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A DEA-financial technology: prior to portfolio analysis with DEA

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Abstract

In this paper, we question the definition of a financial technology that results from the application of a traditional methodology with DEA to the analysis of portfolios of financial assets. We acknowledge the previous applications and show how two approaches have been adopted until now in the literature: a 'DEA-production' approach inherited from production theory and a 'DEA-benchmarking' approach inherited from operational research. We show how these approaches define the technology regarding financial assets; we also identify which underlying criteria are used for input and output selection. As a basis for a new 'DEA-financial' approach, we propose to identify a 'financial production process' that differs from the traditional risk-return relationship but is rather based on the generation of a distribution of returns by an initial investment. This identification of a financial production process ensures the proper selection of input and output variables and addresses several issues recently raised by Cook, Tone & Zhu (2014).

Keywords : Data envelopment analysis; Input; Output; DEA-financial technology; Portfolio

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1. Introduction

In a recent article Cook, Tone & Zhu (2014) list several modeling issues raised by such an ill-adapted transposition of the methodology with DEA to various fields of research. In order to bring adequate solutions to these issues and ensure proper modeling, they also list a series of questions that should be answered prior to any analysis with DEA, amongst which the selection and definition of input and output variables. Before even discussing issues related to model orientations, we focus in this article on the first step that should precede any analysis with DEA: the definition of a technology. It is realized through the identification of a production process that ensures a proper selection of inputs and outputs, followed by the definition of a set of regularity conditions that ensure a proper characterization of the technology. In this article we focus on the identification of inputs and outputs but simply mention for now how it can impact the traditional set of axioms associated with DEA[1].

Since Murthi, Choi & Desai (1997) identified DEA as an "*extremely useful technique for measuring efficiency*" of mutual funds, many analyses of portfolios of financial assets have used the whole methodology with DEA that had initially been designed for production theory and further developed in operations research. Though these works have contributed to elaborate a general approach for measuring single-period portfolio efficiency (see for instance Briec & Kerstens (2010)) some adjustments to the traditional methodology used in this approach can still be proposed in order to make it suited to the analysis of financial portfolios, by so much as an appropriate definition of the underlying financial technology.

In order to understand what has been done until now in terms of identification of inputs and outputs in the previous works that used DEA for portfolio analysis, we provide a review of the various measures that have been identified as input and output variables since Murthi, Choi & Desai (1997) and show that the multiplicity of these measures reveals a lack of clear guidance on how to define a 'production' process, if any, regarding the investment in financial portfolios. We identify two approaches that have been adopted by most authors in this field: a 'DEA-production' that relies on a relationship of the production-kind between the risk of an investment and its return, and a 'DEA-benchmarking' approach that ignores the process of return generation but merely uses DEA as a tool for decision-making problems based on

[1] The traditional set of axioms associated to the use of DEA can be found in Färe (1988).

multiple criteria. We also identify which criteria are used for the selection of inputs and outputs variables under each of these approaches and show their implication on two key factors: the ‘real’ process of return generation on the one hand and the possibility to take into consideration uncommon preferences of decision-makers on the other hand. We then propose as a basis for a ‘DEA-financial’ approach the definition of a ‘financial production process’ that generates from an initial investment a distribution of returns that can then be characterized by various moments (such as its mean or variance). The latter being either considered as desirable or undesirable outputs, the analysis remains open to both risk-averse investors and risk-lovers under the DEA-financial approach. We also briefly discuss various matters related to the identification of inputs and outputs such as the appropriate number of input and output variables or the inclusion of additional costs (transaction costs, front loads, etc.) to the set of input and output variables.

2. Input and output measures used for the assessment of portfolio performance with DEA

Cook, Tone & Zhu (2014) remind the importance of ensuring that the input and output measures selected for the study “*properly reflect, to the greatest extent possible, the “process” under study*”. The choice of output variables has always been quite consensual in the literature that uses DEA for the assessment of portfolio performance. On the contrary, the multiplicity of measures that have been proposed to account for input variables shows that no consensus has been reached regarding either the theoretical framework under which the portfolios had to be evaluated[2], and consequently that no clear definition of the technology related to the investment in portfolios of financial assets has been agreed on so far.

On the one hand, desirable outcomes have always been included in the set of output variables and the choice measures of reward such as the average return on an initial investment as an output obtains a consensus. Over various measures of average return one can find either mean or compounded return on past performance[3], as well as expected return on future performance. Returns can either be expressed as gross or net returns[4] and as sometimes as excess return above the market’s performance, either before or after tax. Minimum returns can also be found in some studies (see Wilkens & Zhu (2001), Glawischnig & Sommersguter-Reichmann (2010)) as well as the number of days/months with positive returns on a daily/monthly distribution of returns, (see Gregoriou & Zhu (2007)), upper (or higher) partial

[2] Theoretical frameworks usually belong to two categories: either a mean-variance (or extended mean-variance) frameworks or an expected utility framework, with stochastic dominance criteria. However, some mix of both can be found and additional criteria such as those listed in this section are often included in the analyses.

