.33	0.16	0.12	0.08	0.05	0.02
Prop. of var.	0.67	0.16	0.08	0.04	0.02	0.01	0.01	0.01	0
Cumul. prop.	0.67	0.84	0.92	0.95	0.97	0.98	0.99	1	1
<i>5-year spread</i>									
Eigenvalue	6.68	1.56	0.38	0.13	0.09	0.07	0.05	0.02	0.01
Prop. of var.	0.74	0.17	0.04	0.01	0.01	0.01	0.01	0	0
Cumul. prop.	0.74	0.92	0.96	0.97	0.98	0.99	1	1	1
<i>10-year spread</i>									
Eigenvalue	6.83	1.62	0.27	0.12	0.06	0.05	0.02	0.02	0.01
Prop. of var.	0.76	0.18	0.03	0.01	0.01	0.01	0	0	0
Cumul. prop.	0.76	0.94	0.97	0.98	0.99	0.99	1	1	1

The *Consensus Forecasts* are produced monthly by *Consensus Economics*, which surveys financial and economics forecasters. The survey is released around the middle of the month.²⁴ Note that the survey implicitly targets yields-to-maturity of coupon bonds and not zero-coupon bonds. However, our zero-coupon yields remain very close to coupon yields over the estimation sample. The remaining discrepancy will be attributed to the deviation between the survey-based forecasts and the model-based ones (the $\varepsilon_{j,h,t}$'s introduced in equation 12 below).

Nevertheless, all of our factors can not be based on 10-year yields since we would then miss the drivers of the deformation of the term structure of interest rates. In other words, we also have to consider factors that will be able to capture the changes in the slope and the curvature of the yield curves.²⁵

Taking all these remarks into account, we use the following factors: the first three are the level, the slope and the curvature of the German yield curve;²⁶ the last two factors are the first two principal components of the 10-year-maturity spreads (vs. Germany) of France, Italy, the Netherlands and Spain. Eventually, survey-based forecasts are available for three out of five factors (the first factor, i.e. the 10-year German yield, and the last two factors, associated with 10-year spreads vs. Germany).

The factors $y_{1,t}, \dots, y_{5,t}$ that result from this procedure are plotted in the upper two panels in Figure 2.

5 Estimation

5.1 Main lines of the estimation strategy

As Ang, Piazzesi and Wei (2006) [5] or Moench (2008) [65], our estimation procedure involves two steps. In the first one, we estimate the historical dynamics of factors y_t and regimes z_t by maximizing the log-likelihood using the Kitagawa-Hamilton algorithm. At the end of this first step, the Kitagawa-Hamilton smoother is used to estimate the regime variables z_t and these are taken as fixed in the next step. The latter concerns the joint estimation of the risk-neutral dynamics of (z_t, y_t) and of the specifications of the hazard rates $\lambda_{n,t}$. This second step is based on non-linear-least-squares techniques, taking into account the internal-consistency issue. Then, it remains to

²⁴ The number of respondents varies across time and countries. One average over the estimation period, while more than 20 forecasters contribute to the German forecasts, around 10 take part to the Italian ones. For each yield, we use the mean of the forecasts produced by the different survey contributors.

²⁵ The importance of such factors has been investigated by various empirical studies in the wake of Litterman and Scheinkman (1991) [54].

²⁶ The first (level) factor is the 10-year German rate, the second (slope) factor is the difference between the spread between the 10-year and the 1-year rates, the third (curvature) factor is computed as the difference between (a) the 3-to-10 year and (b) the 1-to-3 year slope of the yield curve (that is, 2 times the 3-yr yield minus the sum of the 1-yr and the 10-yr yields).

perform the decomposition of the hazard rates into credit and liquidity components. This final operation will be detailed in Section 6.

5.2 Historical dynamics of (z_t, y_t)

The historical dynamics of (z_t, y_t) is defined by a Markov-switching VAR (see equations 1 and 2). This set of five equations is augmented with equations linking survey-based forecasts to their model-based equivalent. These six additional equation read:

$$E_{j,h,t}^{CF} = E_t(y_{j,t+h}) + \varepsilon_{j,h,t}, \quad j \in \{1, 4, 5\}, \quad h \in \{3, 12\}, \quad (12)$$

where $E_{j,h,t}^{CF}$ is the h -period ahead survey-based forecast, $E_t(y_{j,t+h})$ is its equivalent model-based forecast, and the $\varepsilon_{j,h,t}$'s are the measurement errors, assumed to be normally i.i.d.. The model-based forecasts stem from:

$$E_t(y_{t+h}) = [\mu P^h + \Phi \mu P^{h-1} + \dots + \Phi^{h-1} \mu P] z_t + \Phi^h y_t. \quad (13)$$

The parameters are estimated by maximizing the associated log-likelihood. Two kinds of constraints are imposed in the estimation. First, we impose some constraints on the matrix of regime-switching probabilities. The probability of remaining in the crisis regime is then calibrated so as to imply an average length of the crisis of 2 years; this length being consistent with the findings of Cecchetti, Kohler and Upper (2009) [17] who investigate worldwide banking crises over the last 30 years.²⁷ Second, we constrain the unconditional means of the factors. Except for the first factor, the unconditional means of the factors are set to their sample means. The mean of the first factor (10-year German yield) is set to 4.75%. Indeed, its sample mean, which is of 4.10%, is low compared to the average of the long-term forecasts for this yield, the latter being expected to be less affected by short-sample biases.²⁸ Finally, as in Kim and Orphanides (2005) [51], we let the estimation to decide the standard deviations of the measurement errors $\varepsilon_{j,h,t}$ in equations (12).

Parameter estimates are reported in Table 4 and Table 5 (in Appendix F). The second regime, that we identify as a “crisis” regime, is characterized by particularly high standard deviations of the innovations ε_t , especially for the shocks affecting $y_{4,t}$ and $y_{5,t}$ (see Table 5).

The grey-shaded areas in Figure 2 indicate the crisis periods. These periods are estimated as

²⁷ Which translates into $\pi_{C,C} = 95\%$. Cecchetti et al. study 40 systemic banking crises since 1980. This constraint is imposed because preliminary unconstrained estimations resulted in probabilities of remaining in each of the regimes that was implausibly high.

²⁸ For comparison, the average of the 10-year-Bund yield over the last 20 years is approximately 5%. Twice a year, in April and October, the *Consensus Forecasts* present long-term forecasts of macroeconomic variables (up to 10 years ahead). Over the last 10 years, the average of the long-term forecasts of the 10-year German yield is of 4.78%.

those for which the smoothed probabilities of being in the crisis regime are larger than 50%. Three crisis periods are estimated: a first between September 2008 and August 2009, a second between December 2009 and January 2010 and a last that starts in April 2010 and that lasts till the end of the sample (March 2011).

Figure 3 displays survey-based forecasts of three of the factors ($y_{1,t}$, $y_{4,t}$ and $y_{5,t}$) together with their model-based equivalent, computed using equation (13). Except for the 12-month ahead forecasts of the fifth factor (bottom right panel in Figure 3), the model is able to reproduce most of the survey-based forecasts' fluctuations.

5.3 Risk-neutral dynamics

The vector θ of parameters defining the risk-neutral dynamics –that is, matrices μ^* , Φ^* , $\{\pi_{i,j}^*\}$ – and those defining the default intensities –the α 's and the β 's– is estimated by means of non-linear least squares. Basically, we aim at minimizing the sum of squared measurement errors, or SSME, across countries and maturities (1, 2, 5 and 10 years).²⁹ In addition, we have to deal with internal consistency conditions. These conditions arise from the fact that our pricing factors $y_{1,t}, \dots, y_{5,t}$ are known linear combinations of the yields; the latter being in turn some combinations of the factors (see equation 11). To maintain internal consistency, the model has to correctly “price” the factors (that reflect observed bond-portfolios' prices). The internal-consistency restrictions involve highly non-linear transformations of the parameters. As a consequence, numerically minimizing the SSME under the consistency constraints would considerably slow down the optimization procedure.³⁰ We therefore resort to an alternative solution that consists in augmenting the SSME with a term penalizing deviations from internal-consistency restrictions. More precisely, denoting observed yields by $\tilde{R}_{n,t,h}$, modeled yields by $R_{n,t,h}(\theta)$, observed factors by $\tilde{y}_{i,t}$ and modeled factors by $y_{i,t}(\theta)$, the estimator $\hat{\theta}$ results from:

$$\hat{\theta} = \arg \min_{\theta} \sum_{n,t,h} \left(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta) \right)^2 + \chi \sum_{t,i} (\tilde{y}_{i,t} - y_{i,t}(\theta))^2. \quad (14)$$

where χ is a parameter defining the relative penalization of the deviations between modeled (\tilde{y}_t) and observed (y_t) factors.

The loss function that we aim at minimizing (see equation 14) being highly non-linear in the underlying model parameters, it is necessary to find good starting values so as to achieve convergence

²⁹ The measurement errors are defined as the deviations between modeled and actual yields. In addition to sovereign yields, KfW's yields are also used in the estimation.

³⁰ See e.g. Duffie and Kan (1996) [32] for a simple example. Considering only one debtor and no regime-switching, Joslin, Singleton and Zhu (2011) [49] find a parameterization of their Gaussian model that automatically satisfies internal consistency restrictions.

Fig. 2: The five factors y_t and the estimated regime variable z_t

Notes: These plots show the factors $y_{1,t}, \dots, y_{5,t}$ that are used in the analysis. The first factor is the 10-year zero-coupon German yield (minus 4.75%). The second factor is a proxy of the yield-curve slope (difference between the 10-year German yield and the 1-month yield). The third is a proxy of the yield-curve curvature (10-year German yield + 1-month yield - 2 times the 3-year German yield). The fourth and fifth factors are the two first PCs of a set of four 10-year spreads vs. Germany (France, Italy, the Netherlands and Spain). The shaded areas correspond to periods for which the smoothed probability of being in the crisis regime is above 50% (using Kim's algorithm, 1993 [50]).

