Inflation risks and inflation risk premia*

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Keywords: *Term structure models, inflation compensation, inflation risk premia, density forecasts, inflation risks*

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Abstract

This paper investigates the link between the inflation risks from macroeconomic forecasts and the inflation risk premia in financial instruments. We provide new quantitative evidence on both (i) the term structure of inflation compensation and inflation risk premia in the euro area bond market; (ii) inflation uncertainty and skewness in density forecasts at short and long horizons. Our main finding is that it is not the inflation uncertainty but the perceived asymmetries in inflation risks (i.e. the relative risk of inflation overshooting its expected path) which contain powerful information to interpret the dynamics of long-term inflation risk premia. Our results hold during the financial crisis period as well as after controlling for macroeconomic and financial factors.

1 Introduction

The yield spread between nominal and inflation-linked bonds, commonly referred to as the break-even inflation rate (BEIR), has become a key indicator of inflation expectations. Most major economies have issued inflation-linked debt in recent years, and detailed references to BEIRs in central bank publications and speeches (e.g. Bernanke, 2007, Trichet, 2005), research on the anchoring of inflation expectations (e.g. Gürkaynak, Levin and Swanson, 2009), and market commentary (e.g. Wall Street Journal, 2010) are increasingly common.

Yet, BEIRs are not a direct measure of inflation expectations. They reflect the overall inflation compensation that investors request to hold nominal bonds, comprising both the expected level of inflation and a premium to compensate for inflation risks. Understanding the inflation risk premium is therefore crucial for research and policy purposes. Despite the substantial research progress in recent years, the interpretation of the inflation risk premia dynamics remains far from straightforward.

In parallel, the discussion of the risks surrounding the inflation forecasts has become an increasingly important element in macroeconomic forecasting. Attention used to be restricted to point predictions, but nowadays most central bank statements and also market commentary (e.g. J.P. Morgan, 2010) usually elaborate at some length on the "risks" associated to the inflation outlook.

The goal of this paper is to investigate whether the perceived inflation risks in macroeconomic forecasts help understand the inflation risk premia embodied in the term structure of interest rates. To that end, we first present new evidence on both the term structure of inflation risk premia and the risks surrounding the inflation outlook in the euro area. We then use our inflation risk measures to interpret the dynamics of inflation risk premia at different horizons.

To estimate euro area BEIRs and inflation risk premia, we built a no-arbitrage term structure model along the lines of Ang, Bekaert and Wei (2008). To better identify the inflation risk premia, we also use inflation-linked bond yields to pin down real yields (e.g. D'Amico, Kim and Wei, 2007, Hördalh and Tristani, 2010), and survey inflation expectations to pin down the level of expected inflation (e.g. Dewachter and Lyrio, 2008, Chernov and Mueller, 2008). We show that the term structures of euro area BEIRs and inflation risk premia have been predominantly upward sloping but relatively flat. One-year forward BEIRs ending in five years averaged 2,25%, only 20 basis points higher than those ending in two years, and the long-term premium oscillated within a relatively narrow range of 0-50 basis points between 1999 and 2010.

To construct quantitative measures of perceived inflation risks we use the ECB's Survey of Professional Forecasters (ECB's SPF). As the Philadelphia Fed's SPF, the ECB's SPF asks panelists to assign probabilities to future inflation falling into pre-specified ranges, i.e. a density forecast in the form of a histogram. The SPF histograms, however, do not provide measures of inflation risks that could be directly used to interpret the dynamics of the inflation risk premia. By fitting a continuos but potentially asymmetric density to the SPF histograms, we obtain a term structure of mean inflation forecasts, and two metrics of the risks surrounding them, namely the degree of uncertainty and the asymmetry of risks. Those two inflation risk measures provide quantitative evidence on the spread of potential inflation outcomes as well as on the relative probability of inflation overshooting its expected path. Importantly for our goal in this paper, the ECB's SPF requests those histograms for three different horizons (one, two and five years ahead), thereby allowing to match inflation risks and inflation risk premia both at short and long term horizons.

Our main finding is that it is the perceived asymmetry in inflation risks, the "*balance of risks*", and not the degree of uncertainty, which is crucial to understand the dynamics of inflation risk premia. Specifically, at long horizons at which almost all the variation in BEIRs stems from variation in the inflation risk premium, the dynamics of inflation risk premia mimics quite closely those of inflation skewness. Upside inflation risks (positive skewness) indicate that there is a higher probability of inflation turning out above rather than below the (mean) expectation. Investors perceiving upside risks therefore face higher risk that inflation erodes their nominal returns, and, other things equal, are likely to request a higher premium to hold nominal assets. Inflation uncertainty is positively correlated at short horizons, but instead displays a negative correlation with the long-term inflation risk premium. These findings may contribute to resolve the otherwise somewhat puzzling low levels of term and inflation risk premia over 2009-10 observed worldwide with the significant increase in inflation volatility and uncertainty during the financial crisis period.

We present several pieces of evidence supporting the explanatory power of the asymmetries in inflation risks. Regression results show that inflation skewness remains statistically significant after controlling not only for inflation uncertainty but also for macroeconomic and financial factors capturing inflationary pressures (wage growth), economic activity (output gap, unemployment rate), business confidence indicators, and standard financial risk indicators (term spread, bond and stock market volatility). Inflation risk premia estimates based on term structure models are potentially problematic, particularly over the recent financial crisis period. We however show that the asymmetries in inflation risks were an important determinant of the inflation risk premium also in the pre-crisis period 1999-2007, for an alternative term structure model specification, as well as for model-free estimates of inflation risk premium based on inflation-linked swap rates.

This paper is closely related to some recent developments on term structure modelling. Latent factor models can fit data well but, being based on unobservable factors, their results often lack direct economic interpretation. Building on Ang and Piazzesi (2003), some recent literature models macroeconomy and the term structure dynamics jointly, so that movements in the yield curve in general and the inflation risk premia in particular can be directly related to the inflation and economic activity variables included in the macroeconomic model. We instead use inflation risk measures to explain inflation risk premium dynamics. We interpret our risk measures as capturing the combined effect of a large number of structural shocks from a rich macroeconomic model, as those underlying the construction of the "fan-charts" popularized by the Bank of England and the Riksbank since the late 1990s, and the macroeconomic forecasts regularly published by many other central banks and economic institutions. To the extent that a limited number of observable economic factors may not adequately represent the information sets of investors (e.g. Favero et al., 2010, Mönch, 2008), and the information content of survey measures is not easily captured by standard DSGE models (Del Negro and Eusepi, 2010), our inflation risks can therefore provide an alternative way to interpret the dynamics of inflation risk premia without facing the risk of misspecifcation of the macroeconomic model.

As regards using survey data for the interpretation of term structure evidence, Wright (2010) provides some cross-country evidence on term premia and argues that a significant part of the decline in term premia observed in the last two decades in most industrialized countries is related to the decline in inflation uncertainty that took place in the 1990s. This paper considers a single economic area, the euro zone, since its inception in 1999, and shows that over the last decade, in which inflation uncertainty remained quite low and stable until the crisis, inflation risks can also explain the dynamics of inflation risk premia through the asymmetry in perceived inflation risks.

The paper is organized as follows. Section 2 introduces our term structure model (full details are in Appendix A), and Section 3 documents some stylized facts about inflation compensation and inflation risk premia in the euro area. Section 4 provides some stylized facts on the perceived inflation risks at different horizons (Appendix B describes our estimation approach in detail). Section 5 provides supporting empirical evidence showing that perceived inflation risks contain powerful information to interpret the dynamics of inflation risk premia, while Section 6 provides some robustness analysis. Finally Section 7 concludes.

2 The term structure of inflation risk premia

The spread between the yield of nominal bond (y_t^n) and the yield of a real bond (y_t^r) of the maturity n reflects the inflation compensation requested by investors to hold nominal bonds. The requested compensation for inflation, or BEIR, however, comprises two very distinct components, namely the (average) level of inflation over the life of the bond ($E_t(\pi_{t,t+n})$) and an additional risk premium (ϕ_t^n) required by bond holders as compensation for the risk of inflation turning out being different from that expectation. Formally ,

$$y_t^n - y_t^r = BEIR = E_t(\pi_{t,t+n}) + \phi_t^n. \tag{1}$$

A serious challenge to interpret developments in nominal yields and to estimate BEIRs, is that not only the inflation risk premium but, to a large extent, also expected inflation and the real yield are unobservable, and therefore need to be identified from the observed bond yields.

2.1 The model setup

To estimate the term structure of inflation risk premia, we employ a discrete-time affine term structure framework that links bond yields to the dynamics of short-term yields and inflation under no-arbitrage restrictions. Apart from modelling regime changes the basic structure of our framework is similar to Ang, Bekaert and Wei (2008, ABW henceforth), so we here only stress the main features of the model and provide full model details in Appendix A.

No-arbitrage alone provides weak identifying restrictions for real yields and the inflation risk premia. In line with recent literature, we therefore incorporate additional information to improve the decomposition of BEIRs. Specifically, to pin down real yields we incorporate inflation-linked bond yields in the estimation (see also D'Amico, Kim and Wei, 2007, DKW henceforth, and Hördalh and Tristani, 2010). To help estimate the inflation risk premia through equation (1), we also incorporate survey data of inflation expectations at both short and longer-term horizons (see Chernov and Mueller, 2008, Dewachter and Lyrio, 2008). This modelling approach has become relatively standard in recent years, and for example Joyce et. al. (2010) model for the U.K. term structure is also very similar to ours.[1]

The model has three state variables: two latent factors l_t^1, l_t^2, and actual inflation π_t as observable factor. As standard in the related literature, the dynamics of the state vector $X_t = (l_t^1 \; l_t^2 \; \pi_t)'$ follows a VAR(1) process $X_{t+1} = \mu + \Phi X_t + \Sigma \epsilon_{t+1}$. The matrices μ, Φ and Σ are specified as follows

$$\mu = \begin{bmatrix} 0 \\ 0 \\ \mu_\pi \end{bmatrix}, \qquad \Phi = \begin{bmatrix} \Phi_{11} & 0 & 0 \\ \Phi_{21} & \Phi_{22} & 0 \\ \Phi_{31} & \Phi_{32} & \Phi_{33} \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sigma_\pi \end{bmatrix}.$$

The real short rate $\hat{r}_t$ is an affine function of the state vector $\hat{r}_t = \delta_0 + \delta_1' X_t$. To make the real rate dependent on the latent factors but not on inflation[2], we restrict the δ_1 vector to $(\delta_{1,1} \;\; \delta_{1,2} \;\; 0)$. To model the term structure of real yields, we specify the real pricing kernel as an exponential function of the market price of risk λ_t:

$$\hat{M}_t = \exp\left(-\hat{r}_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right).$$

The market price of risk, in turn, is a linear affine function of the state variables $\lambda_t = \lambda_0 + \lambda_1 X_t$, with:

$$\lambda_0 = \begin{bmatrix} \lambda_{0,1} \\ \lambda_{0,2} \\ \lambda_{0,3}^* \end{bmatrix}, \qquad \lambda_1 = \begin{bmatrix} \lambda_{1,11} & \lambda_{1,12} & \lambda_{1,13}^* \\ \lambda_{1,21} & \lambda_{1,22} & \lambda_{1,23}^* \\ \lambda_{1,31}^* & \lambda_{1,32}^* & \lambda_{1,33}^* \end{bmatrix}.$$

We consider two specifications of the market price of risks. In our baseline specification we treat inflation as an *unspanned* factor for the term structure of real interest

[1] For our purpose in this paper we chose the most parsimonious structure needed to fit the data. Joyce *et al.* (2010) employ an additional latent factor in their model, but, in the euro area, the first two principal components already explain more than 99% of the variation in the real term structure .

[2] Theoretically, by imposing this restriction we exclude the Mundell-Tobin effect that gives a direct effect of inflation on the real interest rate.

rates and restrict all λ^* elements of λ_1 to zero. This implies that inflation is part of the physical representation of the state vector and can help forecasting future interest rates but does not affect the cross-section of real bond yields today. Recent literature indeed argues that macroeconomic factors are better modeled as unspanned factors in the yield curve.[3] Focusing on the role of inflation in particular, Kim (2009) discriminates between "internal basis" models were the yield curve is projected on latent factors only and "external basis models". In the latter, case macro variables like inflation directly enter into the modeling of the cross-section of bond yields. In this sense, our baseline specification can be consider an example of "internal basis" or pure latent factor model. We however also allow for the role of inflation in the real pricing kernel in our model to be as general as possible by estimating a model version with all λ^* elements of λ_1 unrestricted. Both model specifications provide qualitatively similar estimates of BEIRs and inflation risk premia.