[3] Arithmetic means of the distribution of returns assume withdrawal of gains while geometric means of the distribution of (gross) returns assume reinvestment of past gains.

[4] What we refer to as the ‘gross return’ is similar to the ‘capitalization factor’ of Basso & Funari (2007).

moments (see Gregoriou (2003), Gregoriou & al. (2005)) or consecutive gains (see Gregoriou & Zhu (2007)). McMullen & Strong (1998) also take into consideration the returns over various time-horizons. Traditional performance indicators as the Sharpe and Treynor ratios, the Jensen's alpha or the reward-to-half-variance index (see Basso & Funari (2005)) have been considered as output measures, as well as other desirable outcomes such as a positive skewness of the distribution of returns (see Wilkens & Zhu (2001)), indicators of stochastic dominance or of the ethical orientation of a fund. In these latter cases a qualitative indicator can be added to the set of output variables (see for instance the ethical factor of Basso & Funari (2003) or the stochastic dominance indicator of Basso & Funari (2001, 2005) or Kuosmanen (2005)).

On the other hand, various costs associated to investment as well as undesirable outcomes have always been included in the set of input variables. Among these input variables one can find numerous measures of volatility risk. Thus, Murthi, Choi & Desai (1997) consider standard deviation of the returns as an input variable, as well as the transaction costs that managers incur "*in order to generate the return*" and which can either be expressed and charged as an expense ratio (management fees, marketing and operational expenses), as additional loads for some funds (sales charges, redemption fees) or as the management turnover of the investment. Similarly, McMullen & Strong (1998) consider standard deviation of returns, sales charges, expense ratio and minimum investment as inputs. Eling (2006) also includes the minimum investment or the lock-up period in the set of input variables (see also Nguyen-Thi-Thanh (2006)) as well as an indicator of trading activity or excess kurtosis; taking into account such characteristics is especially accurate when studying very specific categories of financial assets as hedge funds. Morey & Morey (1999) use as an input variable the variance of returns calculated from historical records as a measure of "total risk" of the investment (both systematic and non-systematic risk). Basso & Funari (2001) propose as input variables various risk measures (standard deviation of returns, root of the half-variance or beta coefficient) and additional costs (subscription and redemption costs). According to Choi & Murthi (2001), one may also consider managerial skills, market and institutional factors in the input requirement set. After them, most authors used indicators among those we just listed, either on single or multiple time horizons (for multiple time horizons, see Wilkens & Zhu (2001), Galagedera & Silvapulle (2002), Basso & Funari (2003)), until Glawischnig & Sommersguter-Reichmann (2010) introduced lower partial moments (mean lower return, lower mean semi-variance and lower mean semi-skewness) as new measures of risk and input variables. Eling (2006), Gregoriou & Zhu (2007) or Branda & Kopa (2012) also mention various drawdowns (maximum or average drawdown, standard deviation of drawdown, Value-at-Risk, conditional Value-at-Risk or Modified Value-at-Risk), the beta factor and residual volatility or tracking error. Gregoriou & Zhu (2005) also used the proportion of negative returns in a distribution of returns as an input.

To some extent, the specific nature of each category of assets can explain this lack of consensus on input measures; while some input and output variables are relevant for some categories of funds, they may be of no use in the performance appraisal of others. By way of example, some cost measures as front-end loads, administration costs or management fees are considered as input variables in the study of mutual funds while they can be included in the measure of return (considered as an output variable) in the study of hedge funds (see Eling (2006)). The cause for treating these costs differently lies, in this particular example, in the fact that hedge funds' returns are already expressed net of such costs while mutual funds' returns are not.

Still, for all contributions listed above risk measures are treated as input variables and return measures as output variables, which can be explained by two main reasons. On the one hand, decision-making in production is based on input reduction and output augmentation and decision-making in finance is based on risk reduction and return augmentation. On the other hand the frontier of efficient portfolios is similar in shape to a production frontier; the analogy has then been made for long between efficiency analysis in production and performance analysis in finance. This analogy and the desirability for return and commonly accepted undesirability of risk have led numerous authors to consider the risk-return relationship of financial assets as equivalent to an input-output relationship. The assimilation of any risk measures to input variables then results from the systematic assumption of risk aversion (as well as a preference for more return) and always goes together with a model orientations that implies input reduction (as well as output augmentation).

The multiplicity of measures can also indicate the need to take into account the diversity of investor's preferences and criteria for choice. The set of relevant criteria must however be determined prior to the analysis through the choice of the theoretical framework. However, it seems necessary to remind that taking into account various preferences for each of these criteria matters. Indeed, the set of investors' preferences is not uniform and must in no way be implied by the definition of the technology (with inputs and undesirable variables systematically assimilated to each other). The presence of risk-lovers, for instance, extensively discussed in a dedicated literature, can only be taken into consideration in models that won't consider risk as an input.