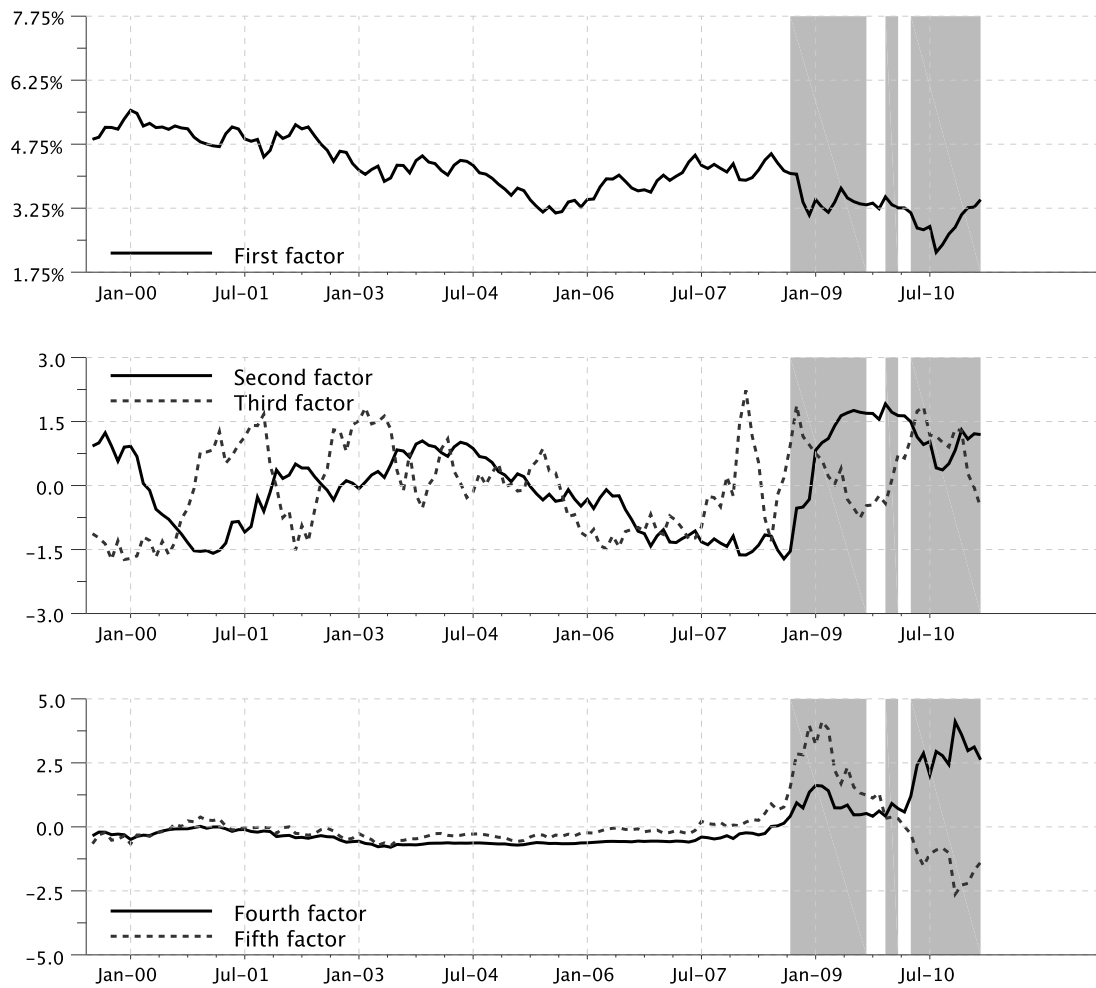
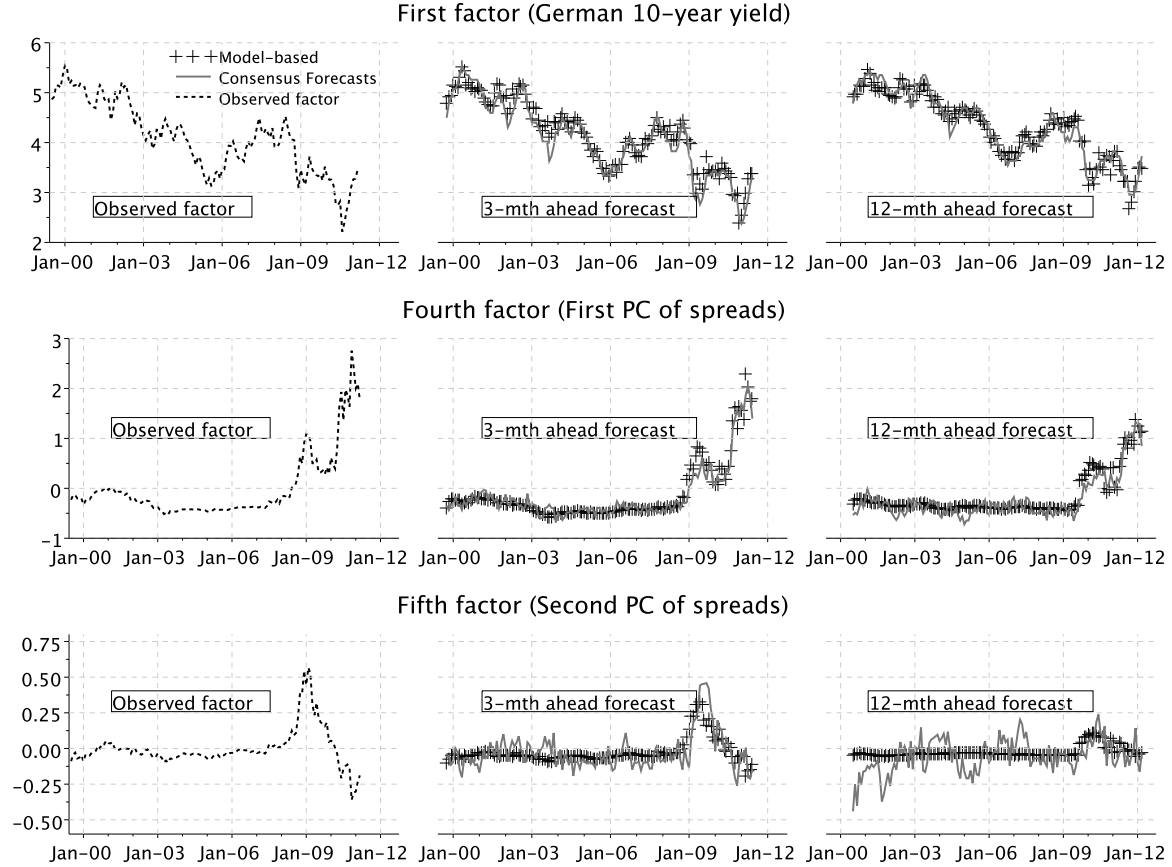


Fig. 3: **Model-based vs. survey-based forecasts**

Notes: The Figures compare survey-based forecasts of the factors (derived from the *Consensus forecasts*) with model-based forecasts. The charts of the left column display the three factors for which some survey-based forecasts are available, namely $y_{1,t}$, $y_{4,t}$, and $y_{5,t}$. The first factor is the German 10-year yield (minus 4.75 percentage points). The fourth and fifth factors are the first two principal components of a set of 10-year spreads vs. Germany for 4 countries (France, Italy, Spain and the Netherlands).



in a reasonable computing time.³¹ We proceed as follows: (a) we consider only the risk-free rates in (14) and we assume that their term-structure depends on the first three factors ($y_{1,t}$, $y_{2,t}$ and $y_{3,t}$) only, (b) we incorporate the risky yields of a subset of debtors (namely Germany, KfW and Portugal) and we (re-)estimate the parameterization of the risk-neutral dynamics (for the five factors y_t) as well the hazard rates of these three entities, (c) we estimate the hazard rates of the remaining entities, one by one, taking the other parameters given. In the final stage, all the parameters are (re)estimated jointly.³²

Table 3 and Table 4 present the parameter estimates. The standard deviation of these estimates are based on a Newey-West (1987) [70] heteroskedasticity and autocorrelation consistent (HAC)

³¹ Optimizations are based on iterative uses of quasi-Newton and Nelder-Mead algorithms (as provided by the Scilab software).

³² The final stage is itself decomposed into several sub-steps: first, the penalty factor χ (for the internal-consistency restrictions) is set to zero. Then, it is progressively increased, till 1, level at which deviations between modeled and actual factors y_t become neglectible.

covariance matrix estimator (see Appendix D).

The parameterizations of the hazard rates, presented in Table 3, stem from the decomposition of the hazard rates between liquidity-related and credit-related components, that will be discussed in the next section. Indeed, the minimization of the loss function specified in (14) leads to estimates of the α_n 's and β_n 's, with $\alpha_n = \alpha_n^c + \alpha_n^\ell$ and $\beta_n = \beta_n^c + \beta_n^\ell$ (α_n 's and β_n 's estimates are not reported). A first look at Table 3 suggests that the estimation results in significant impacts of the factors on the hazard rates. In particular, it turns out that the first three factors, i.e. those related to the risk-free (or German here) yield curve, are important in the spread dynamics. Such finding relates to several studies that pinpoint the relationship between credit spreads and risk-free rates (see, e.g., Manganello and Wolswijk, 2009 [61] on euro-area sovereign data).

6 Results and interpretation

To begin with, the approach results in a satisfying fit of the data. Modeled versus observed spreads are displayed in Figure 4 (grey lines for observed spreads, dotted lines for modeled spreads). On average across countries and maturities (i.e. across 45 series), the ratios of the measurement-error variances over those of the yields are lower than 2%: the average (across countries and maturities) measurement-error standard deviation is around 18 basis points. In the sequel of this section, we focus on two specific issues: liquidity pricing and extraction of default probabilities from bond yields.

6.1 The illiquidity intensity

In our model, we assume that there is a single factor that drives the liquidity pricing in euro-area bond yields. As documented in 4.1, the bonds issued by KfW and those issued by the German government embed the same credit risks –assumed to be nil here– but are not equally exposed to the liquidity-related factor. Accordingly, we simply have:

$$\lambda_t^\ell = \lambda_{KfW,t}. \quad (15)$$

The left part of Table 3 presents the estimated specification of λ_t^ℓ . According to the Student-t ratio, the liquidity factor is significantly linked to the five factors, especially the fifth one (which is the second PC of a set of four 10-year spreads vs. Germany). In addition, the α_ℓ estimates indicate that the liquidity factor jumps upwards in crisis periods. The resulting estimate of the liquidity factor is displayed in the upper plot in Figure 5, together with a 90% confidence interval.³³ It

³³ The computation of this confidence interval is based on the delta method, exploiting the fact that at each point in time, the estimate of λ_t^ℓ is a function of the parameter estimates and of y_t and z_t ($\lambda_t^\ell = \alpha_\ell' z_t + \beta_\ell' y_t$).

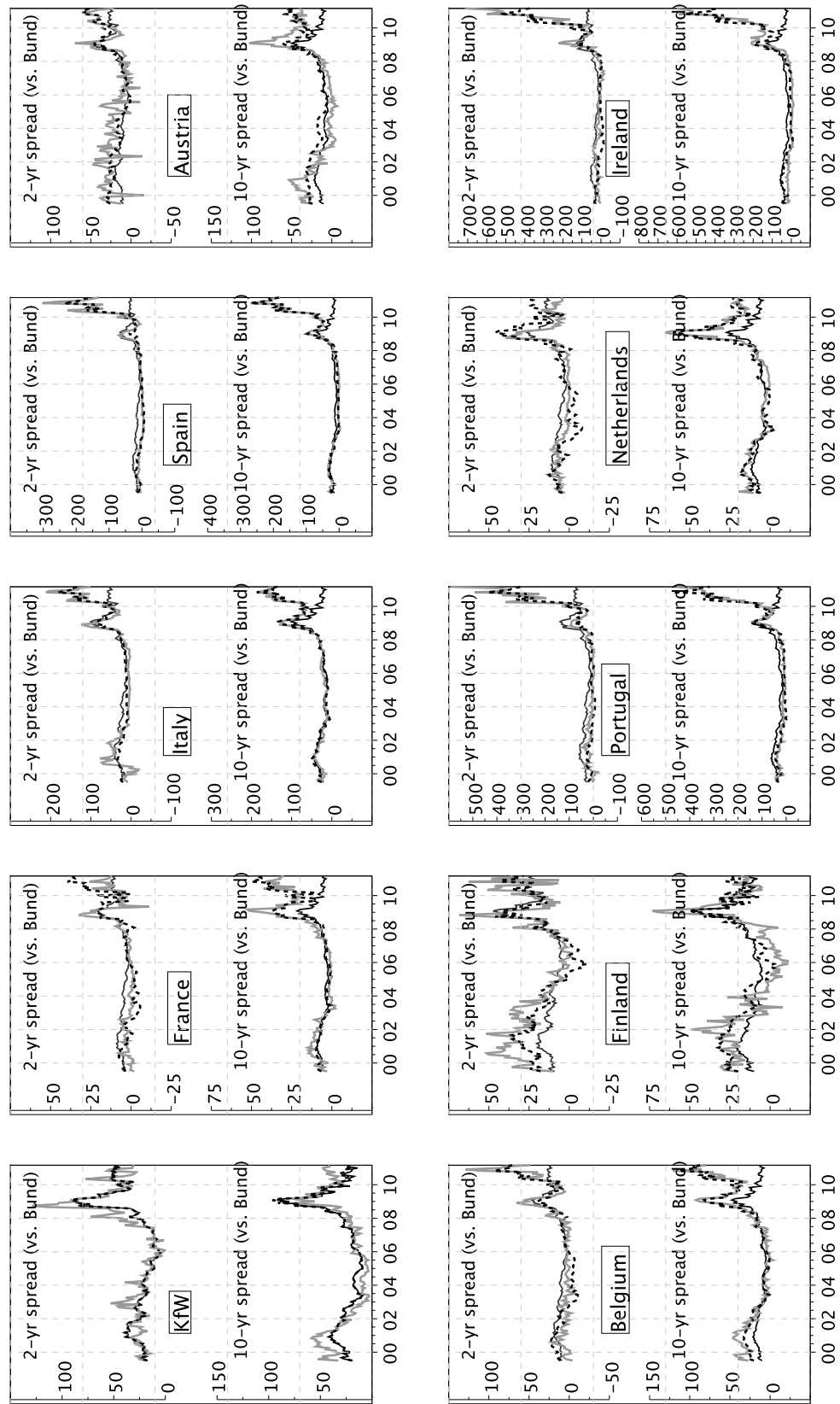
Tab. 3: Estimation of the hazard-rate ($\lambda_{n,t}$) parameterizations