Using the affine term structure framework it is easy to derive closed-form solutions for real and nominal bond prices. For a certain maturity n bond prices are exponentially affine functions of the state vector. For example, in the case of real bond prices, the solutions have the form (solutions for nominal bonds can be found in Appendix A):

$$\hat{P}_t^n = \exp\left(\hat{A}_n + \hat{B}_n' X_t\right).$$

The constants $\hat{A}_n$ and the factor loadings $\hat{B}_n'$ can be recursively computed using the system of Riccati equations (with $\hat{A}_1 = -\delta_0$ and $\hat{B}_1' = -\delta_1$ as initial conditions):

$$\begin{aligned}
\hat{A}_{n+1} &= -\delta_0 + \hat{A}_n + \hat{B}_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}\hat{B}_n\Sigma\Sigma\hat{B}_n' \\
\hat{B}_{n+1}' &= -\delta_1' + \hat{B}_n'(\Phi - \Sigma\lambda_1).
\end{aligned}$$

[3] See among others, Duffee, 2011, Wright, 2009, Joslin *et al.*, 2010.

2.2 Data and estimation method

2.2.1 Data

Our estimation is based on monthly data from January 1995 to September 2010. The nominal zero-coupon yields (3-month, and 1, 2, 3 and 5-year) are from Bloomberg.[4] Real zero-coupon yields (2, 3, and 5-year) since February 2004 are estimated from inflation-linked bond yields following Ejsing et al. (2007). As inflation measure we use year-on-year rates of change in the euro area HICP figures as reported by Eurostat, for they match the ECB´s inflation objective and the survey-based measures of inflation expectations from the ECB's SPF (to be described in detail below).

Table 1 reports some basic statistics of the euro area yield curve data over the sample 1999Q1-2010Q3. In addition to full sample statistics, the table also reports statistics for a pre-financial crisis subsample 1999Q1-2007Q2, that we use as reference period to shelter our analysis from the yield data distortions stemming from the financial turbulences since the summer of 2007. In particular, the time-varying nature of the liquidity premium embodied in nominal and real bond yields worldwide since mid-2007 is difficult to correct for (see discussions in Campbell et al., 2009, DKW, 2007) and can potentially cloud the relationship we aim at unveiling here. Details of our modelling approach are described in detail below, but, for completeness, we also briefly discuss here the main characteristics of the BEIRs.

The euro area nominal, real and BEIR curves were on average upward sloping but relatively flat during the ECB era. Those curves displayed significantly higher volatility at the short-end than over longer horizons, which in the case of the nominal and the BEIR curves reflects the strong anchoring of inflation expectations in the euro area. Yields nonetheless exhibit significant persistence at all horizons.

Looking at the pre-crisis subsample allows for ascertaining some effects of the financial crisis on euro area yield curves. The financial crisis period is characterised by, on average, lower but more volatile yields. The presence of pricing distortions since

[4] Nominal yields before 1999 are yields derived from German government bonds.

2007Q2 is however more evident for real yields, which in addition to higher volatility exhibited *higher* average values, most likely reflecting severe liquidity premia that warns against the direct calculation of BEIRs as spreads between those nominal and real yields. In contrast, the mean and volatility of our model-based BEIRs, on average, remained relatively stable over the whole sample, particularly at longer horizons, which is more reassuring for our analysis in this paper.

Finally, although all the series exhibit some mild non-Gaussian features like skewness and excess kurtosis over the sample, the Gaussian assumption does not seem to be unreasonable as a first approximation to gauge reliable estimates of the euro area term structure of inflation risk premia and its dynamics and to compare them to the dynamics of perceived inflation risks.

2.2.2 State and observation equations

The measurement and transition equations on the state-space representation of the model can be expressed as $w_t = d + ZX_t + \eta_t$ and $X_t = \Phi X_{t-1} + \Sigma\epsilon_t$. The observed data vector w_t contains real and nominal bond yields, inflation and survey inflation expectations. The vector d and the matrix Z reflect the bond price equations that link the state variables (latent factors) and the observed data (see Appendix A for details).

2.2.3 Kalman filtering and optimization

We use Kalman filtering techniques in the estimation because they offer two main advantages in our setting. First, we incorporate additional data in the estimation as they become available. For example,we incorporate yields from inflation-linked bonds only from 2004 onwards given the limited number of bonds to estimate the real term structure before that date (see Ejsing et al., 2007).

Second, we allow for measurement errors in the fitting of most observed variables to account for the characteristics of our data. For example, we fit real and nominal bond yields at all maturities upto a measurement error. Moreover, to capture the fact that the inflation-linked bond yields are likely to be somewhat less liquid than nominal bonds, we allow for a higher measurement error in real bond yields.

Survey inflation expectations are a key element in our setting, but they are only available since 1999, and at a quarterly frequency. The state-space formulation also fits well with those features of the survey data. Fitting survey inflation expectations upto a measurement error we can capture the strong anchoring of inflation expectations nowadays by reducing the influence of actual inflation, whose high volatility otherwise translates into highly volatile inflation expectations at long horizons (see Table 3). Importantly, a better estimation of the term structure of inflation expectations using survey expectations does not come at the cost of fitting financial data: Table 4 shows that the fitting of nominal yields in the model including survey data is as good as, and often somewhat better, than that of the model without including survey information. These results for euro area data corroborate the findings of Kim (2009) and Chernov and Mueller (2008) for the US economy.[5] As inflation is included as observable variable in the state vector, model based inflation expectation are, for all forecast horizons, a function of the states in time t. For example, the model based inflation expectation for inflation five years ahead (60 months) can be computed as follows:[6]

$$E_t^{\text{model}}[\pi_{t+60}] = e_N(I-60)(I-\Phi)^{-1}\mu + e_n\Phi^{60}X_t.$$

Figure 4 displays the model fitting for nominal and real yields and for survey inflation expectations at different horizons. A vertical line differentiates the pre-crisis period 1999Q1-2007Q2 and the full sample, and we will discuss specific modelling challenges for the second period below. Table 2 also lists parameter estimates and confidence bounds computed using MCMC following Chernozhukova and Hong (2003).

[5] Since pinning down inflation expectations is crucial for estimating the inflation risk premia, the use of survey inflation expectations in the estimation of term structure models has become a standard practice (e.g. DKW, 2007, Hördahl and Tristani, 2010, Joyce *et al.*, 2009).

[6] e_3 is a vector of zeros apart from its third element, a one, which selects inflation from the state vector.

3 Inflation compensation and inflation risk premia

This section describes the key features of inflation compensation in the euro area between 1999M1 and 2010M9, and, in particular, the role of inflation expectations and inflation risk premia at different horizons. We focus on one-year forward rates of inflation compensation ending in one, two and five years, because we collect survey inflation forecasts at those horizons, and they are a key element in our model. Moreover, later in this paper we also use survey measures of inflation risks at those horizons to interpret the dynamics of the term structure of euro area inflation risk premia.[7]

Table 1 reports the main characteristics of the inflation compensation (BEIRs), expected inflation and the inflation risk premium (see also Figures 1 to 3). First, the term structure of euro area inflation compensation was predominantly upward sloping but relatively flat: the spread between nominal and real yields at the two year horizon averaged around 200 basis points, and, despite rising with maturity, at the five year horizon it was around 225 basis points, just 25 basis points higher. The difference between the levels of the one year forward BEIRs ending in two and five years peaked at around 30 basis points in early 2000 and early 2002, but from mid-2004 the term structure of inflation compensation flattened considerably, and that maturity spread remained within 10 basis points. Since the start of the financial turbulences in mid-2007 and in particular following their intensification in the autumn of 2008, inflation compensation however turned more volatile. The euro area BEIR curve was inverted for most of 2008 before steepening strongly in 2009 reflecting the volatility of realized inflation and the revisions in short-term expected inflation during the crisis.

The term structure of inflation risk premia in the euro area also exhibits a predominantly upward slope but the spread across maturities is also quite compressed: on average the inflation risk premium was between 5 and 10 basis points within two years and about 25 basis points five years ahead. Moreover, the long-term premia oscillated

[7]Note that Equation (1) also establishes the link among forward inflation compensation, inflation expectations and forward inflation risk premia.

within a relatively narrow range of 0-50 basis points. Our estimates of inflation expectations embodied in bond yields, in line with the evidence from survey data, suggest a strong anchoring of inflation expectations at medium-to-long maturities.

To help interpret euro area BEIRs, Table 5 reports the relative contributions of inflation expectations and the inflation risk premia to the volatility of overall inflation compensation. As reported in Table 1, short horizon BEIRs are more volatile than longer term ones, and about 2/3 of that volatility reflects movements in short-term inflation expectations, with inflation risk premia playing a limited role. In contrast, the volatility of inflation compensation at longer horizons is almost fully driven by the inflation risk premia, while the limited contribution of long-term inflation expectations reflects a strong anchoring of euro area inflation expectations. This result highlights the importance of accounting for this feature of inflation expectations when modelling long-term inflation risk premia. In this regard, our model-based inflation expectations, which combine information from both survey and financial data, suggest that long-term inflation expectations among market participants may be even more firmly anchored that survey data suggest: with a standard deviation of 0.04, our model-based inflation expectations fluctuate less than the survey long-term inflation expectations.

4 The term structure of inflation risks

The goal of this paper is to bring together measures of inflation risk premia and macroeconomic inflation risks. To that end we need quantitative measures of the perceived inflation risk in macroeconomic forecasts. As for the more widely-used US SPF currently run by the Federal Reserve Bank of Philadelphia, the ECB's SPF survey panelists assign probabilities to the forecast variable falling into pre-specified ranges, i.e. a density forecast in the form of a histogram.[8] For example, the 2010 Q3 survey questionnaire asked panelists to attach probabilities to inflation falling within 12 bins of half-a-percentage-point width in the interval (-1%, 4%). Moreover, forecasters provide

[8] For an introduction to the ECB's SPF see Garcia (2003).

such density forecasts over three different horizons of 12 month, 24 month and five years ahead, which allow to construct a term structure of perceived inflation risks. This section presents the main characteristics of such inflation risks.

4.1 Measuring inflation risks

The SPF histograms do not provide direct measures of perceived inflation risks. To quantify them, we fit a continuous but potentially asymmetric density to the SPF histograms: the skew-normal distribution (see Azzalini, 1985). The goal is to provide a full statistical characterization of the potential inflation outcomes as well as the relative probability of inflation turning out above rather than below the (mean) expected rate, both of which can influence the premia requested by bond market investors.

The skew-normal distribution is an extension of the standard normal to account for potential asymmetries through an additional *shape* parameter λ_S, which determines the skew of the distribution.[9] Specifically, as for the general normal distribution, a random variable W that is skew-normal-distributed can be written as $W = \mu + \sigma\left((Z - E[Z])/(\sqrt{V(Z)})\right)$, where $Z \sim SN(\lambda_S)$ is the standard skew-normal The first three central moments of W can then be expressed as

$$
\begin{aligned}
E[W] &= \mu \\
V(W) &= \sigma^2 \\
SK(W) &= \gamma_1 = \left(2b^2 - 1\right) b\delta^3 / \left(1 - b^2\delta^2\right)^{3/2} \\
& \quad \text{where} \quad b = \sqrt{2/\pi} \quad \text{and} \quad \delta = \lambda_S / \sqrt{\left(1 + \lambda_S^2\right)}.
\end{aligned}
$$

The main advantage of the skew-normal distribution for our purpose in this paper is that it provides a one-to-one mapping between its three parameters and the mean, variance and skewness. Furthermore, as it nests the standard normal as a particular

[9]The skew-normal class is built by shifting and re-scaling a standard normal distribution with a density function defined as $f_{\lambda_S}(z) := 2\varphi(z)\Phi(\lambda_S z)$, with $z \in \mathbb{R}$ where φ and Φ are respectively the standard normal density and distribution functions, and $\lambda_S \in \mathbb{R}$ is the *shape* parameter.

case (λ_S=0), we can "*let the data speak*" about potential asymmetries present in the SPF data without restricting our estimates of the other moments of the distribution.[10] Details of our fitting methodology are provided in Appendix B and can also be found in Garcia and Manzanares (2007).

As proxy for market's perceptions of inflation risks we here focus on the key moments of the *combined* density forecast that is constructed by averaging the individual histograms. Such a density forecast by construction combines the subjective perceptions of inflation risks of the individual forecasters and it should better capture the inflation risks perceived in the economy as a whole.

Inflation risk measures can also be constructed by averaging moments of the individual density forecasts. In particular, as regards the uncertainty surrounding inflation expectations, the second moment of the *combined* density forecast is a natural measure of the dispersion surrounding the *consensus* forecast. However, by construction, the second (centered) moment of the combined density forecast incorporates both the average uncertainty surrounding the individual forecasts and also the disagreement with respect to the expected mean inflation expectations across panelists.[11]. As sensitivity analysis we will also consider uncertainty measures based on the average of the second moment of the individual density forecasts.

Regarding asymmetries in perceived inflation risks, we will consider two measures of asymmetry: (i) the skewness of the combined density forecast, calculated as the (normalized) third centred moment[12]; (ii) the distance between the mean and the mode of the combined density forecast, as regularly reported by the Bank of England in its *fan-charts* (see Britton *et al.*, 1998).