Proposing a unified approach for all categories of financial assets regarding the treatment of all these measures of risk, return or other performance indicators may therefore reveal itself a very delicate task, and may not even be of relevance. Consequently the purpose of this article is to propose some general guidelines that can afterwards be applied to any category of financial assets rather than trying to determine precise rules for every category of asset. In this perspective we identify in the next section which criteria have been used until now in the literature to characterize the underlying 'financial technology', and the approaches they resulted in.

3. Identification of input and output variables

3.1. Criteria for the choice of input and output variables

Cook, Tone & Zhu (2014) remind that as long as the technology under study can be related to "*a form "production process", then "inputs" and "outputs" can often be more clearly identified*". However, several criteria that do not necessarily relate to the notion of a "production process" have been either explicitly proposed or implicitly used in the literature that uses DEA on financial portfolios regarding the choice of inputs and outputs – and consequently the definition of a production technology in this field. We present below three of these criteria and question their accuracy regarding portfolio analysis.

• The first criterion relates to the nature of the interaction between input and output variables, and more precisely whether it is characterized by a causal relationship of the production-kind or not. The identification of the causal relationship, if any, considerably eases the choice of input and output variables among the set of relevant variables. Regarding the analysis of financial assets, this first criterion has often led to identify risk measures as input variables since risk had been identified as the source of return in accordance with the intuition behind the CAPM for instance. Yet, this example reminds us that the choice of the theoretical framework under which the portfolios are to be studied is to be handled with special care, and questions whether this framework's assumptions on a production process comply with the use of an approach inherited from production theory. This consideration could however be disregarded if DEA was used as a simple benchmarking tool[5], and in this case the first criterion could be left out. But as part of our attempt to propose a detailed approach adapted to the study of financial assets that defines a production process, we think that this criterion is of particular relevance. We advocate for its use on the grounds that the whole methodology with DEA (including the definition of a set of axioms and further steps) was developed for performance measurement of production units and is therefore particularly suited to the study of DMUs that can be characterized by a production process.

• A second criterion relates to the behavior of decision-makers towards input or output variables inferred from what's assumed to be their preference or aversion to these variables. Though most often implicitly used, this criterion has influenced the choice of all input and output measures listed above. We observed for instance that the inclusion of these variables in the set of input variables has solely been driven by the observation that investors generally consider them as undesirable variables. Once the assumption of aversion to (preference for) some variables has been made and the consequent minimization-seeking (maximization-seeking) behavior of the decision-maker has been assumed, undesirable variables are associated

[5] If we consider only the aspects related to the definition of a technology and leave out further implications on the set of axioms or other aspects of the whole methodology.

to the set of input variables while desirable ones are associated to the set of output variables. This second criterion is often considered as a mean to address the need to take into consideration investors' preferences, and more especially the measures that are relevant in their opinion. The use of this criterion therefore makes the definition of the technology rest upon some assumptions on investors' preferences, as risk aversion for instance. Yet, as these preferences may depart from the mainstream (as risk-loving behaviors), using this criterion will necessarily result in the definition of an inappropriate technology to evaluate the assets from their perspective, and these rather unusual behaviors will consequently be excluded from the analysis.

Moreover, once the first criterion has been successfully applied, the second criterion becomes pointless: as long as an underlying production process can be identified, the set of input and output variables that partly defines the technology can no more depend on investor's preferences assumptions. It is instead fully determined by this production process. In such case, undesirable variables can either be handled by their identification as 'bad outputs' or by an appropriate orientation of the distance function (that will be used to reduce them). We therefore disprove the use of such a criterion for the choice of input and output variables for two main reasons: on the one hand, this criterion restrains the analysis to the most common decision-makers preferences only, and on the other hand it may conflict with the first criterion whenever a production process generates some undesirable output. It therefore ignores both unusual investors' preferences and the existence of undesirable outputs.

• A third criterion relates to the explanatory power of the input and output measures in the assets' efficiency scores (see Eling (2006)). According to this criterion, any variable adding very little information on the assets' performance should be left aside while those measures that provide substantial additional information should be added to the set of input and output variables. Indeed, one main drawback to the addition of input or output dimensions in DEA is that it leads to an increase in the number of efficient DMUs. If however a substantial change in efficiency scores was also observed as a consequence, the explanatory power of the additional variables could justify their inclusion to the set of input or output variables. Obviously, this criterion is of no use for when decision-makers can perfectly identify two things: which measures are relevant for them to make a decision (in which theoretical framework they will consequently run the analysis) and which relative importance is to be given to each of these measures. This possibility remains however quite theoretical, and investors often tend to resort to as many performance indicators and risk measures as they can measure to take their decision, even if some of them will have no impact on it. Our opinion on that matter is that it is necessary to make sure that the variables considered as relevant enter into the 'production process' before any application of this criterion in order to avoid confusion between environmental variables and input or output variables. The choice of input and output variables among a set of relevant variables should depend on the technology itself and not on the impact these variables may have on efficiency scores.