Notes: The table reports the estimates of the hazard-rate parameterizations: $\lambda_{n,t} = (\alpha_n^C)' z_t + (\beta_n^C)' y_t + \lambda_{n,t}^{\ell}$, where α_n^C and β_n^C define the credit-related part of the hazard rate and $\lambda_{n,t}^{\ell} = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 (\alpha_{\ell}^{\ell} z_t + \beta_{\ell}^{\ell} y_t)$ is the liquidity-related part (see Section 3.3). The first column of the table reports the parameterization of the illiquidity intensity λ_t^{ℓ} (with $\lambda_t^{\ell} = \alpha_{\ell}^{\ell} z_t + \beta_{\ell}^{\ell} y_t$) that drives all country-specific illiquidity intensities $\lambda_{n,t}^{\ell}$. The estimation data are monthly and span the period from April 1999 to March 2011. Standard errors and Student-t are reported, respectively, in parenthesis and in square brackets below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. The estimates of $\gamma_{\ell,n}^0$ and $\gamma_{\ell,n}^1$ are based on the following penalty factors (see equation 25): $\chi_1 = 4$ and $\chi_2 = 1$.

	KfW (λ_{ℓ})	France	Austria	Netherlands	Finland	Belgium	Italy	Spain	Portugal	Ireland
$\alpha_{\ell,NC}$	0.41*** (0.073) [5.7]	-0.037** (0.017) [-2.3]	0.084** (0.04) [2.1]	-0.027 (0.028) [-1]	0.23*** (0.04) [5.8]	0.014 (0.022) [0.7]	0.088** (0.04) [2.2]	0.071 (0.052) [1.4]	0.06 (0.12) [0.5]	0.014 (0.11) [0.1]
$\alpha_{\ell,C}$	0.64*** (0.16) [3.9]	0.052* (0.028) [1.9]	0.0049 (0.046) [0.1]	0.045 (0.033) [1.4]	0.18** (0.082) [2.2]	0.0072 (0.027) [0.3]	-0.006 (0.042) [-0.1]	0.0046 (0.057) [0.1]	0.102 (0.102) [1]	0.19* (0.106) [1.8]
$\beta_{\ell,1}$	0.11*** (0.035) [3.2]	-0.024* (0.013) [-1.9]	0.035 (0.023) [1.5]	-0.025 (0.017) [-1.5]	0.13*** (0.021) [6.3]	-0.006 (0.021) [-0.3]	-0.12*** (0.025) [-4.7]	-0.097*** (0.025) [-3.9]	-0.23*** (0.044) [-5.1]	-0.3*** (0.07) [-4.2]
$\beta_{\ell,2}$	0.036* (0.02) [1.8]	-0.041*** (0.0108) [-3.8]	0.018 (0.03) [0.6]	-0.031 (0.023) [-1.3]	0.027* (0.015) [1.9]	-0.089*** (0.019) [-4.6]	-0.058* (0.03) [-1.9]	-0.093*** (0.016) [-5.7]	-0.13*** (0.038) [-3.5]	-0.092 (0.065) [-1.4]
$\beta_{\ell,3}$	0.28*** (0.13) [2.2]	-0.14*** (0.037) [-3.9]	-0.22*** (0.074) [-2.9]	-0.22*** (0.048) [-4.5]	0.16* (0.083) [1.9]	-0.22*** (0.054) [-4.1]	-0.24*** (0.081) [-3]	-0.13*** (0.048) [-2.7]	-0.26 (0.18) [-1.4]	-0.78*** (0.22) [-3.6]
$\beta_{\ell,4}$	0.14*** (0.043) [3.1]	0.054 (0.04) [1.3]	0.094** (0.047) [2]	0.044 (0.049) [0.9]	0.082*** (0.03) [2.8]	0.21*** (0.044) [4.8]	0.3*** (0.065) [4.7]	0.47*** (0.066) [7.2]	0.75*** (0.13) [5.6]	0.98*** (0.2) [4.9]
$\beta_{\ell,5}$	0.61*** (0.093) [6.5]	-0.23** (0.105) [-2.2]	-0.28* (0.16) [-1.7]	0.16 (0.16) [1]	-0.098 (0.106) [-0.9]	-0.52*** (0.15) [-3.4]	-0.97*** (0.22) [-4.3]	-1.7*** (0.32) [-5.4]	-3.4*** (0.58) [-5.7]	-3.5*** (0.83) [-4.2]
γ_{ℓ}^0		-0.067*** (0.023) [-2.9]	-0.096*** (0.031) [-3.1]	-0.093*** (0.025) [-3.7]	-0.076*** (0.012) [-6.5]	-0.14*** (0.023) [-6]	-0.22*** (0.027) [-8.2]	-0.2*** (0.034) [-5.9]	-0.37*** (0.058) [-6.4]	-0.49*** (0.14) [-3.4]
γ_{ℓ}^1		0.23** (0.094) [2.4]	0.53*** (0.14) [3.7]	0.29* (0.17) [1.7]	0.51*** (0.061) [8.3]	0.51*** (0.12) [4.4]	1.09*** (0.23) [4.9]	0.73*** (0.2) [3.7]	1.5*** (0.4) [3.8]	1.4** (0.68) [2.1]

Fig. 4: **Actual vs. model-implied spreads vs. Germany**

Notes: These plots compare observed (light-grey solid lines) and model-implied (dotted lines) spreads between the yields of 9 countries (+ KfW, a German agency) and their German counterparts. Two maturities are considered: 2 years (upper plot) and 10 years (lower plot). The black solid line is the model-implied contribution to the spreads of the liquidity factor λ_t^L (these contributions are computed as the spread that would prevail if the credit parts $\lambda_{h,t}^C$ of the debtor intensities were equal to zero). For KfW (upper-left plot), the fact that the dotted line and the black solid line are confounded results from the identification of the liquidity factor λ_t^L .



turns out that this European factor has some comovements with other proxies of liquidity pricing. Two such measures are displayed in Figure 5 (middle and lower plot). A first proxy, inspired by Manganelli and Wolswijk (2009) [61], consists of a dispersion measure of the bond yields of Aaa-rated countries. This proxy is based on the assumption according to which a significant share of the spreads between Aaa-rated countries should reflect liquidity differences since they are all supposed to have a very high credit quality.³⁴ The second liquidity proxy is the bid-ask spread on the 10-year French benchmark bond (lower plot in Figure 5). In addition to concomitant rises in the three proxies in early 2008, one can observe a common decreasing trend between the early 2000 and 2005.

The liquidity-related factor λ_t^ℓ presents three main humps: in the early 2000s, in 2008 and in 2010. The rise in liquidity premia in the early 2000s –concomitant with the collapse of the Internet bubble– is also found in U.S. data by Fontaine and Garcia (2009) [40], Longstaff (2004) [58] or Feldhütter and Lando (2008) [39]. The fact that the liquidity factor is particularly high during crises periods (burst of the dotcom bubble and post-Lehman periods) is consistent with the findings of Beber, Brandt and Kavajecz (2009) [9] who pinpoint that investors primarily chase liquidity during market-stress periods.³⁵

Given the liquidity-related factor λ_t^ℓ , it remains to perform the default/liquidity decompositions of the country-specific hazard rate (see equations 6 and 7). Specifically, we have to estimate the pair of parameters $(\gamma_{\ell,n}^0, \gamma_{\ell,n}^1)$ for each country n (recall that $\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \lambda_t^\ell$). Intuitively, we look for parameters $\gamma_{\ell,n}^0$'s and $\gamma_{\ell,n}^1$'s that are such that (a) an important share of the spread *fluctuations* is explained by the liquidity intensity $\lambda_{n,t}^\ell$ under the constraints that (b) the implied risk-neutral and historical PDs are positive and that (c) the liquidity-related parts of the spreads are positive. In order to achieve this for each country n , we construct a loss function \mathcal{L}_n that quantifies the previous objectives and we look for parameters $(\gamma_{\ell,n}^0, \gamma_{\ell,n}^1)$ that minimize this function. This procedure is detailed in Appendix E.

The estimated $\gamma_{\ell,n}^0$ and $\gamma_{\ell,n}^1$ are shown in the lower panel of Table 3. Note that these parameters are non-linear combinations of the parameters that were estimated in two steps of the estimation procedure. In particular, each $\gamma_{\ell,n}$ is largely dependent on the estimation of $\alpha_{KF\mathcal{W}}$ and $\beta_{KF\mathcal{W}}$ that define the liquidity-related factor λ_t^ℓ . The standard deviations of the estimated $\gamma_{\ell,n}$'s (reported in Table 3) result from the delta method, taking all these dependencies into account.³⁶

³⁴ To compute this proxy, we use sovereign yield data (the same as in the rest of the analysis) of Austria, Finland, France, Germany and the Netherlands, which are the five countries that remain Aaa-rated over the whole period.

³⁵ Such a behaviour is captured in a theoretical framework by Vayanos (2004) [74].

³⁶ We assume that the large covariance matrix of the parameter estimates obtained in the first step and in the second step of the estimation is block diagonal. This would be exact if both steps of the estimations were independent. This is not rigorously the case since the covariance matrices of the factor innovations $(\Omega(z_t)\Omega(z_t)')-$ are the same under both measures.

Fig. 5: Liquidity intensity λ_t^ℓ and liquidity-pricing proxies

Notes: The upper panel presents the estimate of λ_t^ℓ , which is the factor driving the country-specific illiquidity intensities $\lambda_{n,t}^\ell$ ($\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \lambda_t^\ell$, see Section 3.3). The shaded area corresponds to the 90% confidence band based on the covariance matrix of the parameter estimates presented in Table 3 (the delta method is employed, using the fact that at each point in time, the estimate of λ_t^ℓ is a function of the parameter estimates and of y_t and z_t : $\lambda_t^\ell = \alpha_\ell' z_t + \beta_\ell' y_t$). The confidence band does not take into account the uncertainty stemming from the estimation of the regime variable z_t . The middle plot presents a liquidity-pricing measure inspired by Manganelli and Wolswijk (2009) [61]: for each period t , it is the mean of the absolute values of the spreads between the 10-year Aaa-rated-country yields and their average. (The underlying assumption being that most of the spreads between Aaa countries should be liquidity-driven.) The lower plot shows the bid-ask spreads on the 10-year French benchmark bond (computed as the monthly medians of high-frequency trade data provided by Thomson Reuters Tick History).