[10] For example, other three-parameter distributions (i.e. the two-piece normal) do not provide such a direct mapping. In more parsimonious but potentially skewed distributions, like the beta family, two parameters impose undesired constraints on the estimation of the three moments of interest.

[11] We measure inflation uncertainty by the variance rather than the standard deviation because, by construction, the link between the aggregate variance and the average of individual variances that does not hold for the standard deviations. Specifically Agg. variance = Average Uncertainty + Disagreement, with average uncertainty being the main component of the variance of the combined probability forecasts. See Garcia and Manzanares (2007) and references therein for details.

[12] The skewness of the combined probability forecast cannot be proxied by the skewness of the individual means nor by the average skewness across panelists (Garcia and Manzanares, 2007).

4.2 Inflation risks, inflation and inflation expectations

Being an important novelty in our analysis, this section documents the main characteristics of our inflation risk measures. To gauge the information content of the key moments of inflation forecasts, Table 6 reports the correlations among the risk measures discussed above, as well as with actual inflation figures, both headline HICP inflation and HICP excluding energy and unprocessed food ('core' inflation henceforth). For completeness, the standard deviation of those risk measures is displayed, within brackets, along the main diagonal.

Inflation risk measures display limited correlated with mean inflation expectations, or with actual inflation rates, both at short and long horizons. This evidence suggests that inflation risks contain valuable information beyond that of the central tendency of inflation expectations. Average uncertainty, however, is strongly correlated with core inflation (0.7 at the one-year and 0.5 at the five-year horizons), but not with headline inflation (0.4 and 0.1 respectively). To the extent that core inflation can be interpreted as an underlying trend of inflationary pressures, these correlations suggest that it is not the (noisy) monthly movements in inflation but the smoother, underlying trend of inflation which drives inflation uncertainty.

Short-term (one year ahead) inflation expectations are strongly correlated (0.8) with overall HICP inflation, but that correlation weakens with forecast horizon (0.3 for long-term inflation expectations). Core inflation measures are instead little correlated with inflation expectations both at short (0.2) or longer-term horizons (0.0). This evidence underscores the need to incorporate further information beyond actual inflation developments to identify long-term inflation expectations.

The comovement of inflation risks across horizons is also important to assess the information content of the term structure of inflation risks. Correlations between short and longer-term risk measures show a substantial difference between uncertainty and the balance of risks. Inflation uncertainty shows a great deal of comovement across horizons, particularly when we use our average uncertainty measure. The balance of risks surrounding short and longer term inflation forecasts, in contrast, show little correlation, which suggests that asymmetries in inflation risks reflect different information at different horizons. In particular, perceived asymmetries in long-term inflation risks are not strongly influenced by other features of inflation expectations, nor by standard macroeconomic variables like inflation and the output gap usually employed in the macro-finance literature to interpret the dynamics of the inflation risk premia. Long-term inflation risks may instead contain information about the perceived persistence of inflationary shocks and the monetary policy reactions to them. We come back to this point in our discussion in Section 7.

5 Inflation risks and the dynamics of inflation risk premia

We seek for the link between the inflation risks perceived by macroeconomic forecasters and the inflation risk premia embodied in the term structure of interest rates. Our conjecture is that, to the extent that the perceived inflation risks embodied in the SPF density forecasts reflect those perceived by all economic agents in general and by market participants in particular, our measures of inflation risks should provide information about the pricing of inflation risks in bond yields. Survey-based inflation risk measures should therefore help interpret changes in the inflation risk premia.

5.1 Preliminary considerations

Before formally assessing our conjecture, some potential caveats of the analysis of the link between the inflation risks and inflation risk premia should be borne in mind. First, from a modelling perspective, our estimates of inflation risk premia come from a *Gaussian* framework. The evidence presented in the previous section however suggests that perceived inflation risks exhibit some, albeit limited, time variation over the last few years, and marked features, like skewness, that are in contradiction with such modelling assumptions. Handling those inconsistencies is however beyond the scope of this paper. Our working hypothesis is that the Gaussian framework provides a reasonable and useful approximation that necessarily abstracts from some of the complexities of the actual data, but allows to assess the extent to which our inflation risk measures can help interpret the dynamics of the inflation risk premia.

Second, from a conceptual perspective, the SPF density forecasts reflect the marginal probability of inflation. Arguably, risk measures directly relevant to bond market participants would refer to the joint probability distribution of inflation and bond yields, but we believe that our inflation risk measures offer important new insights.

Finally, from a statistical perspective, there is a mismatch in the frequency of the survey information (the SPF is carried out at quarterly frequency) and our model estimates (monthly). That frequency mismatch should not prevent the comparison to be meaningful. We carry out our analysis on quarterly data that match the survey dates with our inflation risk premia estimates.

5.2 Correlation analysis

Table 7 reports the correlations between the inflation risk premia and perceived inflation risks. We consider forward inflation risk premia in one, one-to-two and four-to-five years, and two measures of uncertainty (average uncertainty and the variance of the aggregate distribution) and asymmetry (skewness and the mean-mode distance) at matching horizons of one, two and five years ahead.

The higher the uncertainty about future inflation, the higher the risks to hedge, and, other things equal, the higher should be the risk premia requested by investors. A positive correlation should therefore be expected. Table 7 however shows that inflation risk premia is *negatively* correlated with inflation uncertainty measures, particularly over the longer horizons at which the inflation risk premia determines the dynamics of inflation compensation. Only at short horizons (one and two years ahead), and only for our average uncertainty measure we find the expected positive correlation.[13]

[13] On the one hand that negative correlation reflects the extreme volatility of actual inflation since 2008 as a result of the surge in oil prices first and their fall afterwards in the context of the economic downturn. Indeed, simple inflation uncertainty measures based on the volatility of realized inflation over 12 or 24 month windows provide a similar pattern. On the other hand, the negative correlation

That result may be somewhat puzzling and counterintuitive. Wright (2010) finds that a significant part of the decline in term premia observed in the last two decades in most industrialized countries seems to be related to the decline in inflation uncertainty that took place in the 1990s. That result seems robust to alternative measures of inflation uncertainty, including for survey data as ours, and, in particular, disagreement (measured as the standard deviation of point forecasts). As discussed above, disagreement is a component of our aggregate uncertainty measure, but we believe that the low or even negative correlation we find here reflects that since in our sample, and despite the rise since late 2008, inflation uncertainty has been relatively stable even at long horizons (see Figure 5), while the long-term inflation risk premium has displayed somewhat stronger fluctuations (see Figure 3).

Asymmetries in the perceived inflation risks instead contain crucial information to interpret developments in the premia, particularly at longer horizons. Positive inflation skewness indicates that the risk of high inflation is perceived to be higher than that of lower inflation. Investors perceiving those risks for inflation therefore face higher probability that their nominal assets may loose value than otherwise if inflation turns out to be different from their baseline expectation. Other things equal, it is logical that investors then request a higher premia to hold those nominal assets. Consistent with this intuition, risk asymmetry measures at long horizons are positively correlated with the inflation risk premia.

5.3 Regression analysis

Table 9 provides more formal statistical evidence on the information content of perceived asymmetries in inflation risks for the dynamics of inflation risk premia. Giving the data and modelling challenges associated to the financial crisis, we first focus on the pre-crisis period 1999Q1-2007Q2 as benchmark sample. To gauge the statistical significance of the correlations shown in Table 8, a regression of long-term inflation risk

between long-term inflation risk premium and survey inflation uncertainty measures also holds before the financial crisis, when inflation volatility and uncertainty was quite low.

premium on our two inflation risk measures corroborates the strong significance and positive sign of inflation skewness (see Column I). In line with the correlation results, inflation uncertainty instead enters with a negative coefficient.

We asses the robustness of that significance by controlling for several macroeconomic and financial factors that have been employed to interpret the dynamics of inflation risk premia in recent literature (e.g. Gürkaynak et al., 2009, Ciccarelli and Garcia, 2009) in Columns II–V. Specifically, inflation skewness remains strongly significant after controlling for indicators of inflation pressures at earlier stages of the pricing chain (see Column II for wage growth, but results also hold for PPI). Moreover, significance holds when controlling for core inflation, the volatility of both core and headline inflation, as well as when adding indicators of economic activity (unemployment rate and the output gap, see Column III, or industrial production). Inflation skewness' significance also holds when controlling for confidence indicators (consumer confidence, see Column IV, and also industrial confidence and the PMI index). Column V shows that inflation skewness retains its explanatory power after including in the regression some financial indicators (the term spread in the euro area yield curve and the volatility in the bond and stock markets) that can control for the pricing of risks in financial markets. In general, the control variables are also statistically significant and their estimated coefficients have the expected signs.[14]

Inflation skewness remains strongly significant even after controlling simultaneously for all the inflation pressure, economic activity, confidence and financial indicators considered before (see Column VI). Interestingly, inflation uncertainty retains its negative coefficient (-0.17) but turns statistically non-significant when jointly controlling for macroeconomic and financial factors in the regression.

[14]Coefficients for labour market indicators (wages, negative, and unemployment rate, positive) may look puzzling, but their interpretation needs to take into account that they refer to contemporaneous information while our dependent variable is long-term forward premia (in five years). They are included as control variables because ECB's SPF participants, a substantial part of them from the financial sector, usually mention them in their qualitative assessments of the inflation outlook, sometimes questioning the continuation of the observed wage moderation and the declines in the unemployment rate in the future, which may fit well with those estimated coefficients.

Two additional robustness checks are included in columns VII to X. First, we show that our main finding is also robust to the alternative measure of the asymmetry in inflation risks based on the distance between the mean and the mode of the inflation density forecast over long-term horizons. Specifically, both for inflation risks regressions and in the general regression controlling for the other seven factors, as for the inflation skewness measure, the mean-mode distance is positive and strongly significant (see Column VIII). Second, Table 9 also provides additional evidence that the significance of our inflation risk measures is robust to an alternative specifications of the term structure model used for the estimation of the inflation risk premia. In particular we consider premia estimates from the alternative specification of the market prices of risk discussed in Section 2. Columns IX and X show that the significance of the asymmetries in inflation risks holds, irrespective of being measured as inflation skewness or mean-mode distance, and after controlling for inflation uncertainty and the set of macroeconomic and financial indicators discussed above. In contrast, inflation uncertainty does not seem to incorporate additional explanatory power beyond that contained in the other standard macroeconomic and financial factors we consider.

6 Robustness

The empirical evidence reported in Table 9 further underpins the information content of perceived asymmetries in inflation risks for the dynamics of inflation risk premia over long horizons. This section provides additional robustness checks by considering (i) the extension of the sample by including the financial crisis period; (ii) model-free evidence on the long-term inflation risk premium using inflation-linked swap rates.

6.1 Inflation risks and inflation-risk premia: extended sample period

Modelling the term structure of interest rates during the financial crisis poses some additional difficulties, but the estimation of the inflation risk premium is especially challenging because strong and time-varying liquidity distortions are likely to affect nominal and inflation-linked bonds asymmetrically during the crisis, and in particular since the intensification of the financial crisis in the autumn of 2008 following the bankruptcy of Lehman Brothers (e.g. Campbell et al., 2009). Without accounting for them, the estimation of inflation compensation and the inflation risk premia would be biased and potentially misleading. As explained above, our estimation approach allows for the nominal and inflation-linked bond yields to be fitted with different measurement errors.[15] Those measurements errors indeed show substantial increases not only in the autumn 2008, when portfolio reallocation from inflation-linked into nominal bonds led to strong declines in observed break-even inflation rates, but also in the first half of 2009 at short horizons as well as in the spring of 2010 with the intensification of the sovereign debt concerns in some euro area countries. Specifically, Figure 7 provides evidence supporting the presence of a sizable negative distortion in nominal bond yields and a strong positive one in real yields due to varying market liquidity, whose evolution over the crisis contributes to explain a large part of the volatility of observed BEIRs calculated from them as argued in our discussion of the evidence in Table 1.

Inflation skewness in inflation risks remains statistically significant for the period 1999Q1-2010Q3 after controlling for macro and financial factors (see Table 9, first two columns). Inflation uncertainty also turns out to be statistically significant over the extended sample period in contrast to most of the macroeconomic and financial factors. The regression coefficient associated to inflation uncertainty however remains negative, and seems to capture the significant increase in inflation uncertainty at all horizons during the financial crisis period. Indeed, despite the statistical significance of inflation uncertainty, the overall explanatory power of the regressions decreases when the financial crisis period is included.

[15] While it is possible to introduce an additional factor to capture the liquidity gap between inflation-linked and nominal bonds as in D'Amico *et al.*(2009), our experience suggests that such an approach works well to account for the liquidity premium in the early years of the inflation-linked bond markets (as D'Amico *et al.*, 2009, do for TIPS) but may trigger identification problems in periods of intense market turbulence, as it is the case at the end of our sample, and therefore increase the risk of model misspecification.