3.2. Production process and underlying preferences assumed under the DEA-production and DEA-benchmarking approaches

Following the classification of Cook, Tone & Zhu (2014) who distinguish between DEA problems that represent a form of production process and those that are general benchmarking problems, we identify a DEA-production approach and a DEA-benchmarking approach in the literature that uses a methodology with DEA for efficiency measurement of financial asset. We then show how approach uses the criteria listed in the previous section.

• The DEA-production approach inherited from the economics literature consists in a simple transfer of the methodology with DEA used in production theory to the study of financial assets, from the definition of the technology to the set of axioms. Under this approach, financial assets are treated as production units (see for instance Galagedera & Silvapulle (2002)) and their levels of risk and return are included in the set of input variables and output variables, respectively, even though the approach itself does not require such assimilation. By doing so, it implicitly assumes a relationship of the production-kind between risk and return, the former being 'produced' by the latter. In this regard, the DEA-production approach is consistent with one of the basic results of the capital market theory that assumes a positive relationship between standard deviation (the measure of the CAPM to account for risk) and expected returns on investment and to some extent formalizes the production function by its monotonically increasing capital market line. It also implicitly assumes risk aversion for all decision-makers (and consequently assumes that the level of risk is always to be minimized) and uses this assumption as an additional argument to support the inclusion of risk measures to the set of input variables, in accordance with second criterion.

However, we think that defining the relationship between the level of 2^{nd}-order risk (measured by the standard deviation of returns) and the realized return on investment as a 'production' relationship would lead to an erroneous and incomplete representation of the technology. On the one hand, no functional form can express the expectation of a higher return as a result of a riskier investment. The risk-return relationship is consequently no appropriate support for the representation of the technology. On the other hand, the positivity of the risk-return relationship has been proved wrong on some categories of assets (mainly on alternative investments, but sometimes on more regular assets too). One can cite for instance the study of Glawischnig & Sommersguter-Reichman (2002) who find a negative correlation between their chosen measures of risk and return on 167 alternative funds. But in spite of these results, some authors keep the assumption of monotonicity on the risk-return relationship (see for instance Choi & Murthi (2001)). Any intent to propose an approach suited to the study of any category financial assets should get rid of the monotonicity assumption, as it has been empirically invalidated. At the same time, getting rid of such assumption implies either rejecting the notion of a production process between risk and return or assuming a congested technology (where more risk would generate a decrease in return). The assumption of a production framework on which a higher level of risk would produce a higher return is therefore to be reconsidered for no loss of generality. Our intent is to propose an approach that drops the assumption of a causal link between risk and return and to that regards embraces all categories of investment.

The rationale to treat risk as an input is then similar to what makes some authors treat any detrimental variable as an input. The idea that it incurs a cost, together with the natural assumption that decision-makers try to decrease their costs, leads to consider every variable that is to be decreased as an input. In production theory, the same rationale is used in models that assimilate byproducts to inputs (see Hailu & Veeman (2001)) and impose negative shadow prices on these inputs. This is consistent with the decision to consider byproducts as undesirable products; it however implies that no positive value can be attributed to these byproducts (often referred to as “bad outputs”).

While the simultaneous maximization of benefit and minimization of costs seems quite straightforward, we question both the minimization of various risks and their assimilation to inputs. The minimization of various risks directly stems from the implicit assumption that they are to be considered as costs. Indeed, risk of second-order (as measured by variance or standard deviation) is not the only risk that has been considered as ‘undesirable’ in the literature, but all risks of even orders in case of mixed risk aversion[6]. Thus Nguyen-Thi-Thanh (2006) who introduced the four first moments of the distribution of returns in the analysis assimilated standard deviation and excess kurtosis as inputs and skewness as an output. This assimilation of various risks to costs implies making assumptions on investors’ preferences for these risks that are restricted to risk aversion or mixed risk aversion, while unusual preferences for some risks that are now extensively discussed in a dedicated literature (see for instance mixed risk-lovers[7]) remained ignored in these studies. In our opinion, risk-seeking behaviors should neither be a priori excluded from the set of potential managerial or investment strategies nor negatively valued, and the definition of the technology should not prevent taking them into consideration.