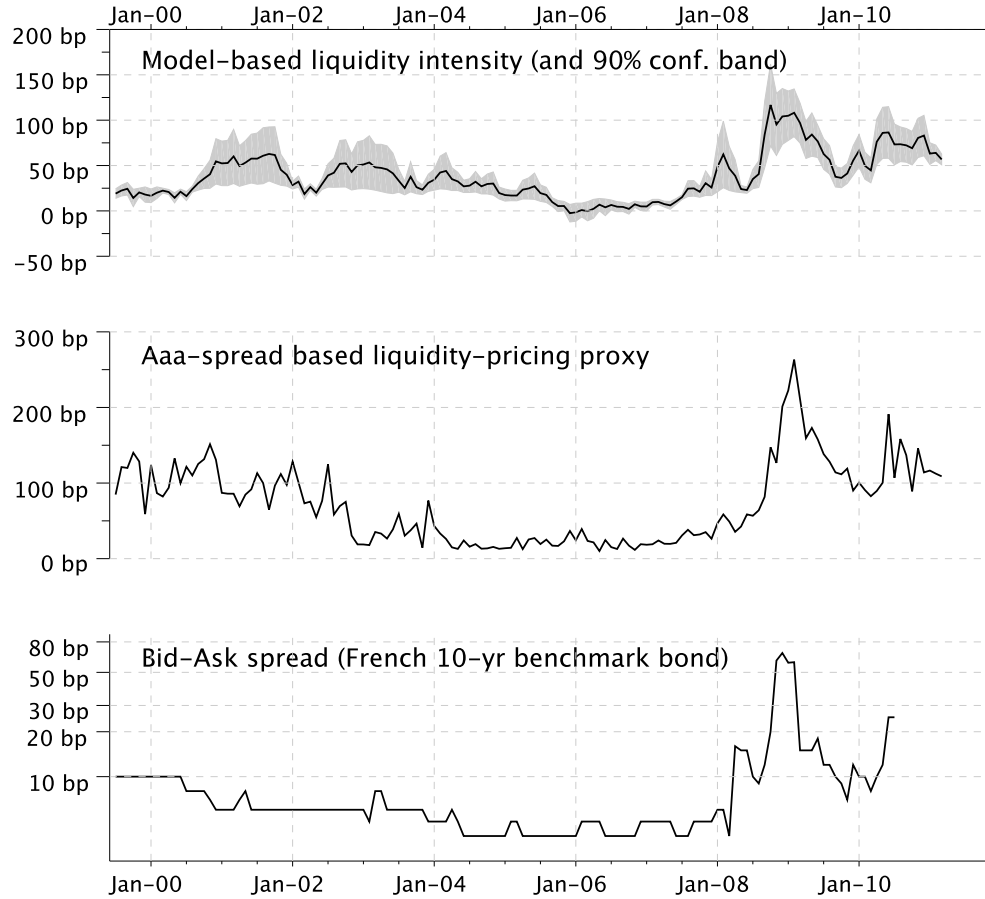


Fig. 6: **Sensitivity to the liquidity factor versus debt outstanding**

Notes: The coordinates of the countries correspond to (x) the sensitivities $\gamma_{\ell,n}^1$ of their hazard rates $\lambda_{n,t}^\ell$ to the liquidity factor λ_t^ℓ (these sensitivities are reported in the lowest row of Table 3) and (y) their total marketable sovereign debt (as of the end of 2009, Source: Eurostat).

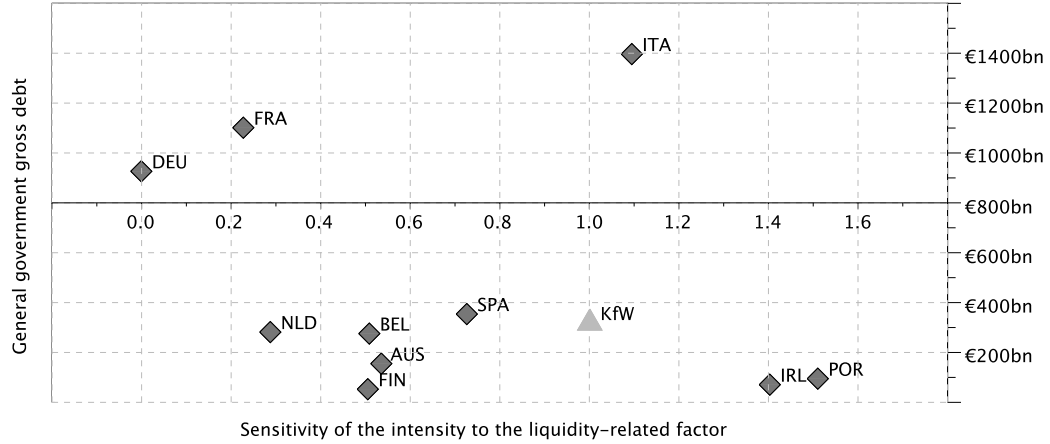


Figure 6 shows a scatter plot where the coordinates of the countries are the sensitivities $\gamma_{\ell,n}^1$ to the liquidity-related factor and the total marketable debt of the different countries. Leaving Italy aside, there seems to be a negative relationship between these sensitivities and the debt outstanding. In spite of the large size of the tradable debt issued by the Italian government, Italy's hazard rate appears to be particularly sensitive to the liquidity factor (among the countries considered in our subset, only Ireland and Portugal are more exposed than Italy to the liquidity factor).³⁷

Moreover, in order to gauge the relative importance of the liquidity-related part of the spreads, we have computed the spreads (versus German yields) that would prevail if the credit part of the countries' hazard rates were equal to zero. Figure 4 presents the resulting spreads (black solid lines). While, for most countries, the liquidity-related part of the spread is less important than the credit-related one (as in Codogno, Favero and Missale, 2003 [22]), it turns out to account for a substantial part of the changes in spreads, especially over the earlier part of the estimation sample.

6.2 Default probabilities

In the remaining of the paper, we show how our results can be exploited to compute the default probabilities implied by the yield data. In the spirit of Litterman and Iben (1991) [53], various methodologies that are widely used by practitioners or market analysts end up with risk-neutral PDs (see, e.g. Chan-Lau, 2006 [18]). In our framework, we can compute both the risk-neutral and

³⁷ To some extent, such a finding is consistent with the results of Chung-Cheung, de Jong and Rindi (2004) [21] according to which transitory costs would be more important in the Italian market, dominated by local traders.

the actual (or real-world) PDs. Our results suggest that there are deep differences between the risk-neutral and the historical dynamics of the factors driving hazard rates. Therefore, risk-neutral and actual PDs may differ considerably (see also Berg, 2009 [11]). This is illustrated in the sequel.

In our framework, the actual PD between time t and time $t + h$ is given by

$$\begin{aligned} P(d_{n,t+h} = 1 | d_{n,t} = 0) &= E_t(\mathbb{I}_{\{d_{n,t+h}=1\}} | d_{n,t} = 0) \\ &= 1 - E_t(\mathbb{I}_{\{d_{n,t+h}=0\}} | d_{n,t} = 0) \\ &= 1 - E_t(\exp(-\lambda_{n,t+1}^d - \dots - \lambda_{n,t+h}^d)). \end{aligned} \quad (16)$$

We are then left with the computation of the survival probability $E_t(\exp(-\lambda_{n,t+1}^d - \dots - \lambda_{n,t+h}^d))$. Recall that $\exp(-\lambda_{n,t}^c) = \exp(-\lambda_{n,t}^d) + \zeta[1 - \exp(-\lambda_{n,t}^d)]$. When $\lambda_{n,t}^d$ is small, the first order approximation leads to:

$$\lambda_{n,t}^d \simeq \frac{1}{1-\zeta} \lambda_{n,t}^c = \frac{1}{1-\zeta} [(\alpha_n^c)' z_t + (\beta_n^c)' y_t]. \quad (17)$$

Up to this approximation, the survival probability is a multi-horizon Laplace transform of a compound auto-regressive process of order one. In the same way as for the yields, the recursive algorithm detailed in Appendix A.2 can be used in order to compute these probabilities. In the computation, we use a constant recovery rate of 50%, which is a rough average of the recovery rates observed for sovereign defaults over the last decade (see Moody's, 2010 [68]).

Figure 7 shows the model-based 5-year probabilities of default (i.e. the probabilities that the considered countries will default during the next 5 years). Confidence intervals at the 95% level are also reported. The computation of the confidence intervals is based on the first-order Taylor expansion of the default probabilities. More precisely, equations (16) and (17), completed with the decomposition process of the hazard rates (as detailed in 6.1) show how the PDs depend on the estimated parameters. This relationship is numerically differentiated. Having an estimate of the asymptotic distribution of the parameters, we can deduce these of the PDs.³⁸ As can be seen in Figure 7, when the estimation leads to negative PDs, these turn out to be not, or weakly, significant.

³⁹ Except for the most indebted countries and for the recent period (Italy, Spain, Portugal and Ireland, during 2009-2010), the PDs are not often statistically different from zero.

Figure 8 presents the model-implied term-structure of PDs as of January 2006 and January

³⁸ While these incorporate (notably) the uncertainty regarding the estimation of the liquidity effect on the yields, they do not account for the uncertainty concerning (a) the estimation of the regime variables z_t and (b) the recovery rate.

³⁹ Since, by construction, the outputs are PDs *with respect to Germany*, negative figures can not be ruled out. However, given the safe-haven status of Germany, such values are unlikely (which is exploited in our estimation process by penalizing negative PDs).

2011. For most countries and especially for the more indebted ones, the term-structure of the PDs is higher and steeper in early 2011 than in January 2006.

Finally, it is worth noting that even when taking into account the uncertainty regarding the estimated real-world PDs, the gap between these and their risk-neutral counterparts is significant in many cases, particularly for the most recent years (see Figure 9). Note that risk-neutral PDs are extensively used, notably by market practitioners. This mainly stems from the fact that risk-neutral PDs are relatively easy to compute, using basic methods inspired by the one proposed by Litterman and Iben (1991) [53]⁴⁰. To illustrate, Figure 10 compares the PDs estimates derived from our model with alternative estimates, as of the end of 2011 Q1. Two kinds of alternative estimates are considered: (a) PDs that are based on the Moody’s credit ratings and the associated matrix of long-run credit-rating-migration probabilities and (b) risk-neutral probabilities computed by CMA Datavision (2011) [28]. Figure 10 shows that our estimates lie somewhere between the two others.⁴¹ In addition, it turns out that our risk-neutral PDs (the triangles) are relatively close to the risk-neutral CDS-based ones computed by CMA.⁴²

7 Conclusion

In this paper, we present a no-arbitrage model of the joint dynamics of euro-area sovereign yield curves. In addition to five Gaussian shocks, the model includes a regime-switching feature that makes it possible to distinguish between tranquil and crisis periods. Such a regime-switching feature is well suited to account for the recent/current economic and financial market stress times. As a source of systematic risk, the regime shifts are priced by investors. Quasi-explicit formulas are available, which makes the model tractable and the estimation feasible. The model is estimated over the last twelve years. The resulting fit is satisfying since the standard deviation of the yields pricing errors –across countries and maturities– is of 18 basis points. Our estimation suggests that the regimes are key in explaining the fluctuations of yields over the last three years.⁴³ Further, some credit and liquidity intensities are estimated for each European country included in our dataset. The liquidity intensities are driven by a single European factor whose identification is based on

⁴⁰ In particular, these methods do not care about liquidity-pricing effects.

⁴¹ The fact that our estimates tend to be higher the rating-based ones was expected: while the rating-based probabilities are “through-the-cycle” ones (see e.g. Löffler, 2004 [56]), our probabilities take the specific crisis context of 2011Q1 into account.

⁴² The remaining differences between the latter two risk-neutral estimates may be attributed to (i) the fact that we consider spreads w.r.t. Germany in our methods while the CMA’s method involves “absolute” CDS, (ii) the absence of treatment of liquidity-pricing effects in the CMA methodology (while empirical evidence suggests that CDS contain liquidity premia, see Buhler and Trapp, 2008 [15]) or also to (iii) our methodology’s measurement errors (see Figure 4).

