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# The Value of Risk Reduction: New Tools for an Old Problem 

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## The Value of Risk Reduction: New Tools for an Old Problem


#### Abstract

The relationship between willingness to pay (WTP) to reduce the probability of an adverse event and the degree of risk aversion is ambiguous. The ambiguity arises because paying for protection worsens the outcome in the event the adverse event occurs, which influences the expected marginal utility of wealth. Using concepts of prudence (equivalently, downside risk aversion), we characterize the marginal WTP to reduce the probability of the adverse event as the product of WTP in the case of risk neutrality and an adjustment factor. For the univariate case (e.g., risk of financial loss), the adjustment factor depends on risk aversion and prudence with respect to wealth. For the bivariate case (e.g., risk of death or illness), the adjustment factor depends on risk aversion and cross-prudence in wealth.


Keywords: value per statistical life, mortality risk, risk aversion, prudence JEL classification: D8, I1

The relationship between willingness to pay for prevention and risk aversion is complex and even counterintuitive. Paying for prevention may worsen the outcome if the adverse event occurs, even as it reduces the probability of that event. Initiated by the work of Drèze (1962) and Jones-Lee (1974), who studied willingness to pay (WTP) to reduce mortality risk, and of Ehrlich and Becker (1972), who studied WTP to reduce the probability or the magnitude of a loss, a vast theoretical and empirical literature has emerged about the value of reducing the probability of an adverse event. Unsurprisingly, this literature uses concepts of risk theory that had been simultaneously developed by Arrow (1965) and Pratt (1964). Subsequently, new concepts such as downside risk aversion (Menezes et al. 1980) and prudence (Kimball 1990) have emerged. ${ }^{1}$ This paper provides new insight about WTP for prevention, incorporating these newer concepts.

We examine the WTP (compensating variation) to reduce the probability of an adverse health or financial event under expected utility. We begin with the univariate case, in which utility depends on a single attribute (e.g., wealth or health). We extend our result to the bivariate case, in which utility depends on both health and wealth, and investigate WTP to reduce the risk of a bad health outcome. This analysis applies to the case of mortality risk, in which the measure of WTP to reduce risk is described as the value per statistical life (VSL). In both cases, we find that WTP to reduce risk can be expressed as the product of the marginal WTP under risk neutrality and an adjustment factor that depends on both risk aversion and prudence with respect to the attribute at risk.

## 1. Univariate case

Let $x$ be a continuous variable, representing wealth or health, and consider the binary lottery with outcome $x_{0}<x_{1}$, where the probability of the worse outcome $x_{0}$ is $p$ and the probability of the better outcome $x_{1}$ is $(1-p)$. Expected utility is given by

$$
\begin{equation*}
E U=p u\left(x_{0}\right)+(1-p) u\left(x_{I}\right) \tag{1.1}
\end{equation*}
$$

where $u(\cdot)$ is a continuously differentiable utility function with $u^{\prime}>0$ (primes denote derivatives). The marginal rate of substitution between $x$ and $p$ is

$$
\begin{equation*}
\frac{d x}{d p}=\frac{u\left(x_{1}\right)-u\left(x_{0}\right)}{p u^{\prime}\left(x_{0}\right)+(1-p) u^{\prime}\left(x_{1}\right)}=\frac{\Delta u}{E u^{\prime}}>0 . \tag{1.2}
\end{equation*}
$$

If $x$ represents wealth, $d x / d p$ represents the marginal WTP to reduce the chance of the low-wealth outcome and increase the chance of the high-wealth outcome. For example, one

[^0]might rent a safe-deposit box to protect valuables from theft. Alternatively, if $x$ represents health, $d x / d p$ represents the marginal willingness to compromise health to reduce the chance of a bad health outcome. For example, one might choose radiation or other treatments with adverse side effects to reduce the chance that a cancer proves fatal.