6.2 Model-free proxies of the inflation risk premium

Inflation-linked swap rates (ILS henceforth) provide an alternative measure of inflation compensation. As standard swaps, in a zero-coupon ILS one party agrees to pay the realized inflation rate over the swap horizon, the floating leg, in exchange for a given inflation rate, the fixed leg of the swap. The euro area ILS market developed very fast since 2003, partly as a financial innovation to cope with the limitations of the euro area inflation-linked bond market in its early years,[16] and given the wide range of maturities at which they are traded, can be used to gauge inflation compensation over the horizons of our survey risk measures. In contrast to inflation compensation measures from term structure models, ILS-based measures: (i) do not require the estimation of nominal and real term structures; (ii) are less prone to liquidity distortions than BEIRs calculated as the yield spread between conventional and inflation-linked bonds due to the recent turbulences in financial markets. Haubrich *et al* (2011) for example employ them to estimate US inflation expectations and related premia during the financial crisis period.

As for our model-based long-term inflation compensation measures, we calculate one-year forward ILS rates ending in five years.[17] We then obtain two alternative estimates of inflation risk premium out of those inflation compensation measures. The first one, our survey-based premium, is calculated by detracting the five-year-ahead (mean) inflation expectation from the ECB's SPF from the ILS forward swap rate. The second, a *crude* swap-based premium measure, is constructed by detracting a fixed inflation rate of 2% from that ILS forward rate.

Figure 9 shows that the dynamics of the two model-free proxies for the long-term inflation risk premium, as their model-based counterparts, closely mimic the skewness in long-term inflation forecasts. Regression results also confirm the main findings of our analysis for the model-based measures (see Table 9, last four columns). Asymmetries

[16] For an overview of the development of the euro area ILS market and some international comparison see Garcia and Van Rixtel, 2007, and references therein.

[17] Traded inflation-linked swaps, as inflation-linked bonds, do incorparate a three-month indexation lag. Although at the horizons we use them here the effect on the relevant expectations is minimal we do take it into account when matching the survey expectations in our calculations.

in inflation risks are statistically significant for those model-free based measures as they were for the model-based counterparts, and are so after controlling for inflation uncertainty and macroeconomic and financial factors. A comparison of the different regression specifications suggests that, with a negative coefficient, the significance of inflation uncertainty, (at 10%), does not contribute much to the explanatory power of the regression, while confidence indicators and the output gap also seem important to explain the dynamics of the inflation risk premia.

7 Interpreting changes in inflation risk premia

Given the statistical evidence supporting that inflation risks, in particular the perceived asymmetries in inflation risks, contain valuable information to explain changes in inflation risk premia, we here use them to interpret the dynamics of long-term inflation risk premia over the first ten years of the single monetary policy in the euro area.

Figure 8 illustrated the strong comovement of inflation skewness and inflation risk premium. The presence of positive inflation skewness (i.e. upside risks associated to long-term inflation) during the first three years of the ECB era explains the relatively high inflation risk premia (peaking at about 50 basis points in early 2001) requested in the early years of the euro area despite relatively low actual and expected inflation at the time. Between 2001 and 2004, several adverse price shocks (oil price increases, food price spikes associated to the outbreak of animal diseases, etc) pushed actual inflation readings beyond the ECB´s definition of price stability at 2%. Yet, those shocks appeared to be interpreted as mainly of temporary nature, as long-term inflation skewness gradually fell since mid-2001, and even turned negative early 2002 and late 2005, which is consistent with the gradual decline in inflation risk premia and its bottoming at less than 10 basis points in the summer of 2005.

Inflation risk premia rose steadily since 2006 and reached 40 basis points in late 2008, as the surge in oil prices pushed actual inflation readings to its maximum levels in the summer of 2008. Inflation skewness, while overall remaining positive during that period, displayed a milder profile than inflation risk premia. Interestingly, however, those relatively contained upside inflation risks were associated to a higher level of long-term inflation expectations which suggests that professional forecasters appeared to assign more probability to a higher level of inflation than market participants, who instead, at least according to our term structure model, seemed to request a higher premia with little change in the level of inflation expectations.

The intensification of the financial and economic crisis since late 2008 however seemed to reconcile market's and professional forecasters' long-term inflation views. The reassessment of inflation risks was quite strong. Inflation uncertainty rose sharply at all horizons in mid-2008, and after peaking around mid-2009 remained close to its highest levels since then. Despite higher uncertainty, the increasing concerns about the severity and length of the economic recession triggered a significant downward revision to long-term inflation risks. Inflation skewness, which in early 2009 had reached its highest levels for more than 8 years, showed a protracted decline, turned negative and even reached its lowest historical levels in early 2010. This opposite reaction of inflation uncertainty and the perceived asymmetries in inflation risks during the financial and economic crisis period underscores the importance of considering the two risk measures to interpret market developments and the assessment of the inflation outlook.

The reaction of inflation risk premia during the crisis was also strong. In tandem with inflation skewness, the long-term inflation risk premium gradually declined from its peaks at around 40 basis points in late 2008 to near-zero levels in mid-2010, the lowest in the euro area period. Those dynamics fully explain the strongly positive coefficient for inflation skewness measures and the negative coefficients found for inflation uncertainty measures in the previous section.

Model-free proxies for the long-term inflation risk premium also moved closely in tandem with inflation skewness (see Figure 9). Indeed, during the financial crisis period, our inflation-linked swap measures display an even closer comovement to the perceived asymmetries in long-term inflation risks than our model estimates.

8 Concluding remarks

The goal of this paper is to investigate the link between the inflation risks in macroeconomic forecasts and the inflation risk premia embodied in the term structure of interest rates. Using new evidence on the term structure of both inflation risk premia and perceived inflation risks in the euro area, we show that the dynamics of long-term inflation risk premia mimics quite closely that of the perceived asymmetries of inflation risks. Indeed, it is the perceived balance of risks, and not the inflation uncertainty, which helps understand the dynamics of inflation risk premia, particularly over long horizons and the most recent period. Some empirical evidence support the robustness of that result. It holds after controlling for a large number of macroeconomic and financial factors that capture inflationary pressures, economic activity, business confidence and standard financial risk indicators, alternative term structure specifications, and also for model-free proxies of inflation risk premium constructed using inflation-linked swaps.

By bringing together forecast risk measures, mainly based on macroeconomic data, and inflation risk premia, mainly based on financial data, our findings may open new avenues to explore inflation expectation formation, bond pricing and term structure modelling, and contribute to extent the survey forecasts beyond the point predictions.

From the point of view of inflation expectation analysis, further work on the determinants of inflation risks would be welcome, not only to improve our understanding of expectation formation in general but also the role of higher-order moments of inflation expectations in other areas like asset pricing. Indeed, we showed that at short horizons inflation uncertainty helps explain inflation risk premium, but at longer horizons the dynamics of inflation risk premia reflects the balance of inflation risks, which suggest that markets price in risk of inflation overshooting its expected path rather than their spread. The increasing emphasis on inflation risks in central bank communication, and also media and market participants is likely to play a role in that relationship. In any case, explaining the forces at work leading to our main finding can be an important topic of research for theoretical models of asset pricing.

From the perspective of term structure modelling, our findings also suggest that there is substantial information in survey-based inflation risks beyond that included in standard term structure models. Such information can be very useful for the modelling of the term structure of interest rates and its interpretation. Our results may offer a new avenue to combine macroeconomic and financial information without the need to impose restrictions from a rich macroeconomic model onto the term structure. In addition, the stylized facts about the perceived inflation risks that we report here could support new models capable of exploiting the non-Gaussian features of the inflation risks to improve the estimation of term structure models. Moving beyond a Gaussian framework while retaining the tractability and interpretability of term structure models is likely to be quite challenging, but it opens a wide range of new possibilities for term structure modelling.

Appendix

A Term structure model specification

A.1 Bond prices

A.1.1 Real bond prices

The price of a one-period real bond $\hat{P}_t^1$ in time t is the expected value of the pricing kernel in time $t+1$

$$\hat{P}_t^1 = E_t(\hat{M}_{t+1}).$$

Substituting the pricing kernel (stochastic discount factor) by $-r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1}$ and applying basic properties of the normal distribution[18] we get

$$\hat{P}_t^1 = E_t(\exp(-r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1})) = \exp(-r_t) = \exp(-\delta_0 - \delta_1'X_t).$$

Comparing this equation with the exponentially affine pricing equation $\hat{P}_t^1 = \exp(\hat{A}_1 + \hat{B}_1'X_t)$ gives the starting values for the recursive computation of the factor loadings $\hat{A}_1 = -\delta_0$ and $\hat{B}_1' = -\delta_1$.

In general, the price of a $n+1$ maturity bond in time t is the expected value of the (stochastically) discounted price of the same bond at time $t+1$ value of a maturity n bond

$$\hat{P}_t^{n+1} = E_t(\hat{M}_{t+1}\hat{P}_{t+1}^n).$$

Substituting the definition of the pricing kernel and applying the exponentially affine pricing rule gives

$$\hat{P}_t^{n+1} = E_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\exp\left(\hat{A}_n + \hat{B}_n'X_{t+1}\right)\right].$$

[18] If X is a normal distributed random variable, $Y = e^x$ is log-normal distributed with $E(Y) = e^{E(x)+\frac{1}{2}Var(x)}$.

Substitution further for X_{t+1} and rearranging gives

$$\begin{aligned}\hat{P}_t^{n+1} &= E_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1} + \hat{A}_n + \hat{B}_n'(\mu + \Phi X_t + \Sigma\epsilon_{t+1})\right)\right] \\ &= \exp\left(-r_t - \frac{1}{2}\lambda_1'\lambda_t + \hat{A}_n + \hat{B}_n'(\mu + \Phi X_t)\right) E_t\left[\exp\left((\hat{B}_n'\Sigma - \lambda_t')\epsilon_{t+1}\right)\right].\end{aligned}$$

Using properties of the normal distribution gives[19]

$$\hat{P}_t^{n+1} = \exp\left(-r_t - \frac{1}{2}\lambda_1'\lambda_t + \hat{A}_n + \hat{B}_n'(\mu + \Phi X_t) + \frac{1}{2}(\hat{B}_n'\Sigma - \lambda_t')(\hat{B}_n'\Sigma - \lambda_t')'\right)$$

Using the affine pricing rule $\hat{P}_t^{n+1} = \exp\left(\hat{A}_{n+1} + \hat{B}_{n+1}'X_t\right)$ on the right side of this equation and substituting for the market price of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$ and the short rate $r_t = -\delta_o - \delta_1' X_t$ gives

$$\begin{aligned}\exp\left(\hat{A}_{n+1} + \hat{B}_{n+1}'X_t\right) &= \exp\Big(-\delta_0 + \hat{A}_n + \hat{B}_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}\hat{B}_n\Sigma\Sigma\hat{B}_n' \\ &\quad + (\hat{B}_n'(\Phi - \Sigma\lambda_1) - \delta_1')X_t\Big).\end{aligned}$$

Equating the constant terms and the terms multiplied by X_t on both sides of this equation gives

$$\begin{aligned}\hat{A}_{n+1} &= -\delta_0 + \hat{A}_n + \hat{B}_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}\hat{B}_n\Sigma\Sigma\hat{B}_n' \\ \hat{B}_{n+1}' &= -\delta_1' + \hat{B}_n'(\Phi - \Sigma\lambda_1)\end{aligned}$$

A.1.2 Nominal bond prices

Nominal bonds are priced by using the nominal pricing kernel M_{t+1} instead of the real pricing kernel. Both pricing kernels are linked by inflation $M_{t+1} = \hat{M}_{t+1}/\Pi_{t+1}$. This implies for the log pricing kernel[20] $m_{t+1} = \log(M_{t+1}) = \hat{m}_{t+1} - \pi_{t+1}$.

[19] Note that Σ is symmetric, that is $\Sigma' = \Sigma$.