⁴³ Counterfactual experiments –whose results are not reported here– have been conducted to gauge the impact of the crisis regime on model-implied yields: when the crisis periods are replaced by no-crisis ones, simulated (counterfactual) spreads remain flat from 2008 onwards.

the KfW-*Bund* spreads. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than their sovereign counterparts –the *Bunds*– but are less liquid. Therefore, the KfW-*Bund* spread should be essentially liquidity-driven. Our results indicate that a substantial part of intra-euro spreads is liquidity-driven. The remaining parts of the spreads reflect credit-risk pricing. Given some assumptions regarding the recovery process, our framework makes it possible to decompose the credit part of the spreads between actual, or real-world, probabilities of default on the one hand and risk premia on the other hand. To that respect, our results suggest that actual PDs are often significantly lower than their risk-neutral counterparts. According to these results, relying on risk-neutral PDs to assess the market participants expectations regarding future sovereign defaults would be misleading.

Fig. 7: Default probabilities estimates (5-year horizon)

Notes: These plots display the model-implied 10-year default probabilities. Formally, they correspond to the time series of $E_t(\mathbb{I}\{d_{n,t+5yrs} = 0\} | d_{n,t} = 0)$, where E_t denotes the expectation (under the historical measure) conditional to the information available at time t . The grey-shaded area correspond to the ± 1 standard deviation area. These standard deviations are based on the covariance matrices of the parameter estimates obtained in the two consecutive steps of the estimation procedure. They notably incorporate the uncertainty regarding the estimation of the liquidity effect on the yields. See Section 6.2 for mor details about the computation of these default probabilities. (Note that the standard deviations do not account for the uncertainty concerning (a) the estimation of the regime variables and z_t (b) the recovery rate.)

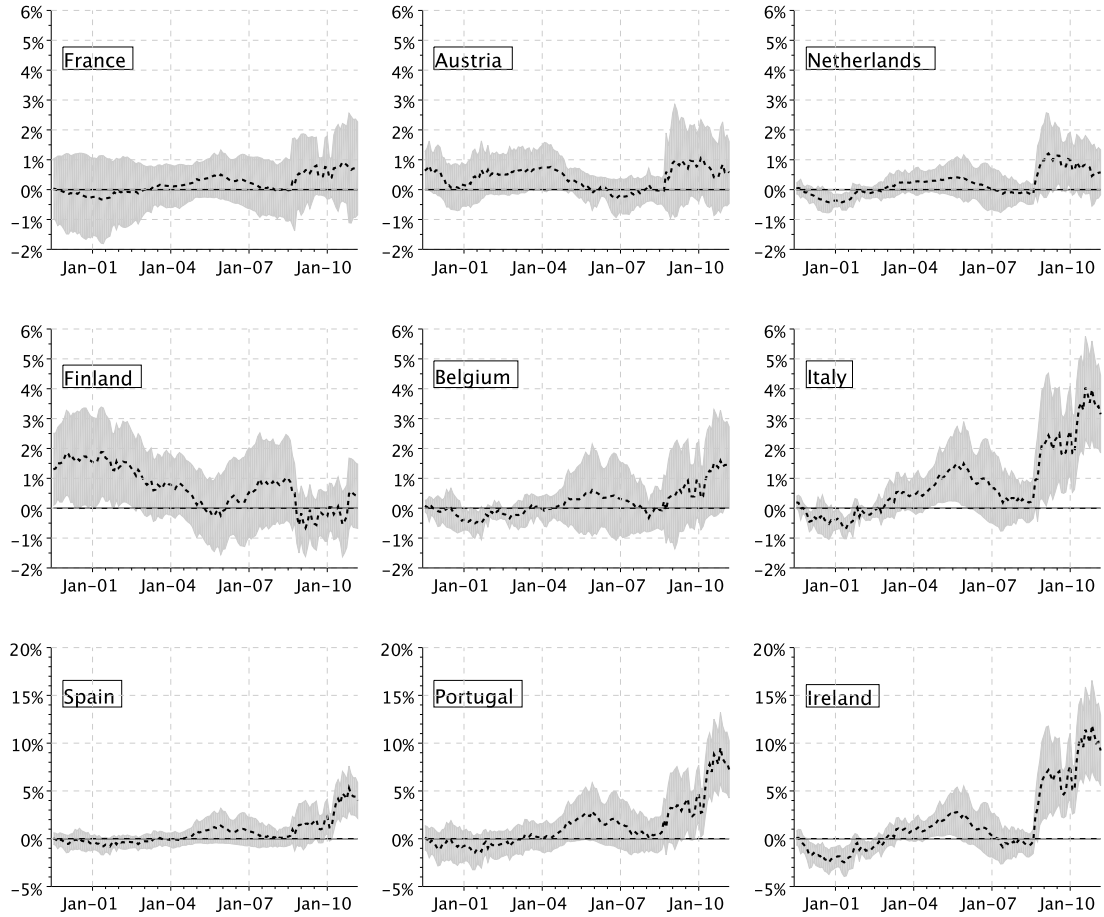


Fig. 8: Term structure of default probabilities

Notes: These plot display the term structure of the default probabilities for two distinct months (January 2003 and October 2010). Formally, for month t and debtor n , the plot shows $E_t(\mathbb{I}\{d_{n,t+h} = 0\} | d_{n,t} = 0)$ for h between 1 month and 10 years (where E_t denotes the expectation –under the historical measure– conditional to the information available at time t). The grey-shaded areas correspond to the ± 1 standard deviation area. These standard deviations are based on the covariance matrices of the parameter estimates obtained in the two consecutive steps of the estimation procedure. They notably incorporate the uncertainty regarding the estimation of the liquidity effect on the yields. See Section 6.2 for mor details about the computation of these default probabilities. (Note that the standard deviations do not account for the uncertainty concerning (a) the estimation of the regime variables z_t and (b) the recovery rate.)

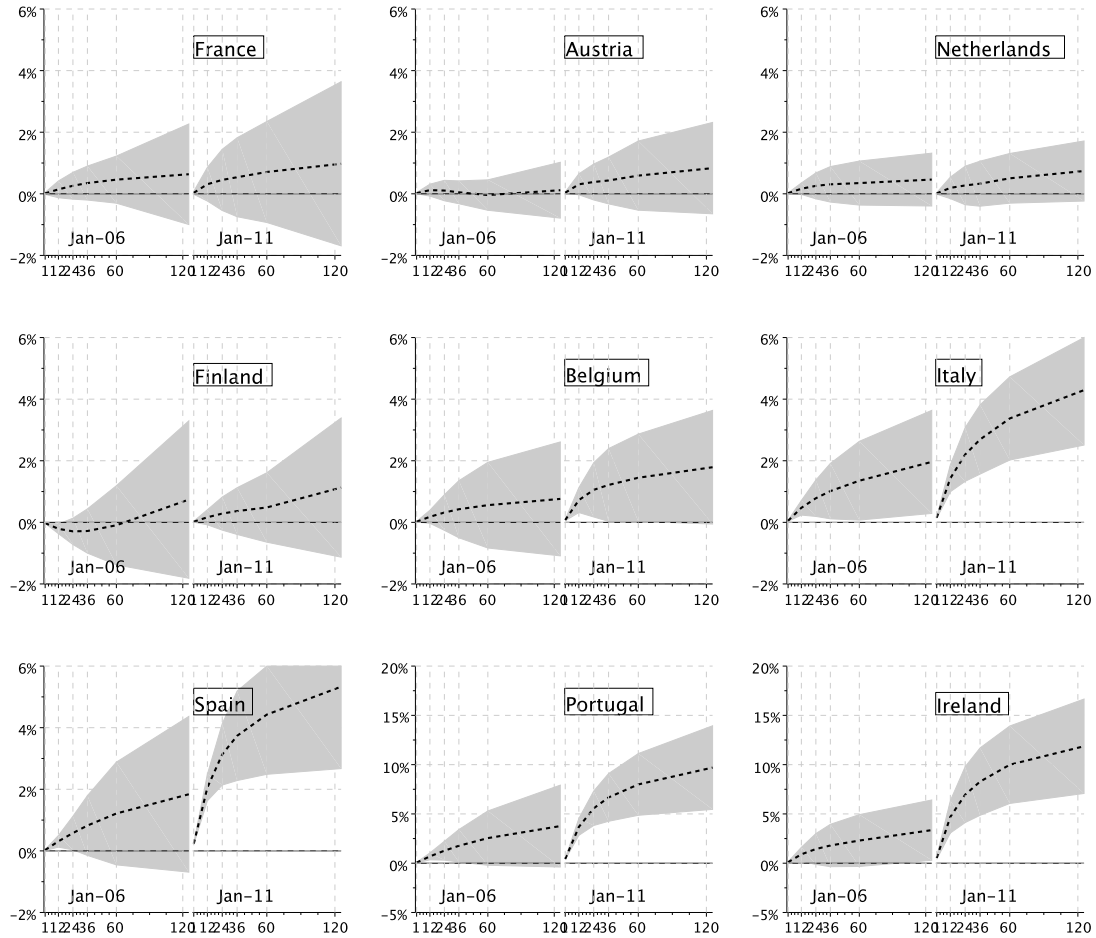


Fig. 9: Default probabilities estimates (5-year horizon)

Notes: These plots display the model-implied 5-year default probabilities under the historical measure (dotted line) and under the risk-neutral measure (black solid line). Formally, the dotted line corresponds to the time series of $E_t(\mathbb{I}\{d_{n,t+5yrs} = 0\} | d_{n,t} = 0)$, where E_t denotes the expectation (under the historical measure) conditional to the information available at time t (see Section 6.2 for the computation of these default probabilities). The black solid line represents $E_t^Q(\mathbb{I}\{d_{n,t+5yrs} = 0\} | d_{n,t} = 0)$. For historical probabilities (dotted line), we report the ± 1 standard-deviation area.