Define $\bar{x}=E(\tilde{x})=p x_{0}+(1-p) x_{1}$ and $\sigma_{x}^{2}=$ variance $(\tilde{x})=p(1-p) L^{2}$, where $L_{x}=$ $x_{1}-x_{0}$. Approximate the terms in the numerator of equation (1.2) using a second-order Taylor series expansion around $\bar{x}$ to obtain:

$$
\begin{align*}
& u\left(x_{1}\right) \approx u(\bar{x})+p L_{x} u^{\prime}(\bar{x})+\frac{\left(p L_{x}\right)^{2}}{2} u^{\prime \prime}(\bar{x}) \text { and }  \tag{1.3}\\
& u\left(x_{0}\right) \approx u(\bar{x})-(1-p) L_{x} u^{\prime}(\bar{x})+\frac{\left([1-p] L_{x}\right)^{2}}{2} u^{\prime \prime}(\bar{x}) . \tag{1.4}
\end{align*}
$$

The denominator of equation (1.2) may be approximated by

$$
\begin{equation*}
E u^{\prime}=u^{\prime}(\bar{x}-\psi) \approx u^{\prime}(\bar{x})-\psi u^{\prime \prime}(\bar{x}), \tag{1.5}
\end{equation*}
$$

where $\psi$ is the prudence premium, which may itself be approximated by

$$
\begin{equation*}
\psi=\frac{\sigma_{x}^{2}}{2}\left(-\frac{u^{\prime \prime \prime}(\bar{x})}{u^{\prime \prime}(\bar{x})}\right) \tag{1.6}
\end{equation*}
$$

(Kimball 1990).
Substituting these approximations into equation (1.2) yields

$$
\begin{equation*}
\frac{d x}{d p} \approx \frac{L_{x} u^{\prime}(\bar{x})+\left(p-\frac{1}{2}\right) L_{x}^{2} u^{\prime \prime}(\bar{x})}{u^{\prime}(\bar{x})+\frac{\sigma_{x}^{2}}{2} u^{\prime \prime \prime}(\bar{x})} \tag{1.7}
\end{equation*}
$$

Dividing the numerator and denominator by $u^{\prime}(\bar{x})$ yields

$$
\begin{equation*}
\frac{d x}{d p} \approx L_{x} \frac{1+\left(\frac{1}{2}-p\right) L_{x}\left(-\frac{u^{\prime \prime}(\bar{x})}{u^{\prime}(\bar{x})}\right)}{1+\frac{\sigma_{x}^{2}}{2} \frac{u^{\prime \prime \prime}(\bar{x})}{u^{\prime}(\bar{x})}} \tag{1.8}
\end{equation*}
$$

Equation (1.8) shows that the marginal rate of substitution between $x$ and $p$ is the product of two terms: the marginal rate of substitution if $u$ is linear (i.e., the potential loss $L_{x}$ ) and an adjustment factor.

The numerator of the adjustment factor depends on the probability of the adverse outcome $p$, the possible loss $L_{x}$, and the coefficient of absolute risk aversion evaluated at the
expected value of $x,\left(-\frac{u^{\prime \prime}(\bar{x})}{u^{\prime}(\bar{x})}\right)$. Interestingly, the effect of risk aversion depends on the difference between the initial probability of loss $p$ and the critical value $1 / 2$. Note that if $p=$ $1 / 2$, the numerator equals one, regardless of the potential loss and the degree of risk aversion. In contrast, if $p<1 / 2$ the numerator is larger than one and is increasing in both $L_{x}$ and the measure of risk aversion. This result is intuitive: for $p<1 / 2$, a decrease in $p$ reduces the variance of final wealth, which is appreciated by a risk-averse decision maker, so he values the decrease by more. ${ }^{2}$ The opposite effect occurs for $p>1 / 2$, for which a decrease in $p$ increases the variance of final wealth. ${ }^{3}$

Notice that the true value of $d x / d p$ must be positive since a decrease in $p$ induces a first-order stochastically dominant improvement in the decision maker's situation. This shift must be compensated by a decrease in $x$ (a first-order stochastically dominant deterioration) to maintain welfare constant. To be useful, an approximation such as the one in equation (1.8) must at least have the same sign as the true value. For a risk-averse decision maker, this is definitely the case when $\mathrm{p} \leq 1 / 2 .{ }^{4}$ From this point forward, we assume the initial probability of loss does not exceed $1 / 2$, which is realistic for most applications.