• The DEA-benchmarking approach inherited from the operations research literature consists in using DEA as a pure benchmarking tool, which renders the identification of a production process pointless. Still, some of the axioms defined in the DEA-production approach are often associated to this DEA-benchmarking approach due to their convenience. It also uses the models developed since Charnes, Cooper & Rhodes (1978). This approach implicitly assumes no production process between any of the input or output variables, but instead determines the set of resources and outcomes regarding the preference of decision-makers for decreases or increases of these variables, respectively. It also often assumes risk aversion for all decision-makers.

[6] See Caballé & Pomansky (1996), Dachraoui, Dionne, Eeckhoudt & Godfroid (2004), Roger (2011).

[7] See for instance Crainich, Eeckhoudt & Trannoy (2013).

The idea behind the adoption of the benchmarking approach is that it is sufficient to evaluate investment strategies relative to each other. This approach considers that the relevance of the chosen set of factors only depends on the decision-makers' own point of view on how to measure financial performance. It considers as input and output variables the parameters that are relevant to this investor and aims at measuring the relative efficiency of the assets according to a unique set of preferences. According to Anderson & al. (2004) for instance, the identification of inputs and outputs under this approach therefore depends on the objective of the study, "*because this study is being performed from the investor's viewpoint*". If the objective of the study is to compare various investment possibilities for a single decision-maker or a group of decision-makers that have perfectly uniform preferences the benchmarking approach is then perfectly adapted.

If however the analysis focuses no more on the viewpoint of one single investor or a uniform group of investors but rather on the performance of the assets, regardless of the decision-makers' individual preferences (which is equivalent to assuming the possibility of various preferences), the benchmarking approach may no longer be adapted. In such case, it seems more accurate to consider that inputs and outputs are inherent to the production process under study no matter what the investor's viewpoint can be. So we recognize the need to take into consideration the investor's viewpoint in the analysis but consider that it should be implemented through a proper selection of the theoretical decision-making frameworks (mean-variance, expected utility, etc.), the latter being the base for the selection of this set of criteria, rather than through the definition of a set of inputs and outputs.

Cook, Tone & Zhu (2014) reminded that the purpose of the performance measurement with DEA has to be determined prior to the analysis and will influence the model orientation. Regarding the study of portfolios of financial assets, determining whether the objective of the study is to compare various investment possibilities from the viewpoint of a single decision-maker or to assess portfolios performance regardless of the decision-makers' individual preferences will then determine which approach is appropriate. Selection the model orientation will then be a direct consequence of the treatment of decision-makers' preferences under these approaches.

As mentioned above, the notions of input and output do not really matter anymore under the DEA-benchmarking approach due to the absence of a production process. Under this approach, it will not therefore be necessary to identify which variables produce or are produced by the others, but rather which variables are to be maximized and which variables are to be minimized. This identification makes the approach be based on underlying preferences of the decision-makers only. In this approach, any variable to be minimized[8] ought to be considered

[8] Variables of the "less-the-better" type according to the terminology used by Cook, Tone & Zhu (2014), or "small-preferred" performance measures in Wilkens & Zhu (2001)

as an input (ignoring the fact that it might be a 'bad' output), and any variable to be maximized[9] ought to be considered as an output.

We point out the fact that when studying financial assets, risk measures have always been minimized since Murthi, Choi & Desai (1997), making the DEA-benchmarking approach having adopted risk aversion as an underlying assumption until now. This assumption is however not intrinsic to the approach and different preferences for risk could therefore easily be introduced.

4. Identification of inputs and outputs under a 'DEA-financial' approach

Cook, Tone & Zhu (2014) remind that any process assimilated to a production process has to be clearly understood prior to the choice of input and output variables. Following their recommendation, we now study how the various inputs and outputs proposed until now in the literature for the study of financial assets can account for a production process. We introduce a 'DEA-financial' approach that under which we assume that the generation of a distribution of returns by an initial investment is the only relationship that can be assimilated to a production process for financial assets, and that risk should rather to be treated as an output. It then relies of the notion of a production process (inherited from the economic approach) and rejects the systematic inclusion of 'undesirable' variables in the set of input variables (inherited from the operational research approach). This DEA-financial approach also takes into account unusual preferences for risk instead of restricting them to risk aversion.

• Most measures proposed in the literature to account for return and second-order risk of financial assets characterize a unique distribution of returns. This distribution of returns is itself generated by the investment of an initial amount in a portfolio of financial assets. It then seems accurate to consider as a production process the transformation of an initial investment (the input) into a distribution of returns (the output) through a production process determined by the composition of this portfolio. Mean return, risk measures or other characteristics can then be derived from this distribution of return. The only relationship of the production kind we take for granted in our approach is therefore the generation of some levels of return and risks by an investment made in financial assets. Under this approach, the technology can be understood as the functioning of the financial market under study that produces some returns from some initial investment. The determinants of this functioning are either unknown or out of the scope of the study, for an approach with DEA aims at studying the outcomes and deals with unknown technologies by modelling them with some regularity conditions.