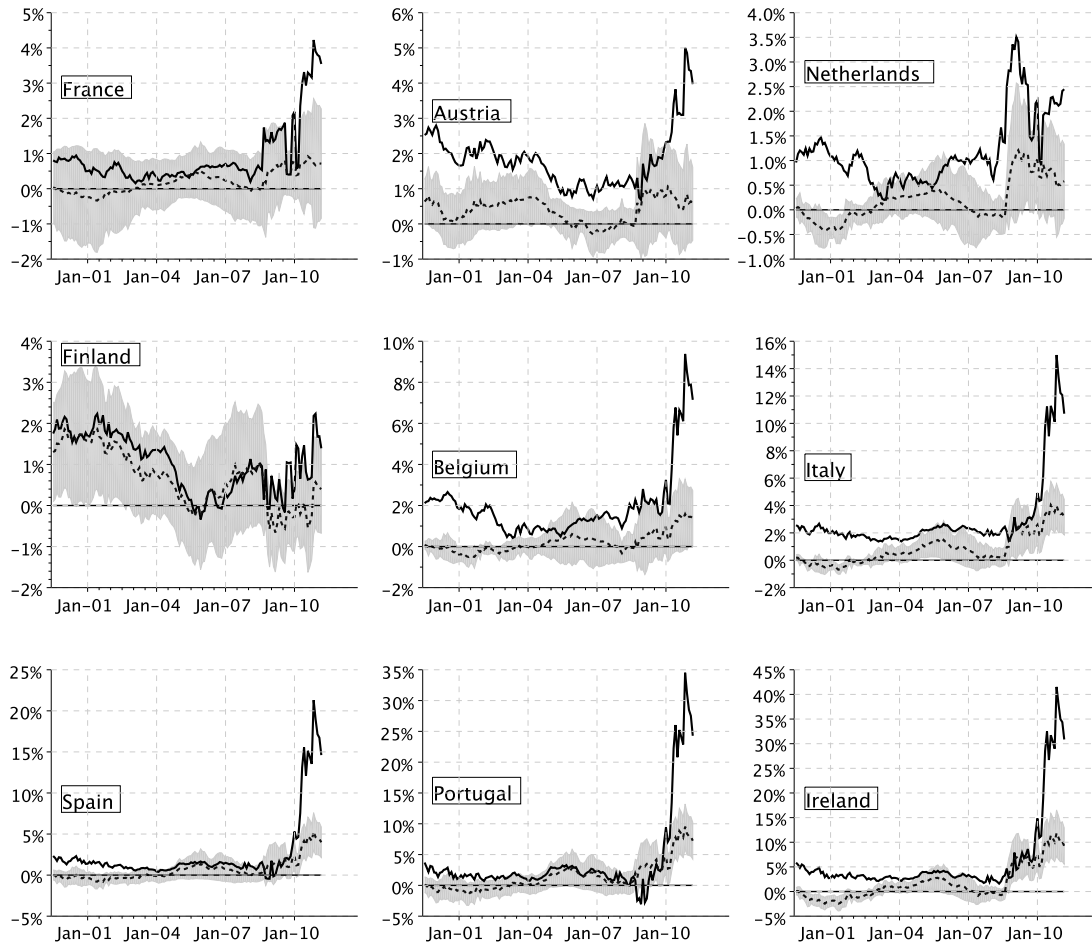
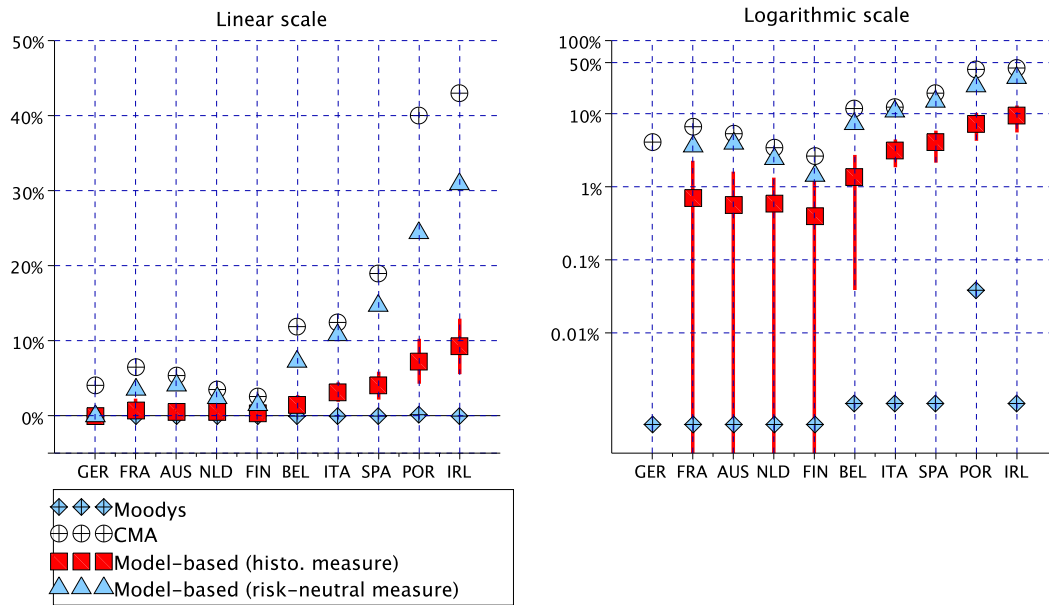


Fig. 10: **Default probabilities estimates (5-year horizon)**

Notes: These plots display different estimates of probabilities of default (PD) of 10 euro-area governments (as of the end of 2011Q1). The two plots show the same data, the right-hand-side chart using a logarithmic scale. The squares and the triangles correspond to outputs of our model. While the squares indicate “real-world” PDs (i.e. the default probabilities obtained under the physical, or historical, measure), the triangles are risk-neutral PDs. The vertical red bars delineate the 95% confidence intervals of the physical PD estimates. The circles indicate the PDs computed by CMA, using an industry standard model and proprietary CDS data from CMA Datavision (2011) [28]. The diamonds correspond to PDs that derive from (a) the Moody’s ratings of the countries (as of 2011Q1) and (b) the matrix of credit-rating-migration probabilities given by Moody’s (2010) [68].



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A Proofs

A.1 Laplace transform of (z_t, y_t)

The risk-neutral conditional Laplace transform of (z_t, y_t) the information available in time $t - 1$ is:

$$\varphi_{t-1}^{\mathbb{Q}}(u, v) = \exp(v' \Phi^* y_{t-1} + [l_1, \dots, l_J] z_{t-1}), \quad (18)$$

where $l_i = \log \sum_{j=1}^J \pi_{ij}^* \exp \{u_i + v' \mu^* e_j + \frac{1}{2} v' \Omega(e_j) \Omega'(e_j) v\}$ and where e_j is the j^{th} column of the identity matrix. Therefore, (z_t, y_t) is compound auto-regressive of order one –denoted by Car(1)– under the risk-neutral measure.

Proof. We have

$$\begin{aligned} \varphi_{t-1}^{\mathbb{Q}}(u, v) &= E_{t-1}^{\mathbb{Q}}(\exp[u' z_t + v' y_t]) \\ &= E_{t-1}^{\mathbb{Q}}(\exp[u' z_t + v' \mu^* z_t + v' \Phi^* y_{t-1} + v' \Omega(z_t) \varepsilon_t]) \\ &= E_{t-1}^{\mathbb{Q}}\left(E_{t-1}^{\mathbb{Q}}\{\exp[u' z_t + v' \mu^* z_t + v' \Phi^* y_{t-1} + v' \Omega(z_t) \varepsilon_t] \mid z_t\}\right) \\ &= \exp(v' \Phi^* y_{t-1}) E_{t-1}^{\mathbb{Q}}(\exp\{u' z_t + v' \mu^* z_t\} \times \\ &\quad E_{t-1}^{\mathbb{Q}}(\exp\{v' \Omega(z_t) \varepsilon_t \mid z_t\})) \\ &= \exp(v' \Phi^* y_{t-1}) E_{t-1}^{\mathbb{Q}}(\exp\{u' z_t + v' \mu^* z_t\} \times \\ &\quad \frac{1}{2} v' \Omega(z_t) \Omega'(z_t) v) \\ &= \exp(v' \Phi^* y_{t-1} + [l_1, \dots, l_J] z_{t-1}). \end{aligned}$$

Using the expression given above for the l_i 's leads to the result. \square

A.2 Multi-horizon Laplace transform of a Car(1) process

Let us consider a multivariate Car(1) process Z_t and its conditional Laplace transform given by $\exp[a'(s)Z_t + b(s)]$. Let us further denote by $L_{t,h}(\omega)$ its multi-horizon Laplace transform given by:

$$L_{t,h}(\omega) = E_t[\exp(\omega'_{H-h+1} Z_{t+1} + \dots + \omega'_H Z_{t+h})], \quad t = 1, \dots, T, \quad h = 1, \dots, H,$$

where $\omega = (\omega'_1, \dots, \omega'_H)$ is a given sequence of vectors. We have, for any t ,

$$L_{t,h}(\omega) = \exp(A'_h Z_t + B_h), \quad h = 1, \dots, H,$$

where the sequences $A_h, B_h, h = 1, \dots, H$ are obtained recursively by:

$$\begin{aligned} A_h &= a(\omega_{H-h+1} + A_{h-1}) \\ B_h &= b(\omega_{H-h+1} + A_{h-1}) + B_{h-1}, \end{aligned}$$

with the initial conditions $A_0 = 0$ and $B_0 = 0$.

Proof. The formula is true for $h = 1$ since:

$$L_{t,1}(\omega) = E_t(\omega'_H Z_{t+1}) = \exp[a'(\omega_H) Z_t + b(\omega_H)]$$

and therefore $A_1 = a(\omega_H)$ and $B_1 = b(\omega_H)$.

if it is true for $h - 1$, we get:

$$\begin{aligned}
L_{t,h}(\omega) &= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1}) E_{t+1} (\omega'_{H-h+2} Z_{t+2} + \dots + \omega'_H Z_{t+H}) \right] \\
&= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1}) L_{t+1,h-1}(\omega) \right] \\
&= E_t \left[\exp(\omega'_{H-h+1} Z_{t+1} + A_{h-1} Z_{t+1} + B_{h-1}) \right] \\
&= \exp[a(\omega'_{H-h+1} + A_{h-1}) Z_t + b(\omega'_{H-h+1} + A_{h-1}) + B_{h-1}]
\end{aligned}$$

and the result follows. \square

B Pricing of defaultable bonds

In the current appendix, we present conditions under which one can derive formulas for nonzero-recovery-rate bond pricing. The set-up is the following: If a debtor n defaults between $t - 1$ and t (with $t < T$, where T denotes the contractual maturity of a bond issued by this debtor), recovery is assumed to take place at time t . In addition, we assume that the recovery payoff –i.e. one minus the loss-given-default– is a constant fraction, denoted by ζ , of the price that would have prevailed in the absence of default. Such an assumption is termed with “recovery of market value” assumption by Duffie and Singleton (1999) [33].

Let us consider the price $B_n^{DR}(T - 1, 1)$, in period $T - 1$, of a one-period nonzero-recovery-rate bond issued by a given debtor (before $T - 1$). Assume that debtor has not defaulted before $T - 1$, then:

$$\begin{aligned}
B_n^{DR}(T - 1, 1) &= \exp(-r_T) E^{\mathbb{Q}} \left[\mathbb{I}_{\{d_{n,T}=0\}} + \mathbb{I}_{\{d_{n,T}=1\}} \zeta \mid \underline{z}_{T-1}, \underline{y}_{T-1}, d_{n,T-1} = 0 \right] \\
&= \exp(-r_T) E^{\mathbb{Q}} \left[E^{\mathbb{Q}} \left(\mathbb{I}_{\{d_{n,T}=0\}} + \mathbb{I}_{\{d_{n,T}=1\}} \zeta \mid \underline{z}_T, \underline{y}_T, d_{n,T-1} = 0 \right) \mid \underline{z}_{T-1}, \underline{y}_{T-1}, d_{n,T-1} = 0 \right] \\
&= \exp(-r_T) E^{\mathbb{Q}} \left[\exp(-\tilde{\lambda}_{n,T}) + \left(1 - \exp(-\tilde{\lambda}_{n,T}) \right) \zeta \mid \underline{z}_{T-1}, \underline{y}_{T-1}, d_{n,T-1} = 0 \right].
\end{aligned}$$

and, defining the random variable $\lambda_{n,T}$ by $\exp(-\lambda_{n,T}) = \exp(-\tilde{\lambda}_{n,T}) + \left(1 - \exp(-\tilde{\lambda}_{n,T}) \right) \zeta$, we have: $B_n^{DR}(T - 1, 1) = E^{\mathbb{Q}} \left[\exp(-r_T - \lambda_{n,T}) \mid \underline{z}_{T-1}, \underline{y}_{T-1} \right]$.