The denominator of the adjustment ratio depends on the riskiness of the lottery (measured by its variance) and a measure of downside risk aversion, $\frac{u^{\prime \prime \prime}(x)}{u^{\prime}(x)}$ (see Modica and Scarsini 2005 and Crainich and Eeckhoudt 2008). This coefficient can be interpreted as follows: the effect of a zero-mean risk $\tilde{\varepsilon}$ on the marginal utility of wealth is measured by

$$
v(x) \approx E\left[u^{\prime}(x+\tilde{\varepsilon})-u^{\prime}(x)\right],
$$

which is the difference between the marginal utility of $x$ with and without $\tilde{\varepsilon}$. Of course $u^{\prime \prime \prime}(x)>0$ implies $v(x)>0$; hence for a prudent/downside-risk-averse decision make, a zeromean risk increases the expected marginal utility.

A second-order approximation of $v(x)$ yields

$$
\begin{equation*}
v(x) \approx \frac{\sigma_{\varepsilon}^{2}}{2} u^{\prime \prime \prime}(x), \tag{1.9}
\end{equation*}
$$

[^1]which, as we have seen, measures a gain in marginal utility ( $\sigma_{\varepsilon}^{2}$ is the variance of $\tilde{\varepsilon}$ ). To transform this gain in marginal utility into its monetary equivalent, one divides, as usual, by the marginal utility of wealth $u^{\prime}(x)$ so that $\left(\frac{u^{\prime \prime \prime}(x)}{u^{\prime}(x)}\right)$ is a measure of the intensity of downside risk aversion. ${ }^{5}$

Downside risk aversion (equivalently, prudence) is characterized by a positive third derivative (i.e., convex marginal utility). The denominator of equation (1.8) is increasing in both the variance of the lottery and downside risk aversion; hence greater downside risk aversion yields a smaller willingness to pay to reduce the probability of the adverse outcome. Intuitively, greater downside risk aversion means that marginal utility rises more as $x$ declines, hence the utility cost of sacrificing $x$ if the adverse outcome occurs is larger. This suppresses the willingness to sacrifice $x$ to reduce $p$.

For the case where $x$ represents wealth, it is conventional to assume that both risk aversion and downside risk aversion are positive. Risk aversion increases the marginal willingness to pay to reduce $p$ (for small probabilities of loss, i.e., $p<1 / 2$ ), but downside risk aversion decreases it.

For the case where $x$ represents health, the curvature of the utility function depends on how health is measured. One possibility is that $x$ measures longevity. Empirical evidence suggests that some individuals are risk neutral, some are risk averse, and some are risk seeking with respect to longevity (Pliskin et al. 1980). Moreover, the sign of risk aversion with respect to longevity may vary with age and longevity; e.g., it seems plausible that young adults might be risk seeking for longevities ranging over middle ages but risk averse over greater longevities. In this case, the adjustment factor may depend on age and the values of $x_{0}$ and $x_{1}$. Alternatively, $x$ may represent quality of health, which is often conceptualized as 'health-related quality of life' (HRQL) and combined with duration to calculate 'qualityadjusted life years' (QALYs), which assume risk neutrality with respect to HRQL (Pliskin et al. 1980, Hammitt 2002). HRQL is measured by various forms of hypothetical questions, including standard gambles between full health and death. The standard gamble form also
${ }^{5}$ Interestingly, a similar presentation can be developed to explain the intensity of absolute risk aversion $\left(\frac{u^{\prime \prime}(x)}{u^{\prime}(x)}\right)$. There $-u^{\prime \prime}(x)$ measures the loss of total utility generated by the presence of $\tilde{\varepsilon}$ and it is divided by $u^{\prime}(x)$ to obtain the monetary equivalent of this utility loss.
assumes risk neutrality with respect to HRQL . If utility is risk neutral with respect to $x$, then the adjustment factor equals one.

## 2. Bivariate case

Now consider a binary lottery on health and WTP to reduce the probability of the adverse outcome. Expected utility is given by

$$
\begin{equation*}
E U=p u\left(w, h_{0}\right)+(1-p) u\left(w, h_{l}\right) \tag{2.1}
\end{equation*}
$$

where $u(w, h)$ is a continuously differentiable utility function for wealth $w$ and health $h, h_{0}<$ $h_{1}$, and $p$ is the probability of the bad outcome. The marginal rate of substitution of wealth for health risk is

$$
\begin{equation*}
\frac{d w}{d p}=\frac{u\left(w, h_{1}\right)-u\left(w, h_{0}\right)}{p u_{1}\left(w, h_{0}\right)+(1-p) u_{1}\left(w, h_{1}\right)}=\frac{\Delta u}{E u^{\prime}}>0, \tag{2.2}
\end{equation*}
$$