[9] Variables of the "more-the-better" type in Cook, Tone & Zhu (2014), or "large-preferred" performance measures in Wilkens & Zhu (2001)

As the measures associated to risk and return both have the same source, it then seems consistent to treat all of them as outputs. Similarly, higher moments of the distribution to be included in the framework (skewness, kurtosis, etc.) would then be considered as outputs. On a general level, any measures characterizing the distribution of returns shall be considered as an output under this approach. Thus, mathematical moments of the distribution (mean return, variance, skewness or kurtosis of returns) as well as indicators of extreme values (Value-at-Risk or maximum drawdown for instance) or even the utility resulting from the distribution will be considered as output variables in our approach. More generally, any measure derived from the time series of returns (or prices) will be considered as an output in our approach, and so will be all mathematical moments of the distribution of returns (the risks of various orders).

A similar understanding of the production process regarding financial assets can be found in Anderson & al. (2004) who consider that any benefit arising from an investment is an output and the investment itself is the input. However, while they consider that the level of risk is “taken” by the investor and is therefore part of the initial investment made in the portfolio, we consider that it cannot be quantified a priori and is therefore not ‘taken’ but rather generated a posteriori. The level of risk, and more generally any measure derived from the distribution of returns, is therefore simultaneous to the generation of this distribution.

• A timing assumption also underlies any production process: output generation must be preceded by the supply of some input, as it results from the latter and the production process necessarily takes some time. This sequence is realized here: all outputs are generated simultaneously after the initial investment has been made. Some outputs might be correlated (risk and return for instance) but in no case can a causal link be deduced from this correlation. Investments in various categories of assets and the subsequent generation of returns on these investments cannot be assimilated to a unique production process. Conversely, we can consider that investments in assets that belong to the same category generate returns on these investments through a unique ‘production process’, even if each investment might not achieve the same results. A comparison of the funds per category will therefore be accurate in our DEA analysis in order to ensure that the assets under study share the same technology of production (or what can be assimilated to a technology of production). Among other categories, one can cite equity, property, real estate, raw materials, metals, money market instruments, investment in securities from various geographical area, bond or stock markets, listed or unlisted securities, etc. Some additional characterization of the output variables can be proposed afterwards in order to differentiate desirable outputs from undesirable ones, but only in case where uniform preferences are assumed among all decision-makers regarding these variables.

As emphasized in Färe & Grosskopf (2003, 2004) regarding the treatment of ‘undesirable’ outputs in production theory, considering byproducts as inputs would lead to inconsistencies with both the traditional set of axioms and physical laws. As these byproducts are technically produced by the inputs, they should be considered as outputs. This argument of technical feasibility can also be put forward to support our choice of treating risk measures as output variables, as we considered an initial investment generates a distribution of return that exhibits some level of risk through a production process that is specific to each category of asset.

• The above redefinition of a proper 'financial production process' mainly questions the causal link between risk and return to conclude that they are both generated by an initial investment. Yet, several other input variables may have to be proposed in addition to this initial investment to complete this definition, such as the mandatory presence of a market, some necessary degree of liquidity or the presence of intermediaries. Indeed, an initial amount available for investment could generate no distribution of return at all if there was no market for instance. One could however argue that the notion of initial investment implicitly assumes that these requirements are met.

• In the DEA-financial approach, we choose not to presuppose any kind of preference for any type of risk prior to the estimation of the technology, in order to maintain our framework as general as possible. We rather take decision-makers' preferences into consideration through the choice of an appropriate distance function than at the early stage of identification of input and output variables.

If the technology that produces a distribution of returns from some initial investment in financial assets was assumed to be a convex, shadow prices associated of input and output dimensions would then account for investor's preferences. Assuming a preference of decision-makers for an increase in an output variable (at any level of input) would then result in imposing a positive shadow price in the model on this variable. Conversely, assuming the aversion of decision-makers to the increase in an input variable (at any level of output) would then result in imposing a negative shadow price in the model on this input or output variable.

Under a financial technology, investors aim at reducing any variable assimilated to a cost and at increasing any variable assimilated to a benefit. If this technology was assumed to be convex, shadow prices associated inputs (as the amount invested, the administrative costs, etc.) should consequently be imposed to be negative. Similarly, shadow prices associated to outputs (as the mean return) should be imposed to be positive. Yet, regarding the outputs, an additional differentiation can be made between 'desirable' ones, often called 'good' outputs, and 'undesirable' ones, often called 'bad' outputs. Shadow prices should then be positive for desirable outputs and negative for undesirable ones.