[20] Using $\pi_{t+1} = \log(\Pi_{t+1})$ and $\hat{m}_{t+1} = \log(\hat{M}_{t+1})$

Using the definition of the real pricing kernel and the fact that inflation π_t is included in the state vector[21] the log nominal pricing kernel can be written as

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1} - e_N'X_{t+1}.$$

In the next step, the price of a one-period nominal bond can be computed by

$$P_t^1 = \exp(A_1 + B_1X_t) = E_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1} - e_N'X_{t+1}\right)\right].$$

Substituting for r_t and X_{t+1} and rearranging terms gives

$$\begin{aligned}\exp(A_1 + B_1X_t) &= E_t\left[\exp\left(-\delta_0 - (\delta_1' + e_N'\Phi)X_t - \frac{1}{2}\lambda_1'\lambda_t - e_N'\mu - (\lambda_t' + e_N'\Sigma)\epsilon_{t+1}\right)\right] \\ &= \exp\left(-\delta_0 - (\delta_1' + e_N'\Phi)X_t - \frac{1}{2}\lambda_1'\lambda_t - e_N'\mu\right) E_t\left[\exp\left(-(\lambda_t' + e_N'\Sigma)\epsilon_{t+1}\right)\right].\end{aligned}$$

Solving the expectations term and substituting for λ_t we get[22]

$$\exp(A_1+B_1X_t) = \exp\left(-\delta_0 - (\delta_1' + e_N'\Phi)X_t - e_N'\mu + \frac{1}{2}e_N'\Sigma\Sigma e_N + e_N'\Sigma\lambda_0 + e_N'\Sigma\lambda_1X_t\right).$$

Equating constant terms and terms multiplied by X_t on both sides gives

$$\begin{aligned}A_1 &= -\delta_0 - e_N'\mu + \frac{1}{2}e_N'\Sigma\Sigma e_N + e_N'\Sigma\lambda_0) \\ B_1' &= e_N'\Sigma\lambda_1 - (\delta_1' + e_N'\Phi)\end{aligned}$$

In general, similar to the case of real bond prices, we have

$$\begin{aligned}P_t^{n+1} = \exp\left(A_{n+1} + B_{n+1}'X_t\right) = E_t[\exp(-\delta_0 + \delta_1'X_t - \frac{1}{2}\lambda_1'\lambda_t - \lambda_t'\epsilon_{t+1} \\ - e_N'X_{t+1} + A_n + B_n'X_{t+1})].\end{aligned}$$

[21] As inflation π_t is the last variable in the state vector X_t which is of dimension N and the vector e_N (which contains only zeros with the exception of the element N which is one) can be used to extract π from the state vector.

[22] We use the relationship $(\lambda_t'+e_N'\Sigma)(\lambda_t'+e_N'\Sigma)' = (\lambda_t'+e_N'\Sigma)(\Sigma'e_N+\lambda_t) = e_N'\Sigma\Sigma e_N+\lambda_t'\lambda_t+2e_N'\Sigma\lambda_t$.

After substituting for X_{t+1} we get

$$\begin{aligned}\exp\left(A_{n+1}+B'_{n+1}X_t\right) &= E_t[\exp(-\delta_0+\delta'_1X_t-\frac{1}{2}\lambda'_1\lambda_t+((B'_n-e'_N)\Sigma-\lambda'_t)\epsilon_{t+1}\\ &+ A_n+(B'_n-e'_N)\mu+(B'_n-e'_N)\Phi X_t)].\end{aligned}$$

Solving for the expectation term[23] and substituting for λ_t gives after rearrangements

$$\begin{aligned}\exp\left(A_{n+1}+B'_{n+1}X_t\right) &= \exp\Big[-\delta_0+A_n+(B_n-e'N)(\mu-\Sigma\lambda_0)\\ &+\frac{1}{2}(B_n-e'N)\Sigma\Sigma(B_n-e'N)+((B_n-e'N)\Phi-\delta'_1-(B_n-e'N)\Sigma\lambda_1)X_t\Big].\end{aligned}$$

Again, after equating constant terms and terms multiplied by X_t on both sides of the equation we get

$$\begin{aligned}A_{n+1} &= -\delta_0+A_n+(B'_n-e'_N)(\mu-\Sigma\lambda_0)-\frac{1}{2}(B'_n-e'_N)\Sigma\Sigma(B'_n-e'_N)'\\ B'_{n+1} &= (B'_n-e'_N)(\Phi-\Sigma\lambda_1)-\delta'_1\end{aligned}$$

A.2 State-space representation

In order to use Kalman filter estimation techniques, we first express the affine term structure model in a state-space form:

$$\begin{aligned}w_t &= d+ZX_t+\eta_t && \text{(measurement equation)}\\ X_t &= \Phi X_{t-1}+\Sigma\epsilon_t && \text{(state equation).}\end{aligned}$$

The vector d and the matrix Z, which link the state variables (latent factors) with the observed data, are constructed by using the coefficients $\hat{A}_n$, $\hat{B}'_n$, A_n, and B'_n described in section A.1. The transformations of the bond prices into bond yields are done by using the relation $P^n_t=\exp(-y^n_tn)$ or $y^n_t=-\log(P^n_t)/n$.

[23] We use the relationship $((B'_n-e'_N)\Sigma-\lambda'_t)((B'_n-e'_N)\Sigma-\lambda'_t)'=(B'_n-e'_N)\Sigma-\lambda'_t)(-\lambda_t+\Sigma(B'_n-e'_N)')=\lambda'_t\lambda_t+(B'_n-e'_N)\Sigma\Sigma(B'_n-e'_N)'-2(B'_n-e'_N)\Sigma\lambda_t$

The vector of observed data w_t contains the bond yields and, if the model includes them, the SPF inflation expectations:

$$\mathbf{w}_t = \begin{bmatrix} y_t^3 \\ y_t^{12} \\ y_t^{24} \\ y_t^{36} \\ y_t^{60} \\ \pi_t \\ \hat{y}_t^{24} \\ \hat{y}_t^{36} \\ \hat{y}_t^{60} \\ E_t^{\mathrm{SPF}}[\pi_{t+12}] \\ E_t^{\mathrm{SPF}}[\pi_{t+24}] \\ E_t^{\mathrm{SPF}}[\pi_{t+60}] \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} -A_3/3 \\ -A_{12}/12 \\ -A_{24}/24 \\ -A_{36}/36 \\ -A_{60}/60 \\ 0 \\ -\hat{A}_{24}/24 \\ -\hat{A}_{36}/36 \\ -\hat{A}_{60}/60 \\ e_3(I-\Phi^{12})(I-\Phi)^{-1}\mu \\ e_3(I-\Phi^{24})(I-\Phi)^{-1}\mu \\ e_3(I-\Phi^{60})(I-\Phi)^{-1}\mu \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} -B_3/3 \\ -B_{12}/12 \\ -B_{24}/24 \\ -B_{36}/36 \\ -B_{60}/60 \\ 0 \\ -\hat{B}_{24}/24 \\ -\hat{B}_{36}/36 \\ -\hat{B}_{60}/60 \\ e_3\Phi^{12} \\ e_3\Phi^{24} \\ e_3\Phi^{60} \end{bmatrix}.$$

The structure of the variance-covariance matrix of the measurement errors is as follows

$$\begin{bmatrix} \sigma_\epsilon^2(1) & 0 & \dots & 0 \\ 0 & \sigma_\epsilon^2(2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_\epsilon^2(12) \\ & & & \end{bmatrix},$$

We impose the following restrictions in its main diagonal elements

$$\begin{aligned} \sigma_\epsilon^2(1), \dots, \sigma_\epsilon^2(5) &= \tilde{\sigma}_\epsilon^2(1), \\ \sigma_\epsilon^2(6) &= 0, \\ \sigma_\epsilon^2(7), \dots, \sigma_\epsilon^2(9) &= \tilde{\sigma}_\epsilon^2(2), \\ \sigma_\epsilon^2(10), \dots, \sigma_\epsilon^2(12) &= \tilde{\sigma}_\epsilon^2(3), \end{aligned}$$

which implies that (i) inflation π_t is measured without error; (ii) data freely determines the variances of the measurement errors for nominal yields, real yields and survey inflation expectations. The transition equation of the state-space is identical to the transition equation described in the main text.

B Measuring inflation expectations and inflation risks

As for the more widely-used US SPF currently run by the Federal Reserve Bank of Philadelphia, survey panelists of the ECB's SPF assign probabilities to future inflation falling into pre-specified ranges, i.e. a density forecast in the form of a histogram. Every survey round, the density forecast (histogram) reported by each panelist are aggregated to construct a *combined* density forecast, which reflects the average probability assigned to each interval. Such histograms however do not provide quantitative measures of the perceived inflation risks. To obtain accurate proxies for inflation uncertainty and skewness, we fit a continuos but potentially asymmetric density to them.

This appendix provides some additional of our fitting methodology. We interpret those SPF histograms as a discretized version of a continuos density forecast. The probabilities assigned to each survey interval should therefore correspond to the integrals of the underlying density function over each of the pre-specified intervals (α_{i-1}, α_i), $i = 1, \ldots, I$, where $\alpha_0 := -\infty$, and $\alpha_I := \infty$. In practice, however, it is unlikely that survey participants discretize their density forecasts by computing those integrals. As working hypothesis, we then assume that the discretization resembles a sampling experiment based on "draws" from their continuous density forecast. The reported probabilities reflect how many of those draws lie within each of the intervals, and therefore interpret the reported probabilities as the realization of a multinomial random variable with I classes. In this framework, the observed frequencies $(\widehat{p})$ are a sufficient statistic for estimating the theoretical probabilities (p, with p_i denoting the probability mass of the theoretical density in the interval (α_{i-1}, α_i)).

Our inference problem is to find the parameter vector ϱ that defines the theoretical density function by matching the reported frequencies of the SPF histograms. In this context, least squares, the fitting criterion usually employed in existing literature on SPF data, is not efficient.[24] In search for an efficient criterion, following Cressie and Read (1988), we use a small departure from maximum likelihood estimation within the family of "power divergence estimators".[25] Indexed by the parameter $\tau \in R$, that family is defined as the estimators obtained by minimizing $I^{\tau}(\widehat{p}, p) = [1/\tau(\tau+1)] \sum_{i=0}^{I} [\widehat{p}_i[\widehat{p}_i/p_i(\varrho)]^{\tau} - 1]$ with respect to ϱ. While exhibiting optimal large sample properties, more robust power distance estimators underperform with respect to maximum likelihood estimation in terms of efficiency in small samples (in the multinomial framework, a small number of draws to discretize the density forecast and report the SPF histogram). Within that family of estimators, the researcher can choose an efficient fitting criterion by taking into account the small sample properties of the power divergence estimators and the characteristics of the SPF data. Specifically, an inspection of the SPF data suggests that (numerical) robustness to *inliers* (i.e. intervals with much lower observed probability than the theoretical density suggests, for example related to rounding) is fundamental. Monte Carlo simulations specifically designed to match those particularities of the SPF data confirm that a small departure from maximum likelihood estimation (that is, a positive but relatively low value of the parameter τ (τ=0.2)) is optimal for the SPF data (see Cressie and Read, 1988, and Garcia and Manzanares, 2007).

As underlying density we employ a potentially skewed distribution, Azzalini's (1985) skew-normal family whose main features are described in the main text. Monte Carlo evidence confirms that these two methodological contributions, namely our fitting criterion and the skew-normal density, lead to significant accuracy gains in the estimation of the key moments of the SPF histograms (Garcia and Manzanares, 2007).

[24] Although widely-used to fit densities to the SPF histograms, the least squares criterion (although consistent) is not efficient: it assigns equal weight to the fitting errors for each interval, while an efficient criterion would weight fitting errors differently according to the probability assigned to each interval to improve the estimation.

[25] The Pearson and the Neyman Chi-Square criteria, the Hellinger distance, and the Kullbach-Leibler divergence belong to this family of estimators, and maximum likelihood is a limiting case when $\tau \rightarrow 0$.

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Table 1: Summary statistics of euro area yield curve data

	Central moments (Full-sample: 1999Q1-2010Q3)				Central moments (Pre-crisis: 1999Q1-2007Q2)				Autocorrelation (Full-sample)		
	Mean	**STDev.**	**Skew**	**Kurtosis**	**Mean**	**STDev.**	**Skew**	**Kurtosis**	**Lag 1**	**Lag 2**	**Lag 3**
Nominal yields											
1-year yield	2.98	1.18	-0.37	-0.61	3.15	0.93	0.58	-0.69	0.97	0.94	0.89
2-year yield	3.17	1.07	-0.31	-0.66	3.35	0.87	0.42	-0.96	0.97	0.93	0.88
5-year yield	3.74	0.85	-0.13	-0.73	3.88	0.75	0.24	-1.14	0.97	0.93	0.90
Real yields											
2-year yield	1.10	0.75	0.00	-0.87	0.77	0.51	0.42	-1.07	0.95	0.90	0.85
5-year yield	1.36	0.56	0.12	-0.67	1.23	0.34	-0.34	-1.02	0.93	0.86	0.81
Forward BEIRs											
1-year BEIR	1.98	0.46	-0.56	1.33	2.04	0.23	-0.39	0.90	0.95	0.88	0.78
1-year forward BEIR ending in two years	2.03	0.24	-0.49	-0.38	2.07	0.16	0.44	-0.87	0.96	0.90	0.82
1-year forward BEIR ending in five years	2.23	0.14	0.23	-0.34	2.25	0.14	0.33	-0.59	0.92	0.81	0.73

Note: table statistics refer to the monthly data over the period 1999M1-2010M9 used in the estimation of our term structure model. Nominal yields are (AAA) zero coupons as reported in Bloomberg; real yields are zero-coupon adjusted for inflation seasonality estimated following Ejsing et al. (2007), and their statistics refer to the period 2004Q1-2010Q3 due to the limited number of inflation-linked bonds available for their estimation before 2004; forward break-even inflation rates (or inflation compensation) are based on our benchmark term structure model as described in Section 2.