where $u_{j}$ denotes the derivative of $u$ with respect to its $\mathrm{j}^{\text {th }}$ argument.
Define $\bar{h}=E(\tilde{h})=p h_{0}+(1-p) h_{l}$ and $\sigma_{h}^{2}=$ variance $(\tilde{h})=p(l-p) L_{h}{ }^{2}$, where $L_{h}$ $=h_{l}-h_{0}$. As in the univariate case, approximate the terms in the numerator by

$$
\begin{align*}
& u\left(w, h_{1}\right) \approx u(w, \bar{h})+p L_{h} u_{1}(w, \bar{h})+\frac{\left(p L_{h}\right)^{2}}{2} u_{22}(w, \bar{h}) \text { and }  \tag{2.3}\\
& u\left(w, h_{0}\right) \approx u(w, \bar{h})-(1-p) L_{h} u_{1}(w, \bar{h})+\frac{\left([1-p] L_{h}\right)^{2}}{2} u_{22}(w, \bar{h}) . \tag{2.4}
\end{align*}
$$

The terms in the denominator of equation (2.2) may be approximated by

$$
\begin{align*}
& u_{1}\left(w, h_{1}\right) \approx u_{1}(w, \bar{h})+p L_{h} u_{12}(w, \bar{h})+\frac{\left(p L_{h}\right)^{2}}{2} u_{122}(w, \bar{h}) \text { and }  \tag{2.5}\\
& u_{1}\left(w, h_{0}\right) \approx u_{1}(w, \bar{h})-(1-p) L_{h} u_{12}(w, \bar{h})+\frac{\left([1-p] L_{h}\right)^{2}}{2} u_{122}(w, \bar{h}) . \tag{2.6}
\end{align*}
$$

Substituting these approximations into equation (2.2) yields, after simplification,

$$
\begin{equation*}
\frac{d w}{d p} \approx L_{h} \frac{u_{2}(w, \bar{h})}{u_{1}(w, \bar{h})} \frac{1+\left(\frac{1}{2}-p\right) L_{h}\left(-\frac{u_{22}(w, \bar{h})}{u_{2}(w, \bar{h})}\right)}{1+\frac{\sigma_{h}^{2}}{2} \frac{u_{122}(w, \bar{h})}{u_{1}(w, \bar{h})}} . \tag{2.7}
\end{equation*}
$$

As in the univariate case, $d w / d p$ is the product of two terms: the marginal rate of substitution if utility is linear, $L_{h} \frac{u_{2}(w, \bar{h})}{u_{1}(w, \bar{h})}$, and an adjustment factor. The first term can be
interpreted as follows. $L_{h} u_{2}(w, \bar{h})$ is the loss in total utility generated by the loss in health $L_{h}$ , which is converted into a monetary equivalent through division by $u_{1}$, the marginal utility of wealth.

The numerator of the adjustment factor depends on the probability of the adverse outcome $p$, the potential loss $L_{h}$, and the coefficient of absolute risk aversion with respect to health evaluated at the mean outcome $\left(-\frac{u_{22}(w, \bar{h})}{u_{2}(w, \bar{h})}\right)$. As in the univariate case, if $p=1 / 2$ the numerator equals one. If $p<1 / 2$, the numerator increases in the magnitude of the potential loss $L_{h}$ and the coefficient of risk aversion, and if $p>1 / 2$ the numerator decreases in $L_{h}$ and risk aversion.

The denominator of the adjustment factor depends on the riskiness of the lottery (measured by its variance) and a coefficient of cross-prudence in wealth, analogous to the Crainich and Eeckhoudt (2008) coefficient of prudence $\frac{u^{\prime \prime \prime}(x)}{u^{\prime}(x)}$ in equation (1.8). As shown by Eeckhoudt et al. (2007), cross-prudence in wealth implies an individual will be more downside risk averse with respect to wealth when his health is risky rather than certain, hence reducing WTP.

As discussed in Section 1, the curvature of the utility function for health is uncertain. If $h$ represents HRQL, utility is risk neutral in $h$ and hence the adjustment ratio is equal to one. If $h$ represents longevity, utility may be risk averse, risk neutral, or risk seeking, with perhaps different signs for different values of $h$.