We apply a similar treatment to risk as the one used for byproducts in 'weakly disposable DEA models'. These models consider undesirable byproducts as outputs and allow for both positive and negative shadow prices on byproducts. This possibility that is so accurate in a framework where decision-makers preferences cannot be restricted to the most common ones however faces criticisms in the literature on production theory (see for instance, Hailu &

Veeman (2001) who argue that "*only non-positive pollutant shadow prices are theoretically acceptable*"). The rationale behind such an argument lies in the very definition of a 'bad output' that implicitly assumes its undesirability. For this reason, we choose in our approach not to refer to risk or any other output variable as a 'good' or 'bad' outputs but rather identify them as 'intended' or 'joint' outputs. On the one hand, variables we choose to call the 'intended outputs' are those that are targeted by the production process, as the return on investment. These intended outputs are traditionally referred to as 'good outputs' in production theory. On the other hand, variables we choose to call will have what we call the 'joint outputs' are those that were not intended to be produced but still were generated through the production process. These joint outputs are traditionally referred to as 'bad outputs' in production theory.

Though we agree on the positivity of shadow prices associated to intended outputs, we leave the characterization of joint outputs as 'good' or 'bad' to the choice of decision-makers according to their own preferences. We then impose no a priori assumption of negativity on shadow prices associated to joint outputs that could potentially be positively valued by some decision-makers.

In order to get a better understanding of the reason why some risk measures could have a positive shadow price, let's illustrate this case by examples found in production theory. Smoke is typically a joint production of power generation with coal, while the electricity itself is the intended output (see Baumol & Oates (1988)). Likewise, all byproducts in agricultural productions (greenhouse gases emissions and nitrogen surplus in livestock production, wool for sheep meat producers, meat for milk producers, etc.) are unintended outputs. But even if these byproducts are not intentionally generated throughout the production process, some of them could still be recycled and used as new inputs for instance, or sold and generate some income. When applied to financial assets, the same reasoning can hold: some investors may positively value some types of risk, provided their potential impact on final return.

• Under the DEA-benchmarking approach, performance evaluation of financial is not limited to a risk-return analysis but can rather be seen as a cost-benefit approach on which risk and return are usually considered as costs or benefits among many others, while remaining the core indicators of each category. The inclusion of various additional costs (in addition to risk) in the analysis has become a consensus while the additional benefits (as the ethical indicator of Basso & Funari (2003)) are seldom found in the literature. What has to be determined is whether these costs should be considered as part of the production process (possibly as joint productions) or as environmental factors. If these costs are considered as part of the production process and therefore as specific characteristics of the investment that define the technology, they are to be treated either as inputs or as joint productions. Galagedera & Silvapulle (2002) consider "*experience, scale of operation and level of investor confidence as fund-specific operational characteristics*". Operational expenses could be considered as inputs or control variables if more operations could generate extra return. This is the most extensively used solution and it allows a studying the sources of efficiency by examining the slacks of each additional cost.

If these costs were considered as joint productions (based on the notion of bad outputs, for instance subscription and redemption fees) they can be included in the set of output variables. this solution was suggested for instance by Jensen (1967) who recommended keeping gross returns (that include brokerage commissions and management expenses) in the analysis. Another solution could consist in deducing these costs from the returns. In order avoid the loss of information that such treatment could generate, Choi & Murthi (2001) suggested for instance to consider them separately, even though gross returns already include the additional costs.

These costs can also be determinants of inefficiency that could potentially explain the distance to the efficient frontier but would not intervene in the "production" process as defined by the technology (fund managers' skills for instance, or minimum investment thresholds). For instance, Galagedera & Silvapulle (2002) consider "*each fund's major sector*" as "*explanatory variables associated with fund management strategy*".

The possibility of including various additional costs to the set of input or output variables is a key feature of DEA. As no production process is clearly defined under the DEA-benchmarking approach, these costs can simply be considered as input variables. Yet, these additional costs could only be environmental variables and would in this case be used as determinants of inefficiency rather than elements of the production process. All DMUs that compose a technology set are assumed to operate in a similar environment, as environment necessarily impacts their performance. However, as noticed in Dyson & al. (2001), "*this assumption can rarely be safely made, and, as a consequence, environmental variables are often brought into the analysis to supplement the input/output set*". Taking various costs into consideration is then possible even in cases where they are not considered as part of the production process.

5. Additional matters related to the identification of inputs and outputs

• One additional issue to the identification of input and output variables that is often raised in the literature and also mentioned in Cook, Tone & Zhu (2014) relates to the appropriate number of DMUs to constitute a sample when the study is performed with DEA. A minimum number of observations is required to build a technology set and depends on the number of input and output variables chosen to characterize the technology under study. The reciprocal question relates to the maximum amount of variables to be allowed in the set of input and output variables, knowing that an additional variable most often results in an increase in the number of efficient DMUs. This phenomenon is sometimes referred to as the *curse of dimensionality* and is of course specific to non-parametric estimators (see Simar & Wilson

(2000)). On this subject, numerous authors recommend restricting as much as possible the number of input and output variables on the sole basis that it should ensure a better reflection of the actual production function. In search of more accuracy regarding the actual frontier, it seems consistent to integrate as many observations as possible in the sample. Still, Golany & Roll (1989) point out that the increase in the number of DMUs can also hamper the degree of homogeneity within the sample, as it increases the chances for exogenous variables to impact efficiency scores.