Further, let us consider the price of the same bond in period $T - 2$. Assuming that there was no default before $T - 2$:

$$\begin{aligned}
B_n^{DR}(T - 2, 2) &= \exp(-r_{T-1}) \times \\
&\quad E^{\mathbb{Q}} \left[\left(\mathbb{I}_{\{d_{n,T-1}=0\}} + \mathbb{I}_{\{d_{n,T-1}=1\}} \zeta \right) E^{\mathbb{Q}} \left[\exp(-r_T - \lambda_{n,T}) \mid \underline{z}_{T-1}, \underline{y}_{T-1} \right] \right. \\
&\quad \left. \mid \underline{z}_{T-2}, \underline{y}_{T-2}, d_{n,T-2} = 0 \right] \tag{19}
\end{aligned}$$

$$= \exp(-r_{T-1}) \times \tag{20}$$

$$E^{\mathbb{Q}} \left[E^{\mathbb{Q}} \left[\left(\mathbb{I}_{\{d_{n,T-1}=0\}} + \mathbb{I}_{\{d_{n,T-1}=1\}} \zeta \right) \exp(-r_T - \lambda_{n,T}) \mid \underline{z}_{T-1}, \underline{y}_{T-1} \right] \right. \tag{21}$$

$$\left. \mid \underline{z}_{T-2}, \underline{y}_{T-2}, d_{n,T-2} = 0 \right] \tag{22}$$

$$= E^{\mathbb{Q}} \left[\exp(-r_T - \lambda_{n,T} - r_{T-1} - \lambda_{n,T-1}) \mid \underline{z}_{T-2}, \underline{y}_{T-2} \right] \tag{23}$$

where $\lambda_{n,T-1}$ is defined through $\exp(-\lambda_{n,T-1}) = \exp(-\tilde{\lambda}_{n,T-1}) + \left(1 - \exp(-\tilde{\lambda}_{n,T-1}) \right) \zeta$.

Applying this methodology recursively, it is easily seen that the price of a nonzero-recovery-rate defaultable bond of maturity h is given by (assuming no default before t , i.e. conditionally on $d_{n,t} = 0$):

$$B_n^{DR}(t, h) = E^{\mathbb{Q}} \left[\exp(-r_{t+h} - \dots - r_{t+1} - \lambda_{n,t+h} - \dots - \lambda_{n,t+1}) \mid \underline{z}_t, \underline{y}_t \right] \tag{24}$$

where the $\lambda_{n,t+i}$'s are such that $\exp(-\lambda_{n,t+i}) = \exp(-\tilde{\lambda}_{n,t+i}) + \left(1 - \exp(-\tilde{\lambda}_{n,t+i}) \right) \zeta$.

C Sovereign yield data

The estimation of the model requires zero-coupon yields. However, governments usually issue coupon-bearing bonds. For Germany, France, Spain and Netherlands, we bootstrap constant-maturity coupon yield curves provided by Barclays Capital.⁴⁴ For Belgium, we use zero-coupon yields computed by the National Bank of Belgium and made available by the BIS. For remaining countries, we resort to a parametric approach (see BIS, 2005 [13], for an overview of zero-coupon estimation methods). The yield curves are derived from bond pricing data on regularly replenished populations of sovereign bonds. We choose the parametric form originally proposed by Nelson and Siegel (1987) [69]. Specifically, the yield of a zero-coupon bond with a time to maturity m for a point in time t is given by:⁴⁵

$$R_t^m(\theta) = \beta_0 + \beta_1 \left(-\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_2 \left[\left(\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \exp\left(-\frac{m}{\tau_1}\right)\right]$$

where Θ is the vector of parameters $[\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]'$. Assume that, for a given country and a given date t , we dispose of observed prices of N coupon-bearing bonds (with fixed coupon), denoted by $P_{1,t}, P_{2,t}, \dots, P_{N,t}$. Let us denote by $CF_{k,i,t}$ the i^{th} (on n_k) cash flows that will be paid by the k^{th} bond at the date $\tau_{k,i}$. We can use the zero-coupon yields $\{R_t^m(\Theta)\}_{m \geq 0}$ to compute a modeled (dirty) price $\hat{P}_{k,t}$ for this k^{th} bond:

$$\hat{P}_{k,t}(\Theta) = \sum_{i=1}^{n_k} CF_{k,i,t} \exp\left(-\tau_{k,i} R_t^{\tau_{k,i}-t}(\Theta)\right).$$

The approach then consists in looking for the vector Θ that minimizes the distance between the N observed prices and modeled bond prices. Specifically, the vector Θ_t is given by:

$$\Theta_t = \arg \min_{\Theta} \sum_{k=1}^N \omega_k (P_{k,t} - \hat{P}_{k,t}(\Theta))^2$$

where the ω_k 's are some weights that are chosen with respect to the preferences that one may have regarding the fit of different parts of the yield curve. Intuitively, taking the same value for all the ω_k 's would lead to large yield errors for financial instruments with relatively short remaining time to maturity. This is linked to the concept of duration (i.e. the elasticity of the price with respect to one plus the yield): a given change in the yield corresponds to a small/large change in the price of a bond with a short/long term to maturity or duration. Since we do not want to favour a particular segment of the yield-curve fit, we weight the price error of each bond by the inverse of the remaining time to maturity.⁴⁶

Coupon-bond prices come from Datastream.⁴⁷ In the same spirit as Gurkaynak et al. (2005) [43], different filters are applied in order to remove those prices that would obviously bias the obtained yields. In particular, the prices of bonds that were issued before 1990 or that have atypical coupons (below 1% or above 10%) are excluded. In addition, the prices of bonds that have a time to maturity lower than 1 month are excluded.⁴⁸

⁴⁴ For details about bootstrapping methods, see e.g. Martellini, Priaulet and Priaulet (2003) [62]

⁴⁵ We use the Nelson-Siegel form rather than the extended version of Svensson (1994) [73] because the latter requires more data to be estimated properly (and for some countries and some dates, we have too small a number of coupon-bond prices).

⁴⁶ Using remaining time to maturity instead of duration has not a large effect on estimated yields as long as we are not concerned with the very long end of the yield curve.

⁴⁷ Naturally, the number of bonds used differ among the countries (from 19 bonds for the Netherlands to 175 bonds for Germany).

⁴⁸ The trading volume of a bond usually decreases considerably when it approaches its maturity date.

D Computation of the covariance matrix of the parameter estimates

The second step of the estimation deals with the parameters defining the risk-neutral dynamics of (z_t, y_t) and the parameterization of the hazard rates. In this appendix, we detail how the covariance matrix of these estimates is derived. The non-linear least square estimator $\hat{\theta}$ is defined by (this is equation 14):

$$\hat{\theta} = \arg \min_{\theta} \sum_{n,t,h} \left(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta) \right)^2 + \chi \sum_{t,i} (\tilde{y}_{i,t} - y_{i,t}(\theta))^2$$

where $y_{i,t}(\theta)$ is the i^{th} entry of the vector of “theoretical” factors, in the sense that it is a linear combination of the “theoretical” yields $R_{n,t,h}(\theta)$, that are themselves a combination of observed factors \tilde{y}_t .

This estimator must satisfy the first-order conditons:

$$\sum_{n,t,h} \frac{\partial R_{n,t,h}(\theta)}{\partial \theta} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta)) + \chi \sum_{t,i} \frac{\partial y_{i,t}(\theta)}{\partial \theta} (\tilde{y}_{i,t} - y_{i,t}(\theta)) = 0,$$

where the left-hand side of the previous equation is of dimension $k \times 1$ (the dimension of vector θ). The Taylor expansion of the previous equation in a neighborhood of the limit value θ_0 leads to (after multiplication by $1/\sqrt{T}$):

$$\begin{aligned} 0 &\simeq \frac{1}{\sqrt{T}} \left[\sum_{n,t,h} \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) + \chi \sum_{t,i} \frac{\partial y_{i,t}(\theta_0)}{\partial \theta} (\tilde{y}_{i,t} - y_{i,t}(\theta_0)) \right] + \\ &\quad \sqrt{T} (\hat{\theta} - \theta_0) \left[\frac{1}{T} \sum_{n,t,h} \left[\frac{\partial^2 R_{n,t,h}(\theta_0)}{\partial \theta \partial \theta'} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) - \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} \left(\frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} \right)' \right] + \right. \\ &\quad \left. \frac{1}{T} \chi \sum_{t,i} \left[\frac{\partial^2 y_{i,t}(\theta_0)}{\partial \theta \partial \theta'} (\tilde{y}_{i,t} - y_{i,t}(\theta_0)) - \frac{\partial y_{i,t}(\theta_0)}{\partial \theta} \left(\frac{\partial y_{i,t}(\theta_0)}{\partial \theta} \right)' \right] \right]. \end{aligned}$$

Since $E(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) = 0$ and $E(\tilde{y}_{i,t} - y_{i,t}(\theta_0)) = 0$ (for any i), we have

$$\begin{aligned} \frac{1}{T} \sum_{n,t,h} \frac{\partial^2 R_{n,t,h}(\theta_0)}{\partial \theta \partial \theta'} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) &\xrightarrow{a.s.} 0, \\ \frac{1}{T} \sum_{t,i} \frac{\partial^2 y_{i,t}(\theta_0)}{\partial \theta \partial \theta'} (\tilde{y}_{i,t} - y_{i,t}(\theta_0)) &\xrightarrow{a.s.} 0. \end{aligned}$$

Therefore:

$$\begin{aligned} \sqrt{T} (\hat{\theta} - \theta_0) &\simeq \left[\frac{1}{T} \sum_{n,t,h} \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} \left(\frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} \right)' + \frac{1}{T} \chi \sum_{t,i} \frac{\partial y_{i,t}(\theta_0)}{\partial \theta} \left(\frac{\partial y_{i,t}(\theta_0)}{\partial \theta} \right)' \right]^{-1} \times \\ &\quad \frac{1}{\sqrt{T}} \left[\sum_{n,t,h} \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) + \chi \sum_{t,i} \frac{\partial y_{i,t}(\theta_0)}{\partial \theta} (\tilde{y}_{i,t} - y_{i,t}(\theta_0)) \right]. \end{aligned}$$

Hence, the asymptotic distribution of $\sqrt{T} (\hat{\theta} - \theta_0)$ is estimated by $\hat{\mathcal{J}}^{-1} \hat{\mathcal{Z}} \hat{\mathcal{J}}^{-1}$ where:

$$\hat{\mathcal{J}}^{-1} = \left[\frac{1}{T} \sum_{n,t,h} \frac{\partial R_{n,t,h}(\hat{\theta})}{\partial \theta} \left(\frac{\partial R_{n,t,h}(\hat{\theta})}{\partial \theta} \right)' + \frac{1}{T} \chi \sum_{t,i} \frac{\partial y_{i,t}(\hat{\theta})}{\partial \theta} \left(\frac{\partial y_{i,t}(\hat{\theta})}{\partial \theta} \right)' \right]^{-1}.$$

The second matrix, denoted by $\hat{\mathcal{I}}$, is the estimate of the covariance matrix of $1/\sqrt{T} \sum_t \gamma_t(\theta_0)$ where

$$\gamma_t = \sum_{n,h} \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) + \chi \sum_i \frac{\partial y_{i,t}(\theta_0)}{\partial \theta} (\tilde{y}_{i,t} - y_{i,t}(\theta_0)).$$

To compute $\hat{\mathcal{I}}$, we use the Newey-West (1987) [70] HAC estimator. This estimate is given by:

$$\hat{\mathcal{I}} = \sum_{i=-(T-m+1)}^{i=T-m-1} \kappa\left(\frac{i}{m}\right) \hat{cov}(\hat{\gamma}_t, \hat{\gamma}_{t+i})$$

where $\hat{\gamma}_t = \gamma_t(\hat{\theta})$ and where \hat{cov} denotes the sample covariance operator. In practice, we use the Bartlett kernel $\kappa(x) = 1 - |x|$ and a bandwidth of 5.