The cross-partial derivative $u_{12}(w, h)$ is often assumed to be positive, especially in the case where a low value of $h$ represents death or severe disability (Dreze 1962, Jones-Lee 1974, Hammitt 2002). Evidence of how the marginal utility of wealth varies with health state has been obtained from survey responses about WTP to reduce health risk. Viscusi and Evans (1990) estimate that the marginal utility of income is reduced by a factor of 0.77 to 0.92 by workplace injuries that average one month of work loss. Sloan et al. (1998) estimate reductions by factors of 0.1 and 0.5 for multiple sclerosis. Finkelstein et al. (2013) use selfreported well-being, income, and health data to conclude that the marginal utility of income decreases by a factor of 0.75 to 0.90 for an increase of 1.3 chronic diseases. Domeij and Johannesson (2006) find that assuming the marginal utility of income is smaller when health is worse is consistent with observed savings patterns over the lifecycle. In contrast, Evans and Viscusi (1991) cannot reject the hypothesis that $u_{1}(w, h)$ is constant or decreasing with $h$
when evaluating mild, temporary injuries associated with household use of toilet cleanser and insecticide.

The denominator of equation (2.7) contains an expression, $\frac{u_{122}}{u_{1}}$, that is not familiar. Its detailed interpretation is given in the Appendix. It suffices to say here that the numerator $u_{122}$ (cross-prudence) describes whether the value of $u_{12}$ (i.e., the marginal rate of substitution between wealth and health) increases or decreases with health. As shown in the Appendix, it is the bivariate equivalent of the term $u^{\prime \prime \prime}$ appearing in the univariate model (equation (1.8)). Note that to be expressed in monetary units, $u_{122}$ must be divided as usual by the marginal utility of wealth $u_{1}$.

Eeckhoudt and Hammitt (2004) studied how the marginal rate of substitution of wealth for mortality risk (VSL) depends on risk aversion with respect to wealth. Their analysis used the standard state-dependent expected utility model in which $h$ is a binary variable representing the states living and dead (Drèze 1962, Jones-Lee 1974). They found that VSL is independent of local risk aversion with regard to wealth. Moreover, transforming a utility function to increase risk aversion with regard to wealth cannot be accomplished without altering other critical factors, such as how the marginal utility of wealth varies with wealth. Hence a transformation of the utility function that increases risk aversion with respect to wealth can increase, decrease, or leave unchanged VSL, depending on which other characteristics are held constant.

## 3. Conclusion

WTP to reduce the probability of an adverse event depends on risk aversion and on how the marginal utility of wealth is affected by spending for prevention. An increase in risk aversion induces a change in how the marginal utility of wealth varies as a function of wealth. We show how these effects can be explained using the concept of downside risk aversion (prudence).

For both univariate and bivariate cases, we find that WTP to reduce the probability of an adverse event can be approximated by the product of two terms: WTP under risk neutrality, and an adjustment factor that depends on both risk aversion ( $u^{\prime \prime}$ ) and prudence ( $u^{\prime \prime \prime}$ ) in the univariate case or cross-prudence ( $u_{122}$ ) in the multivariate case. Under the reasonable assumption that the probability of loss is less than or equal to one-half, the adjustment factor is increasing in risk aversion and decreasing in prudence or cross-prudence.

Because an increase in risk aversion cannot be achieved without simultaneously altering how prudence or cross-prudence vary with wealth, the effect of an increase in risk aversion is ambiguous, as previously demonstrated using alternative methods by Dionne and Eeckhoudt (1985) for the univariate case and by Eeckhoudt and Hammitt (2004) for the bivariate case of mortality risk. By distinguishing the effects of risk aversion and prudence, our analysis provides a new perspective on the net effect of an increase in risk aversion on WTP to reduce the probability of a financial or health loss.

## Appendix

The purpose of this appendix is to formally prove the interpretation of the term $\frac{u_{122}}{u_{1}}$ that appears in the expression for WTP under bivariate utility (equation (2.7)).

A preference for 'combining good with bad' (see Eeckhoudt et al. 2009) induces in the bivariate case a preference for lottery A over B:

where $\tilde{\varepsilon}$ is a zero-mean risk (a 'bad' for a risk-averse decision maker) and $-l$ is a sure loss (a 'bad' if marginal utility is positive).