Table 2: Parameter estimation (benchmark model especification)

	Estimated parameters	99% Confidence bounds	
		lower bound	upper bound
$\mu_\pi * 1200$	0.1570	0.0362	0.2088
Φ_{11}	0.9591	0.8651	0.9993
Φ_{22}	0.9592	0.9288	0.9999
Φ_{33}	0.9206	0.8888	0.9849
Φ_{21}	0.0896	0.0295	0.1903
$\Phi_{31} * 1200$	0.0022	0.0004	0.0138
$\Phi_{32} * 1200$	-0.0023	-0.0030	-0.0001
$\sigma_\pi * 12 * 10^5$	0.0292	0.0055	0.0895
$\lambda_{0,1}$	-0.3039	-0.7257	-0.0055
$\lambda_{0,2}$	0.4344	0.0690	0.7599
$\lambda_{1,11}$	-0.0047	-0.0102	-0.0014
$\lambda_{1,12}$	0.0170	0.0078	0.0358
$\lambda_{1,21}$	0.0429	0.0037	0.0564
$\lambda_{1,22}$	-0.0513	-0.0639	-0.0031
$\delta_0 * 1200$	1.2243	1.2243	1.2243
$\delta_{1,1} * 100$	0.0123	0.0004	0.0110
$\delta_{1,2} * 100$	-0.0112	-0.0239	-0.0035
$\tilde{\sigma}^2_\epsilon(1) * 12 * 10^5$	0.0129	0.0026	0.0162
$\tilde{\sigma}^2_\epsilon(2) * 12 * 10^5$	0.0090	0.0038	0.0504
$\tilde{\sigma}^2_\epsilon(3) * 12 * 10^5$	0.0149	0.0091	0.1011

Note: the Table entries show the estimates for the key parameters of the model. Confidence bounds are constructed by MCMC approach as advocated by Chernozhukov and Hong (2003).

Table 3: Volatility of survey and model inflation expectations

Horizon	Survey inflation expectations	Model without surveys	Model including surveys
in one year	20	25	18
in two years	11	17	8
in five years	5	8	3

Note: the table entries are the standard deviations of survey inflation expectations and the corresponding estimates from our benchmark term structure model especification for each horizon, in basis points. Our survey data are form the ECB´s Survey of Professional Forecasters. Results from two versions of the term structure model, including and not including survey data in the estimation, are reported. The higher volatility of the model inflation expectations without survey data translates into a too low a volatility of inflation risk premia estimates.

Table 4: The fitting of nominal bond yields under different model specifications

Horizon	Model without surveys	Model including surveys
One year ahead	12	13
Two year ahead	6	7
Five year ahead	10	7

Note: the Table entries report the root mean-square fitting errors for the nominal yields in models including and not including survey inflation expectations. Using survey data does not increase fitting errors significantly.

Table 5: Decomposition of inflation compensation (BEIRs)

Panel A: Average levels of inflation expectations and risk premia

One-year forward	Expected inflation	Inflation risk premium
in one year	1.94	0.03
in two years	1.91	0.08
in five years	1.90	0.23

Panel B: In-sample variance decomposition of inflation compensation

	Expected inflation	Inflation risk premium
in one year	0.70	0.30
in two years	0.65	0.35
in five years	0.15	0.85

Note: Figures are averages of the monthly estimates of inflation compensation (BEIRs), expected inflation and inflation risk premia from our term structure model over the sample 1999M1-2010M9. The relative contributions of each component to the variance of inflation compensation are calculated according to

$$\frac{cov(BEIR_t^n, E_t(\pi_t^n))}{Var(BEIR_t^n)} + \frac{cov(BEIR_t^n, \phi_t^n)}{Var(BEIR_t^n)} = 1$$

Table 6: The comovement of inflation, expected inflation and inflation risks

Panel A: Short-term expectations (one year ahead)

	Level	**Uncertainty**		**Asymmetry**	
	Mean	Aggregate Uncertainty (from combined prob. forecast)	Average Uncertainty (from individual prob. forecasts)	Skewness	Mean-mode
Mean	(0.27)				
Aggregate Uncertainty (combined prob. forecast)	-0.5	(0.12)			
Average Uncertainty (individual prob. forecasts)	0.5	0.5	(0.10)		
Skewness	0.2	-0.1	0.1	(0.18)	
Mean-mode	0.3	-0.1	0.1	1	(0.12)
Headline inflation	0.8	-0.6	0.4	0.2	0.2
Core inflation	0.5	-0.2	0.5	0.2	0.2
Output Gap	0.7	-0.5	0.4	0.2	0.2

Panel B: Long-term expectations (five years ahead)

	Level	**Uncertainty**		**Asymmetry**	
	Mean	Aggregate Uncertainty (from combined prob. forecast)	Average Uncertainty (from individual prob. forecasts)	Skewness	Mean-mode
Mean	(0.06)				
Aggregate Uncertainty (combined prob. forecast)	-0.1	(0.07)			
Average Uncertainty (individual prob. forecasts)	0.0	0.8	(0.06)		
Skewness	0.2	-0.1	-0.3	(0.13)	
Mean-mode	0.2	-0.3	-0.4	1	(0.10)
Headline inflation	0.2	-0.5	-0.4	0.1	0.1
Core inflation	0.2	-0.5	-0.3	0.1	0.1
Output Gap	0.1	-0.7	-0.7	0.3	0.3

Note: figures within brackets are standard deviations; other entries report correlation coefficients. Our survey data are estimated from the ECB´s Survey of Professional Forecasters (SPF) by fiiting a continuos distribution to the survey density forecasts (histograms) as described in Section 4 and Appendix B. Agg. Uncertainty refers to the variance of the combined (or aggregate) SPF density forecast constructed by averaging the individual density forecasts; Ave. Uncertainty refers to the average of the variances of the individual forecasts. Risk measures estimated following Garcia and Manzanares (2007). Core inflation refers to the HICP index excluding energy and unprocessed food prices, as published by Eurostat. Output gap is calculated based on a quadratic trend.

Table 7: Inflation risk premia and inflation risks

	Inflation uncertainty					
	One year ahead		Two years ahead		Five years ahead	
One-year premium	Agg. Uncertainty	Ave. uncertainty	Agg. Uncertainty	Ave. uncertainty	Agg. Uncertainty	Ave. uncertainty
in one year	**-0.5**	**0.4**	-0.6	0.4	-0.5	-0.4
ending in two years	-0.6	0.3	**-0.7**	**0.3**	-0.7	-0.7
ending in five years	-0.4	0.3	-0.4	0.3	**-0.7**	**-0.7**

	Asymmetry in inflation risks		
One-year premium	One year ahead	Two years ahead	Five years ahead
in one year	**0.2**	0.2	0.1
ending in two years	0.2	**0.2**	0.4
ending in five years	0.2	0.3	**0.5**

Note: perceived inflation risks are calculated from the ECB´s Survey of Professional Forecasters (SPF) by fiiting a continuos distribution to the survey density forecasts (histograms) as described in Section 4 and Appendix B. Agg. Uncertainty refers to the variance of the combined (or aggregate) SPF density forecast constructed by averaging the individual density forecasts (histograms); Ave. Uncertainty refers to the average of the variances of the individual density forecasts. We measure uncertainty by the variance rather than the standard deviation because, by construction, the link between the variance of the combined density forecast and the average of individual variances that does not hold for the standard deviations. Specifically Agg. Variance = Ave. Uncertainty + Disagreement, with average uncertainty being the main component of the variance of the combined probability forecasts (see Garcia and Manzanares, 2007, and references therein for a detailed discussion).

Table 8: Inflation risks and long-term inflation risk premium in the euro area

Dependent variable is one-year forward inflation risk premium ending in five years (1999Q1-2007Q2)

	Benchmark term structure model								Alternative model specification	
Regressor	I	II	III	IV	V	VI	VII	VIII	IX	X
Constant	0.47	0.58	0.52	0.40	0.47	0.33	0.47	0.33	0.19	0.19
	(0.09)	(0.06)	(0.06)	(0.12)	(0.09)	(0.11)	(0.090)	(0.11)	(0.19)	(0.19)
Inflation skewness	**0.26*** **	**0.25*** **	**0.21*** **	**0.17** **	**0.21** **	**0.15*** **			**0.25*** **	
(in five years)	(0.09)	(0.07)	(0.07)	(0.08)	(0.076)	(0.041)			(0.078)	
Mean-mode							**0.35** **	**0.19*** **		**0.31** **
(in five years)							(0.13)	(0.062)		(0.11)
Inflation uncertainty	-0.73***	-0.85***	-0.82***	-0.37	-0.86***	-0.17	-0.73***	-0.16	-0.30	-0.28
(in five years)	(0.23)	(0.17)	(0.18)	(0.28)	(0.15)	(0.16)	(0.23)	(0.17)	(0.29)	(0.31)
Wages		-0.025***				-0.003***		-0.003***	-0.004***	-0.004***
		(0.004)				(0.7E-3)		(0.7E-3)	(0.001)	(0.001)
Unemployment rate			0.21***			0.11***		0.10**	0.17*	0.16*
			(0.058)			(0.082)		(0.049)	(0.086)	(0.09)
Output gap			7.24***			5.86**		5.57***	5.59**	5.12**
			(2.56)			(1.43)		(1.53)	(2.26)	(2.40)
Consumer confidence				0.005*		0.01***		0.010***	0.015***	0.015***
				(0.002)		(0.003)		(0.003)	(0.004)	(0.005)
Term spread					0.056*	0.09***		0.086***	0.14***	0.14***
					(0.031)	(0.012)		(0.013)	(0.019)	(0.020)
Bond market volatility					-0.035*	-0.025		-0.025	-0.035	-0.035
					(0.019)	(0.019)		(0.020)	(0.031)	(0.032)
VIX					0.01***	0.006***		0.005***	0.007***	0.007***
					(0.001)	(0.001)		(0.001)	(0.002)	(0.002)
Adjusted R-square	0.30	0.72	0.60	0.31	0.65	0.87	0.30	0.86	0.82	0.81
RSS	0.12	0.04	0.06	0.11	0.05	0.02	0.12	0.02	0.04	0.04
Joint significance (p-value)	0.006	0.000	0.000	0.011	0.000	0.000	0.006	0.000	0.000	0.000

Note: each column reports a regression of the long-term inflation risk premium on our two measures of perceived asymmetries in inflation risks (inflation skewness and the distance between the mean and the mode of the density forecast) inflation uncertainty and different combinations of macroeconomic and financial factors. Perceived inflation risks are estimated from the ECB´s Survey of Professional Forecasters (SPF) by fiiting a continuos distribution to the survey density forecasts (histograms) as described in Section 4 and Appendix B. Inflation uncertainty is measured as average uncertainty in the individual density forecasts. Wages refers to the negotiated wage index calculated by the ECB to incorporate the latest wage aggreements reached in the euro area as a whole. Unemployment rate is year-on-year change in the euro area-wide figure as published by Eurostat. The output gap is estimated by fitting a quadratic trend to euro area real GDP data between 1995-2010. Consumer confidence is from the EC surveys. Term spread is measured as the 2- and 10-year bond yield spread. The additional market risk proxies are the bond market and the stock market (VIX) volatility.

*** indicates significance at 1%, ** at 5% and * at 10%, with HAC robust (Newey-West) standard errors reported within brackets.