E Disentangling credit from liquidity risks: the loss function

In that appendix, we details the loss function introduced in 6.1. This function is aimed at being minimized in order to find pairs of $(\gamma_{\ell,n}^0, \gamma_{\ell,n}^1)$ that are such that (a) an important share of the spread *fluctuations* is explained by the liquidity intensity $\lambda_{n,t}^\ell$ under the constraints that (b) the implied risk-neutral and historical PDs are positive and that (c) the liquidity-related parts of the spreads are positive. Actually, an additional “shadow” parameter is introduced in the loss function to account for the fact that objective (a) focuses on the fluctuations and not on the level the spread (this will be clarified below). We consider linearized versions of the spreads in order to facilitate the optimization. This considerably fasten the optimization to the extent that (1) it avoids computations of multi-horizon Laplace transforms defined by (26) at each evaluation of the loss function and (2), it implies that analytical derivatives of the loss functions are available (which is particularly welcome when implementing the delta method to get standard deviations of the estimated $\gamma_{\ell,n}^0$ and $\gamma_{\ell,n}^1$). Formally, we define the following loss function \mathcal{L}_n for each country n :

$$\begin{aligned} \mathcal{L}_n(\delta_0, \delta_1, \delta_2) = & \sum_t \left\{ \left(\overline{\lambda_{n,t}}^{\mathbb{Q}} - \left\{ \delta_0 + \delta_1 \overline{\lambda_{n,t}}^{\ell \mathbb{Q}} \right\} + \delta_2 \right)^2 \right. \\ & + \chi_1 \left[\left(\left[\overline{\lambda_{n,t}}^{\mathbb{P}} - \left\{ \delta_0 + \delta_1 \overline{\lambda_{n,t}}^{\ell \mathbb{P}} \right\} \right]_- \right)^2 + \left(\left[\overline{\lambda_{n,t}}^{\mathbb{Q}} - \left\{ \delta_0 + \delta_1 \overline{\lambda_{n,t}}^{\ell \mathbb{Q}} \right\} \right]_- \right)^2 \right] \\ & \left. + \chi_2 \left(\left[\delta_0 + \delta_1 \overline{\lambda_{n,t}}^{\ell \mathbb{Q}} \right]_- \right)^2 \right\} \end{aligned} \quad (25)$$

where $[x]_-$ is equal to x if $x < 0$ and 0 otherwise, and where the operator $\overline{\bullet}^{\mathbb{Q}}$ is defined by (for any time series x):

$$\overline{x_t}^{\mathbb{Q}} = \frac{1}{h} E_t^{\mathbb{Q}}(x_{t+1} + \dots + x_{t+h}). \quad (26)$$

When x is replaced by the hazard rate λ_n , we get a linearized approximation of the spread vs. Germany at maturity h . The operator $\overline{\bullet}^{\mathbb{P}}$ is the equivalent expectation computed under the historical measure.⁴⁹ The maturity h is supposed to be a benchmark maturity that is privileged regarding objectives (a) to (c). We use $h = 60$ months.

Using this loss function, the estimation of the $\gamma_{\ell,n}^0$'s and the $\gamma_{\ell,n}^1$'s is based on the following optimization:

$$(\gamma_{\ell,n}^0, \gamma_{\ell,n}^1, \gamma_n^2) = \arg \min_{\delta_0, \delta_1, \delta_2} \mathcal{L}_n(\delta_0, \delta_1, \delta_2).$$

⁴⁹ If the relationship between spreads and intensities were linear, then $\gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \overline{\lambda_{n,t}}^{\ell \mathbb{Q}}$ would be the part of the h -period spread (country n vs. Germany) corresponding to liquidity effects. Though the linearity assumption does not strictly hold, the approximation is reasonable as long as the λ 's remain small.

The three parts of the loss function (the second part including two terms) reflect the three criteria (a), (b) and (c) mentioned above. (a) The more the fluctuations of $\overline{\lambda_{n,t}^Q}$ can be tracked by those of $\overline{\lambda_{n,t}^L}$, the lower the first part of the loss function is. In this first term, the shadow parameter δ_2 is introduced because we want this first part of \mathcal{L}_n to focus on the *fluctuations* and not on the *level* of the intensities. Without the shadow parameter δ_2 , we would arbitrarily favour those specifications of the liquidity intensity that imply close-to-zero-mean default-related spreads. (b) The second part of the loss function penalizes the specifications of the liquidity intensity that generate negative default compensations (under both measures). (c) The third term implies an additional cost when the liquidity-related part of the spread is negative.

Generating positive PDs is arguably a more important objective than getting positive liquidity compensations. As a consequence, χ_1 is taken higher than χ_2 . We use $\chi_1 = 4$ and $\chi_2 = 1$ (see equation 25) for all countries except for Finland, for which we set these parameters to zero. With $\chi_1 = 4$ and $\chi_2 = 1$, we get positive and statistically significant Finnish PDs in the early 2000s. It may be due to the fact that the liquidity of Finnish bonds has increased over the last decade; but in our framework, we can not increase the liquidity spreads in the early 2000s without producing deeply negative PDs in the late 2000s (penalized when $\chi_1 = 4$).

F Historical and risk-neutral dynamics of y_t and z_t

Tab. 4: Parameters defining the historical and risk-neutral dynamics (Part 1/2)

Notes: The table reports the estimates of the parameters defining the dynamics of the factor under historical and risk-neutral measures. The estimation data are monthly and span the period from April 1999 to March 2011. Standard errors and Student-t are reported, respectively, in parenthesis and in square brackets below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level.

The historical-dynamics parameterization is estimated by maximizing the log-likelihood (equation 3). The covariance matrix of the parameter estimates is based on the Hessian of the log-likelihood function. The risk-neutral dynamics of the factors is estimated together with the hazard-rate specifications reported in Table 3 using non-linear least squares. For the latter, the covariance matrix of the parameter estimates is computed using the Newey-West (1987) [70] adjustment (see Appendix D).

	Non-Crisis	Crisis		$\Phi_{i,1}$	$\Phi_{i,2}$	$\Phi_{i,3}$	$\Phi_{i,4}$	$\Phi_{i,5}$
μ_1	0.0054 (0.0086) [0.63]	-0.0052 - -	$\Phi_{1,i}$	0.98*** (0.0032) [310]	0.0089*** (0.0024) [3.7]	0.017*** (0.0053) [3.3]	-0.015* (0.0081) [-1.9]	-0.12*** (0.033) [-3.6]
μ_2	0.003 (0.019) [0.15]	-0.0028 - -	$\Phi_{2,i}$	-0.012 (0.013) [-0.94]	1.02*** (0.0108) [94]	0.17*** (0.017) [10.4]	-0.049 (0.031) [-1.6]	0.25** (0.12) [2]
μ_3	0.086* (0.048) [1.8]	-0.082* - -	$\Phi_{3,i}$	0.029 (0.024) [1.2]	-0.054*** (0.02) [-2.7]	0.88*** (0.034) [26]	0.091* (0.054) [1.7]	0.28 (0.17) [1.6]
μ_4	-0.057*** (0.0054) [-10.7]	0.054*** - -	$\Phi_{4,i}$	0.0023 (0.0033) [0.72]	0.00092 (0.0025) [0.37]	-0.019*** (0.0046) [-4.2]	0.93*** (0.0076) [124]	-0.083* (0.045) [-1.8]
μ_5	-0.016*** (0.0025) [-6.5]	0.015*** - -	$\Phi_{5,i}$	0.00054 (0.0015) [0.36]	0.0012 (0.0013) [0.96]	-0.0033 (0.0024) [-1.4]	-0.0043 (0.0033) [-1.3]	0.85*** (0.016) [54]
	Non-Crisis	Crisis		$\Phi_{i,1}^*$	$\Phi_{i,2}^*$	$\Phi_{i,3}^*$	$\Phi_{i,4}^*$	$\Phi_{i,5}^*$
μ_1^*	0.017*** (0.00047) [37]	0.016*** (0.0022) [7.3]	$\Phi_{1,i}^*$	1*** (0.0004) [2511]	0.011*** (0.00026) [42]	0.0026*** (0.00063) [4.1]	0.00015 (-0.00038) [0.17]	0.00027 (-0.00025) [0.058]
μ_2^*	0.044*** (0.0054) [8.2]	0.069*** (0.019) [3.7]	$\Phi_{2,i}^*$	0.013*** (0.003) [4.4]	0.98*** (0.0038) [256]	0.13*** (0.0097) [14]	-0.00042 (-0.0028) [-0.061]	-0.0017 (-0.0036) [-0.051]
μ_3^*	-0.074*** (0.0059) [-13]	-0.109*** (0.025) [-4.3]	$\Phi_{3,i}^*$	-0.021*** (0.0043) [-4.8]	-0.013** (0.005) [-2.6]	0.88*** (0.011) [79]	-0.0006 (-0.0041) [-0.056]	-0.0016 (-0.0047) [-0.031]
μ_4^*	0.0034*** (0.00103) [3.3]	-0.00106 (0.0027) [-0.39]	$\Phi_{4,i}^*$	0.00028 (-0.00045) [0.54]	-0.00016 (-0.0056) [-0.45]	-0.00103 (-0.018) [-0.83]	1*** (0.0015) [689]	0.023*** (0.0051) [4.6]
μ_5^*	-0.00025 (0.00087) [-0.29]	-0.004** (0.002) [-2]	$\Phi_{5,i}^*$	0.000093 (-0.0051) [0.21]	-0.00108*** (-0.0021) [-3.4]	-0.000058 (-0.024) [-0.04]	0 (0.00067) [0.0086]	1*** (0.0033) [308]
Markov-switching probabilities								
$\pi_{NC,NC}$	0.96***		$\pi_{NC,NC}^*$	1***				
$\pi_{C,C}$	0.96		$\pi_{C,C}^*$	1***				

Tab. 5: **Parameters defining the historical and risk-neutral dynamics (part 2/2)**

Notes: See previous table. This table presents the estimated covariance matrices $\Sigma(z_t)$ of the Gaussian shocks $\Omega(z_t)\varepsilon_t$ in equation (1) (we have $\Sigma(z_t) = \Omega(z_t)\Omega(z_t)'$). The upper (respectively lower) part of the table reports the covariance matrix associated with the non-crisis (respectively crisis) regime.

Non-crisis regime					
	$\Sigma_{i,1}$	$\Sigma_{i,2}$	$\Sigma_{i,3}$	$\Sigma_{i,4}$	$\Sigma_{i,5}$
$\Sigma_{1,i}$	0.027*** (0.0036) [7.5]				
$\Sigma_{2,i}$	0.026*** (0.0038) [6.9]	0.036*** (0.0048) [7.5]			
$\Sigma_{3,i}$	-0.031*** (0.0053) [-5.8]	-0.027*** (0.0057) [-4.7]	0.079*** (0.011) [7.1]		
$\Sigma_{4,i}$	-0.0031*** (0.00105) [-3]	-0.0032** (0.0013) [-2.5]	-0.00104 (0.0022) [-0.48]	0.0038*** (0.00064) [6]	
$\Sigma_{5,i}$	-0.00069* (0.00038) [-1.8]	-0.00057 (0.00047) [-1.2]	-0.0011 (0.00082) [-1.4]	0.0012*** (0.00022) [5.5]	0.00055*** (0.000092) [5.9]
Crisis regime					
$\Sigma_{1,i}$	0.069*** (0.019) [3.6]				
$\Sigma_{2,i}$	0.058*** (0.021) [2.8]	0.11*** (0.032) [3.5]			
$\Sigma_{3,i}$	-0.0073 (0.015) [-0.48]	0.0013 (0.02) [0.064]	0.084*** (0.025) [3.3]		
$\Sigma_{4,i}$	-0.041* (0.022) [-1.9]	-0.0103 (0.026) [-0.4]	0.073*** (0.027) [2.7]	0.15*** (0.042) [3.6]	
$\Sigma_{5,i}$	-0.0081 (0.0055) [-1.5]	0.004 (0.0069) [0.59]	-0.0049 (0.0061) [-0.81]	-0.0061 (0.008) [-0.77]	0.0106*** (0.003) [3.6]