In lottery A , the two bad consequences never occur together: they are spread between the two states of the world. In lottery B, on the contrary, either everything is bad (in the first state of the world, where the decision maker faces both $-l$ on $x$ and $\tilde{\varepsilon}$ on $y$, or nothing is bad (in the second state).

For an expected-utility maximize, a preference for A over B implies

$$
\begin{equation*}
\frac{1}{2} u(x-l, y)+\frac{1}{2} E[u(x, y+\tilde{\varepsilon})]>\frac{1}{2} E[u(x-l, y+\tilde{\varepsilon})]+\frac{1}{2} u(x, y), \tag{A.1}
\end{equation*}
$$

which is equivalent to

$$
\begin{equation*}
u(x, y)-E[u(x, y+\tilde{\varepsilon})]<u(x-l, y)-E[u(x-l, y+\tilde{\varepsilon})] . \tag{A.2}
\end{equation*}
$$

Using the concept of 'utility premium' (Friedman and Savage 1948), it is show in Eeckhoudt et al. (2007, p. 121) that inequality (A.1) holds for risk-averse expected-utility maximizers if $u_{122}>0$ and this is termed 'cross-prudence in wealth'.

While this result indicates a direction for preferences as in (A.1) and relates it to the sign of a third cross-derivative of $u$, one may also be interested in the intensity of this preference. In order to do so, we define a positive change in wealth $m$ such that one obtains

$$
\begin{equation*}
\frac{1}{2} u(x-l, y)+\frac{1}{2} E[u(x, y+\tilde{\varepsilon})]=\frac{1}{2} E[u(x-l, y+\tilde{\varepsilon})]+\frac{1}{2} u(x+m, y) . \tag{A.3}
\end{equation*}
$$

In (A.3) the right hand side of (A.1) is increased through the addition of $m$ in the best state of the world until equality prevails. ${ }^{6}$

Rearranging terms in (A.3), one obtains

$$
\begin{equation*}
u(x+m, y)-E[u(x, y+\tilde{\varepsilon})]=u(x-l, y)-E[u(x-l, y+\tilde{\varepsilon})] . \tag{A.4}
\end{equation*}
$$

Because $u(x+m, y) \approx u(x, y)+m u_{1}(x, y)$, (A.4) becomes

$$
\begin{equation*}
u(x, y)+m u_{1}(x, y)-E[u(x, y+\tilde{\varepsilon})] \approx u(x-l, y)-E[u(x-l, y+\tilde{\varepsilon})] . \tag{A.5}
\end{equation*}
$$

Applying the methodology of Eeckhoudt et al. (2007) to the terms other than $m u_{1}(x, y)$, one is left with

$$
m u_{1}(x, y) \approx \frac{\sigma_{\varepsilon}^{2}}{2}\left[u_{22}(x, y)-u_{22}(x-l, y)\right] .
$$

Approximating $u_{22}(x-l, y)$ to first order around $u_{22}(x, y)$,

$$
\begin{equation*}
m u_{1}(x, y) \approx \frac{\sigma_{\varepsilon}^{2} \cdot l}{2} u_{221}(x, y) \tag{A.6}
\end{equation*}
$$

so that finally

$$
\begin{equation*}
m \approx \frac{\sigma_{\varepsilon}^{2} \cdot l}{2} \frac{u_{221}(x, y)}{u_{1}(x, y)} . \tag{A.7}
\end{equation*}
$$

As a result, $m$ measures in monetary terms the intensity of the pain induced by the misallocation of the losses in B as compared with A. From (A.7), this monetary intensity depends upon $\frac{u_{122}}{u_{1}}$ as intuitively claimed in the discussion of (2.7).

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[^2]
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[^0]:    ${ }^{1}$ For a differentiable utility function, prudence and downside risk aversion are equivalent and require $u^{\prime \prime \prime}>0$.

[^1]:    ${ }^{2}$ For a similar explanation in the framework of self-protection, see Eeckhoudt and Gollier (2005).
    ${ }^{3}$ The effect of a change in risk aversion depends on both the numerator and denominator of equation (1.8), because increasing risk aversion (e.g., by taking a concave function of the $u$ ) affects $u^{\prime \prime \prime}$ as well.
    ${ }^{4}$ For $p>1 / 2$, the approximation may but need not yield positive WTP.

[^2]:    ${ }^{6}$ A similar development for the univariate case is presented by Crainich and Eeckhoudt (2008).