Table 9: Inflation risks and long-term inflation risk premium: additional evidence

	Dependent variable is one-year forward inflation risk premium ending in five years					
	Model-based premium during full ECB period (1999Q1-2010Q3)		**Model-free premium (2002Q3-2010Q3)**			
			I/L forward swap rate adjusted by survey expectations		**I/L forward swap rate over 2% rate**	
Regressors						
Constant	0.37	0.36	0.20	0.16	0.08	0.04
	(0.18)	(0.18)	(0.27)	(0.34)	(0.24)	(0.27)
Inflation skewness (five years ahead)	**0.17****		**0.56****		**0.56***	
	(0.08)		(0.27)		(0.30)	
Mean-mode (in five years)		**0.21****		**0.79****		**0.80****
		(0.10)		(0.35)		(0.38)
Inflation uncertainty (five years ahead)	-0.95***	-0.94***	-1.18*	-1.14*	-1.08*	-1.03*
	(0.27)	(0.27)	(0.57)	(0.62)	(0.60)	(0.59)
Wages	- 0.001	- 0.001	-0.005	0.005	-0.007	-0.007
	(0.001)	(0.001)	(0.003)	(0.003)	(0.005)	(0.004)
Unemployment rate	-0.022	-0.024	-0.12	-0.13	-0.09	-0.11
	(0.045)	(0.046)	(0.010)	(0.083)	(0.10)	(0.092)
Output gap	0.66	0.63	4.25	4.37	9.81*	9.94**
	(1.55)	(1.51)	(4.35)	(3.45)	(5.19)	(4.52)
Consumer confidence	0.003	0.003	-0.017	-0.017*	-0.024**	-0.025***
	(0.002)	(0.002)	(0.010)	(0.009)	(0.010)	(0.008)
Term spread	0.031	0.030	0.085	0.093	0.10	0.10
	(0.024)	(0.024)	(0.070)	(0.066)	(0.066)	(0.064)
Bond market implied volatility	0.016	0.015	0.06	0.062	0.050	0.052
	(0.020)	(0.020)	(0.05)	(0.05)	(0.054)	(0.052)
VIX	0.006***	0.006***	-0.008*	-0.008*	-0.010**	-0.010**
	(0.001)	(0.002)	(0.006)	(0.004)	(0.005)	(0.004)
Adjusted R-square	0.60	0.59	0.30	0.35	0.44	0.47
RSS	0.10	0.10	0.53	0.5	0.58	0.55
Joint significance test (p-value)	0.000	0.000	0.039	0.024	0.006	0.003

Note: each column reports a regression of different measures of long-term inflation risk premium on our two measures of perceived asymmetries in inflation risks (inflation skewness measured by the standard normalised third moment and the distance between the mean and the mode of the aggregate SPF probability forecast) and a serie of macroeconomic and financial factors. The model-based inflation risk premium is from our term structure model over the period 1999Q1-2010Q3. Model-free measures of the inflation risk premium are calculated for the period over which euro area I/L swap rates are available. The first model-free measure is constructed by detracting survey long-term inflation expectations from the one-year forward I/L swap rate ending in five years. The second model-free measure is constructed by detracting a constant inflation rate of 2% from that forward I/L swap rate.
Perceived inflation risks are estimated from the inflation probability forecasts of the ECB´s SPF five years ahead following Garcia and Manzanares (2007). Inflation uncertainty is measured as average uncertainty in the individual probability forecasts. Wages refers to the negotiated wage index calculated by the ECB to incorporate the latest wage aggreements reached in the euro area as a whole. Unemployment rate is year-on-year change in the euro area-wide figure as published by Eurostat. The output gap is estimated by fitting a quadratic trend to euro area real GDP data between 1995-2010. Consumer confidence is from the EC surveys. Term spread is measured as the 2- and 10-year bond yield spread. The additional market risk proxies are the bond market and the stock market (VIX) volatility. The output gap is estimated from HP filtered quarterly real GDP data between 1995-2010. Consumer confidence is from the EC surveys. Yield curve slope is measured as the 2- and 10-year yield spread. *** indicates significance at 1%, ** at 5% and * at 10%, with HAC robust (Newey-West) standard errors reported within brackets.

Forward inflation compensation (BEIR), inflation expectations and risk premia

Figure 1: BEIR ending in one year

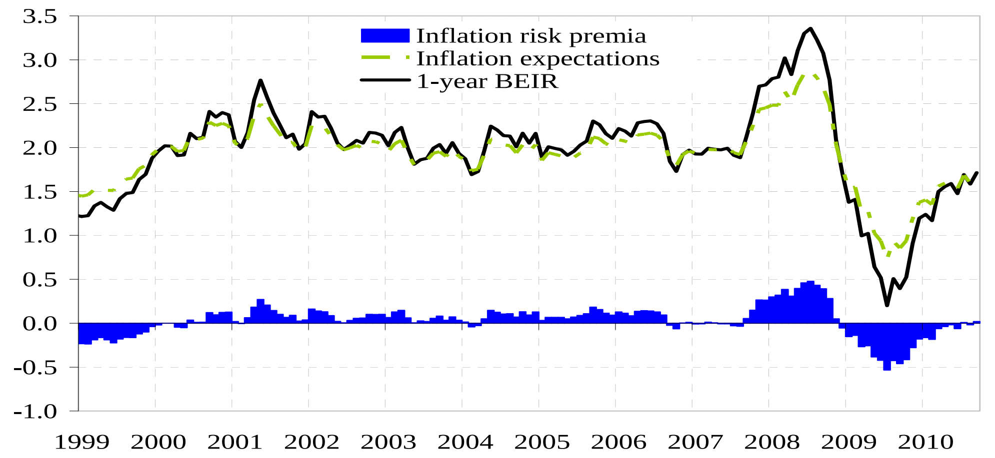


Figure 2: One-year forward ending in two years

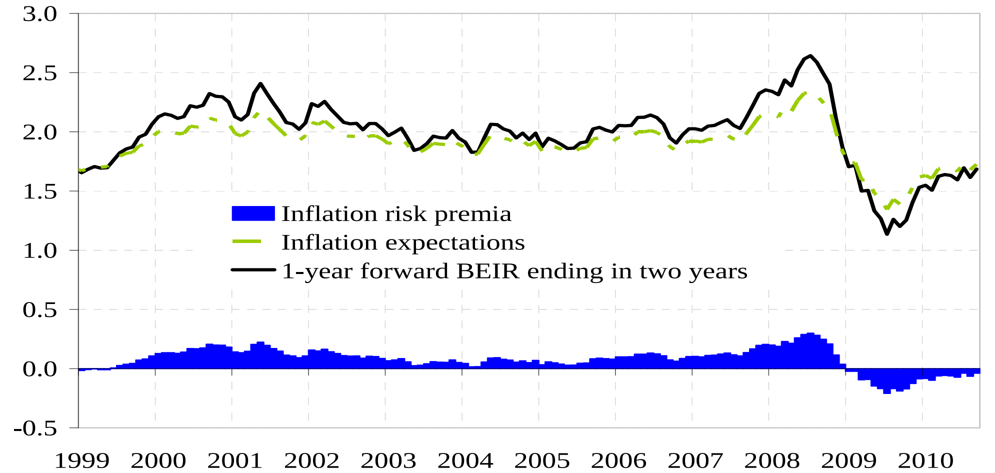
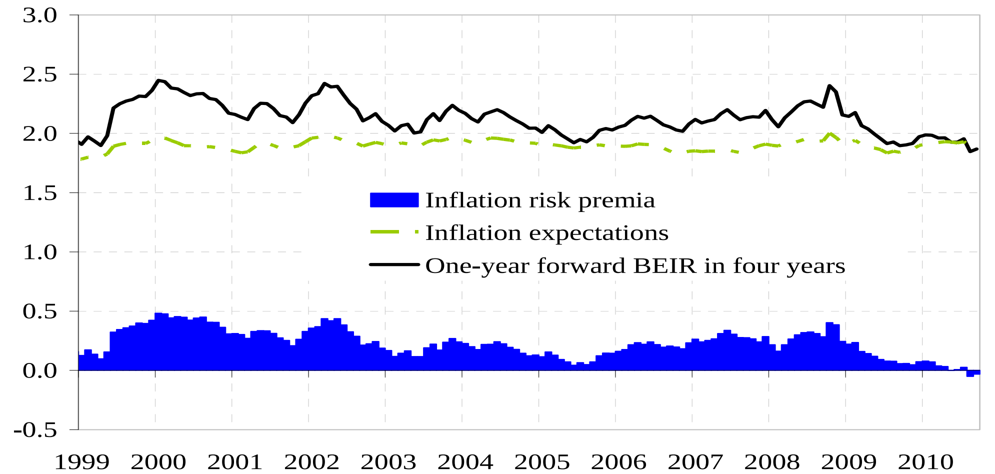


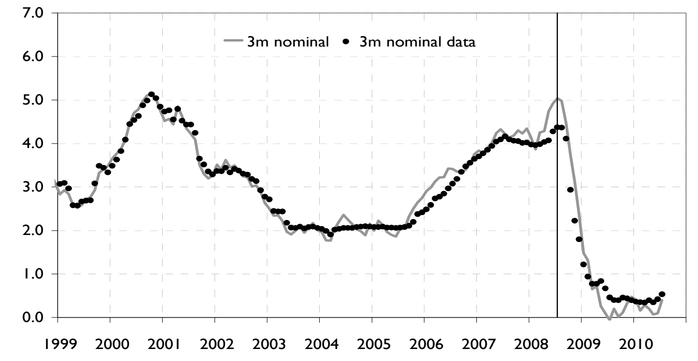
Figure 3: One-year forward ending in five years



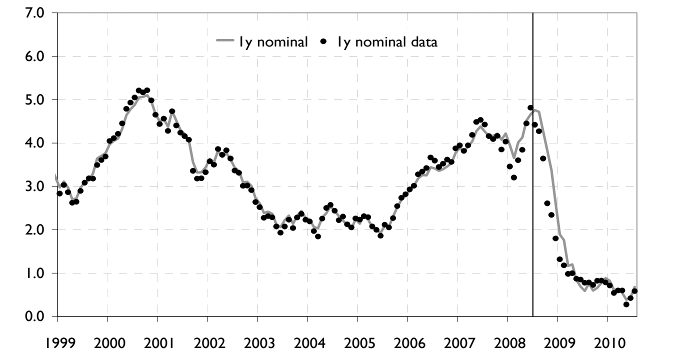
Note: The charts depict the decomposition of the break-even inflation rates (BEIRs, or inflation compensation) estimated from our term structure model into the expected level of inflation and the inflation risk premium associated with it. Data are in percentage points.

Figure 4: Term structure model measurement equations: fitted and observed values

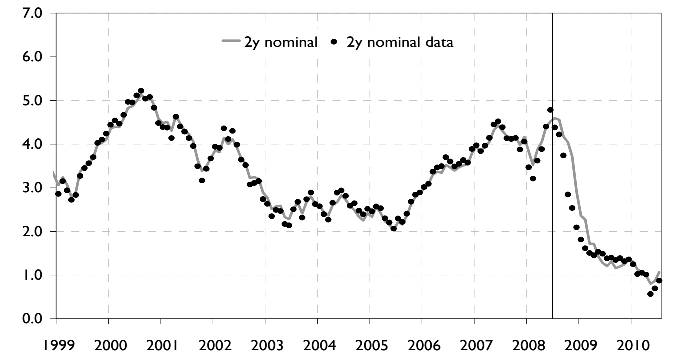
Fitted and observed 3 months nominal bond yields



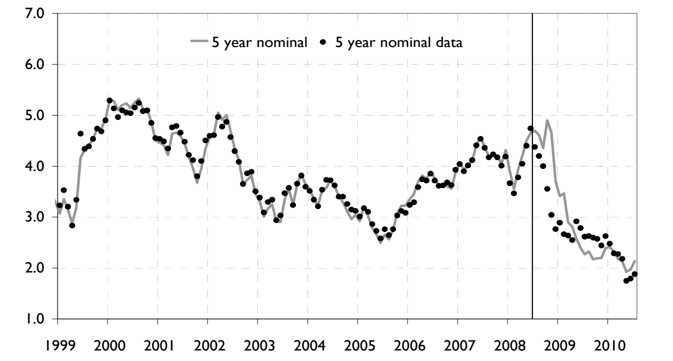
Fitted and observed 1 year nominal bond yields



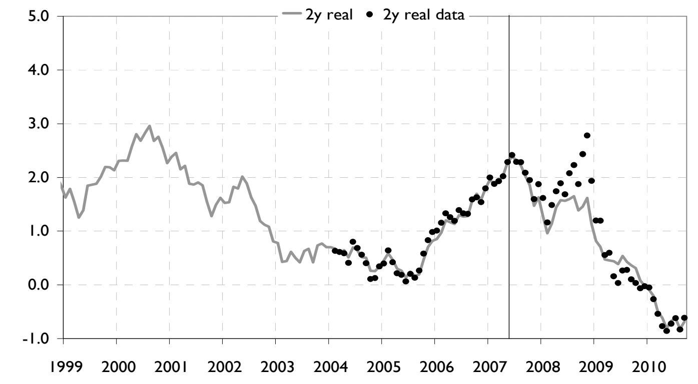
Fitted and observed 2 year nominal bond yields



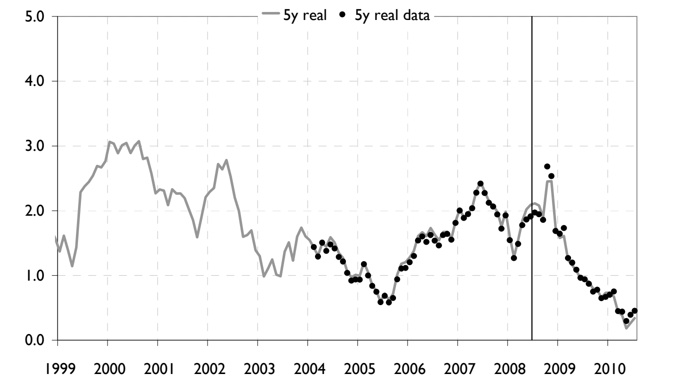
Fitted and observed 5 year nominal bond yields



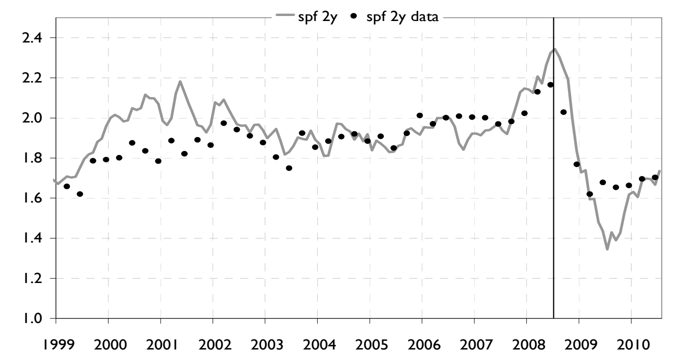
Fitted and observed 2 year real bond yields



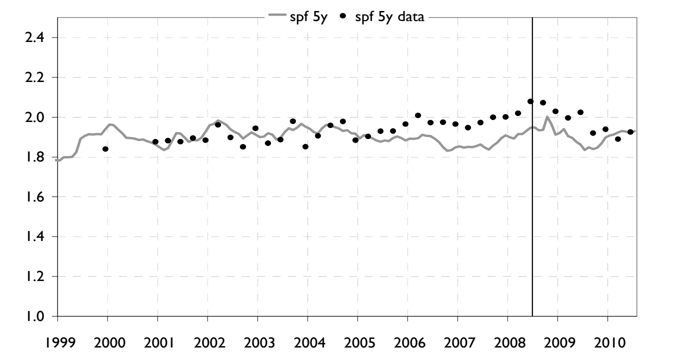
Fitted and observed 5 year nominal bond yields



Fitted and observed 2 year SPF inflation forecast



Fitted and observed 5 year SPF inflation forecast



Note: The charts depict the fitted and the observed values for nominal and real bond yields and inflation expectations in our term structure model. Data are in percentage points. The vertical line at June 2007 distinguishes the pre-crisis from the full sample periods.

Euro area long-term inflation risks (in five years)

Figure 5: Inflation uncertainty measures

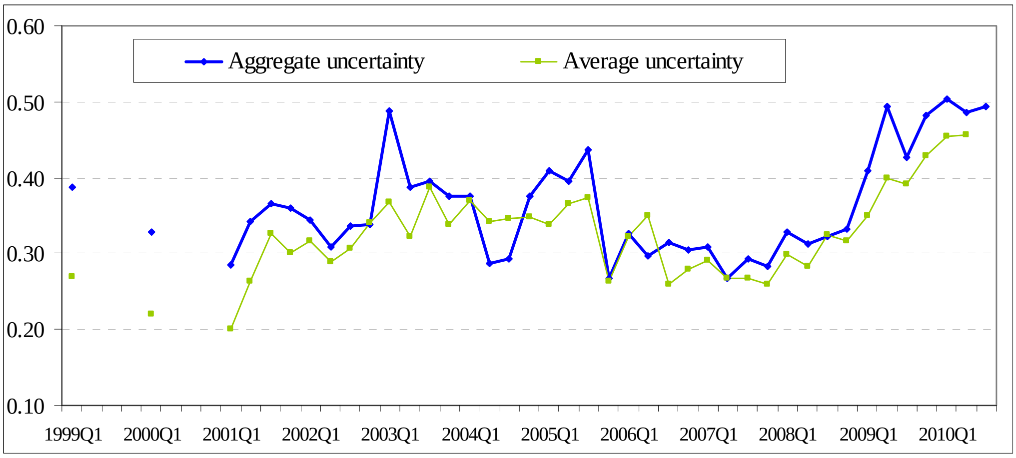
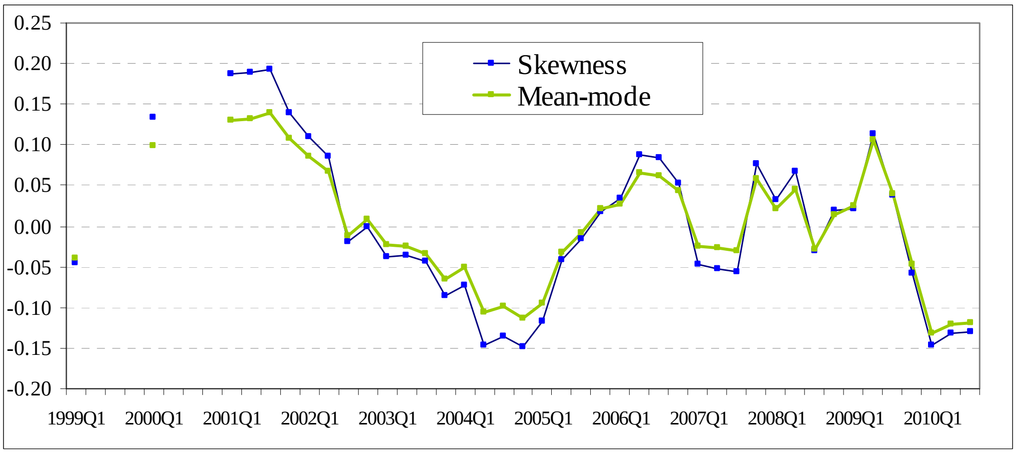
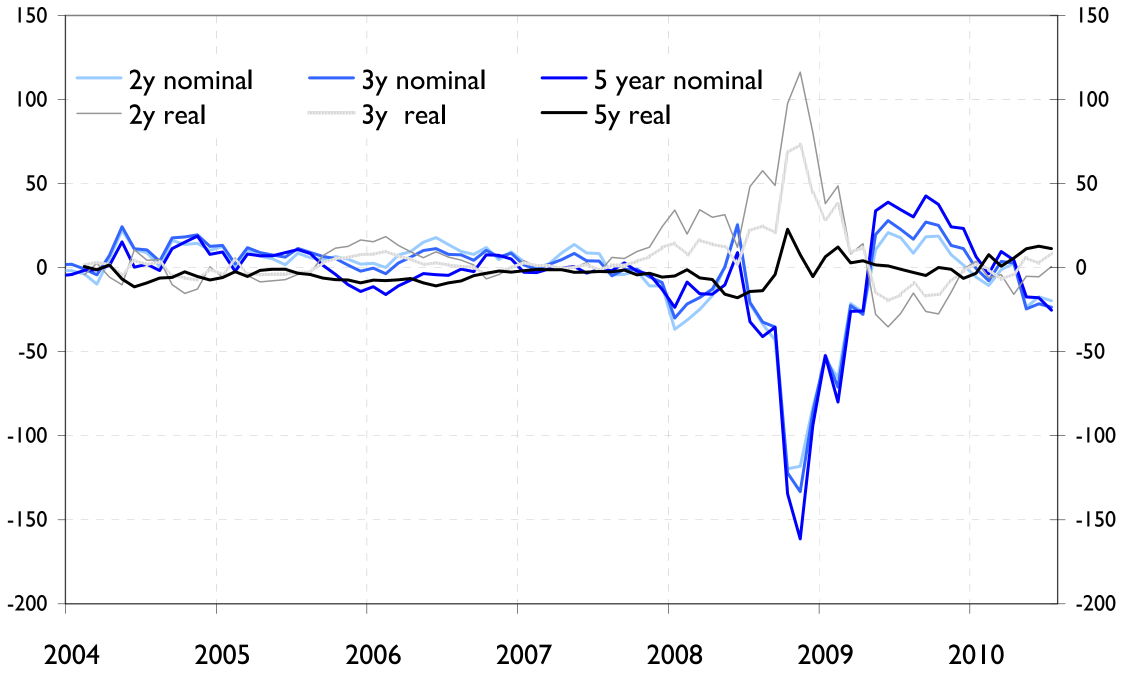


Figure 6: Asymmetries in long-term inflation risks



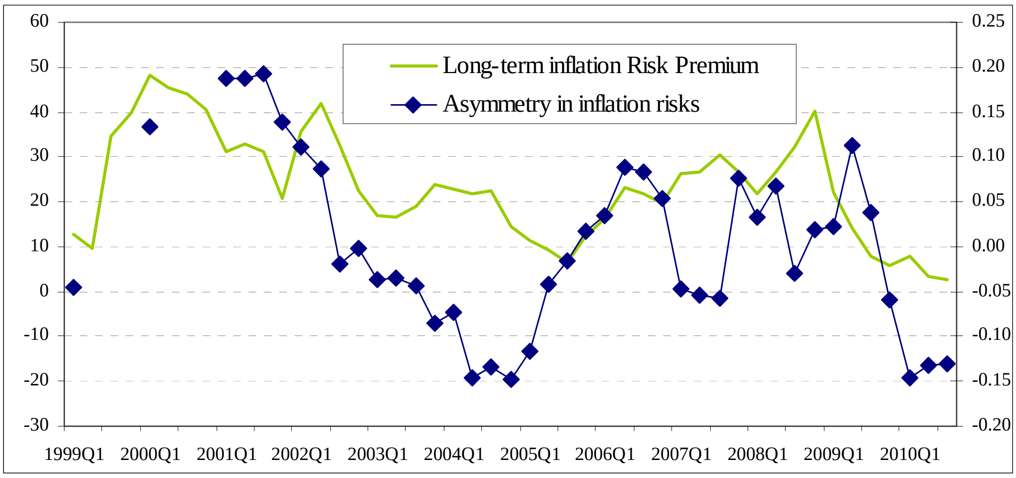
Note: Aggregate uncertainty in Figure 4 refers to the variance of the combined (or aggregate) SPF density forecast constructed by averaging the individual density forecasts (histograms); Average Uncertainty is the average of the variances of the individual density forecasts. By construction, Aggregate Variance is the sum of Average Uncertainty and the Variance of the means of the individual probability forecasts (*disagreement*). Given that additive relationship, we report uncertainty measures based on the second moments of both the individual and the combined probability forecast. Skewness and Mean-mode in Figure 5 measure the asymmetry in the combined (or aggregate) SPF density forecast, three-quarter centred moving averages. Garcia and Manzanares (2007) introduces the methodology for the estimation of the inflation risk measures employed in this paper and discusses them in detail.

Figure 7: Measurement errors for nominal and real yields: observed over fitted values



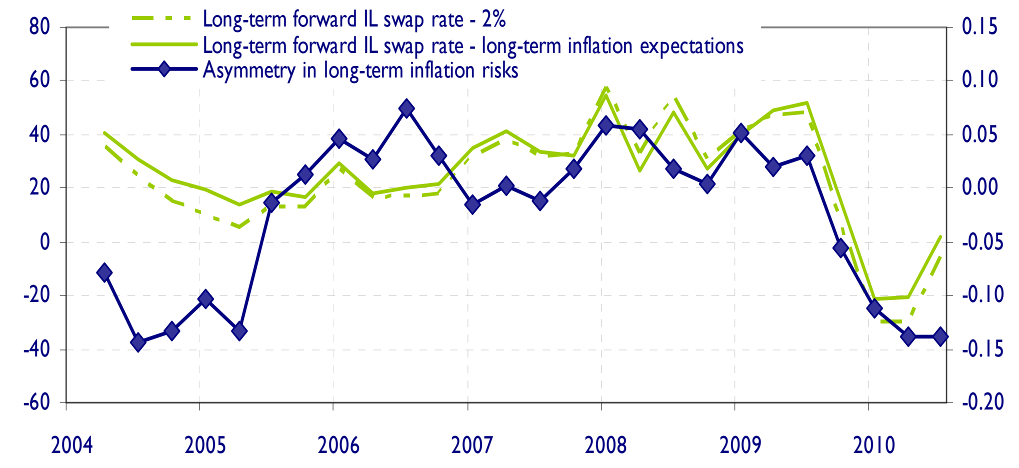
Note: The chart shows the difference between the observed and fitted nominal and real yields for different horizons, in basis points. In contrast to the relatively minor measurement errors between 2004 and 2007, the sharp deterioration in liquidity conditions following the collapse of Lehman Brothers and the intensification of the financial crisis in late 2008 triggered a substantial rise in real yields (as reflected in positive measurement errors) and the fall in nominal yields (as reflected in negative measurement errors) and led to sharp declines in observed break-even inflation rates. Although somewhat attenuated thereafter, distortions remained relatively high for the rest of our sample period.

Figure 8: Long-term inflation risk premia (in basis points, LHS-scale) and asymmetries in inflation risks (RHS-scale)



Note: Three-quarter centred moving averages of inflation skewness surrounding inflation expectations five years ahead estimated following Garcia and Manzanares (2007). Inflation risks are the model estimates for the month in which the survey was conducted, in basis points. Sample 1999Q1-2010Q3.

Figure 9: Model-free measures of long-term inflation risk premium (in basis points, LHS-scale) and asymmetries in inflation risks (RHS-scale)



Note: Three-quarter centred moving averages of inflation skewness surrounding inflation expectations five years ahead estimated following Garcia and Manzanares (2007). Inflation risks are the model estimates for the month in which the survey was conducted, in basis points. Sample 2004Q3-2010Q3.