In the analysis of portfolios of financial assets, intuition would suggest including more indicators of financial performance in the analysis in order to make performance evaluation more accurate and consequently ease the decision-making process. However, as a higher number of indicators and the resulting higher differentiation between the DMUs (specialization of each DMU) necessarily lead to an increase in the number of efficient DMUs, decision-making gets even more complicated with DEA (criteria for decision-making have to be even more sophisticated or more precisely defined). Under the DEA-benchmarking approach, one could argue that no such consideration really matters: each DMU in the sample has been selected in order to be used as a specific benchmark for the DMUs to be assessed. The fact that the actual frontier remains unknown is no concern in such case, as the estimated frontier intentionally serves this purpose. Trying to limit the number of efficient DMUs then also becomes pointless. By contrast, adopting a DEA-production approach or the DEA-financial approach imposes to deal with this matter, except in situations where it is technically feasible to collect complete and unbiased data on a whole universe of financial assets instead of a sample, which is hardly feasible and remains theoretical.

In our opinion, once the theoretical framework under which the study will be performed has been selected, the corresponding criteria for decision-making are known and a list of performance indicators or risk measures related to these criteria can be established. Some variables in this list may however provide redundant information, and their inclusion would therefore lead to the drawback mentioned above (an artificial increase in the number of efficient DMUs). In such case, it is recommended to select one of these variables only and leave aside the redundant variables that add little information regarding the criteria for choice under consideration. By way of example, an indicator such as the Value-at-Risk requires both skewness and kurtosis in its calculation; the inclusion of Value-at-Risk in the set of input or output variables and the inclusion of skewness and kurtosis should therefore be mutually exclusive (see Eling (2006)).

Regarding the DEA-financial approach, we think that the appropriate number of inputs and outputs should be determined by the theoretical framework under which the study will be performed and therefore depend on the level of aggregation of the technology. In a mean-variance framework, it would be accurate to consider mean and variance of the distribution of returns only as output dimensions. The level of aggregation of the technology therefore relies on the chose theoretical model of evaluation and should fully determine the identification of inputs and outputs. Regarding that matter, we consequently choose to rely on the definition of the technology that determines the chosen level of aggregation and consequently the number of input and output variables.

• Regarding the inclusion of additional costs to the set of input or output variables, we know that under the DEA-benchmarking approach, performance evaluation of financial is not limited to a risk-return analysis but can rather be seen as a cost-benefit approach on which risk and return are usually considered as costs or benefits among many others, while remaining the core indicators of each category. The inclusion of various additional costs (in addition to risk) in the analysis has become a consensus while the additional benefits (as the ethical indicator of Basso & Funari (2003) for instance) are seldom found in the literature.

What has to be determined is whether these costs should be considered as part of the production process (possibly as joint productions) or as environmental factors. If these costs are considered as part of the production process and therefore as specific characteristics of the investment that define the technology, they are to be treated either as inputs or as joint productions. For instance, Galagedera & Silvapulle (2002) consider "*experience, scale of operation and level of investor confidence as fund-specific operational characteristics*". Operational expenses for instance could be considered as inputs or control variables if more operations could generate extra return. This is the most extensively used solution and it allows a detailed study of the sources of efficiency by examining the slacks of each additional cost.

• Additional matters such as the measures used for inputs and output variables could be discussed, such as the need to avoid a mix of activity levels and ratios. This issue is especially relevant as one could be tempted to use ratios that combine input and output variables instead in order to answer to the need of restraining the number of efficient DMUs in the sample. If such a solution could indeed lower the number of efficient DMUs, this would be at the cost of some loss of information which can be far from desirable: the DMU(s) performing the better on this input or undesirable output variable on the denominator may very well be rated as inefficient, except if the desirable output on the numerator is high enough.

Apart from the loss of information, a ratio also implicitly assumes constant returns to scale, which considerably restricts the scope of the study and constrains the input and output variables to be accounted for by nominal measures (instead of rates of return, for instance). It then account for an input variable or a joint output variable (on the denominator) in addition to the intended output variable (on the numerator), while being assimilated to a desirable output.

Following Cook, Tone & Zhu (2014) who remind that a mix of ratio and raw data can however be permitted in DEA applications, we point out that most measures in our approach are *de facto* ratios, as the source of output is the distribution of returns. Consequently, any output could be expressed as a percentage of the initial investment: return could be expressed as a rate of return on investment, which is the return per unit invested, and second-order risk could be expressed as standard deviation of rates of returns. The models can consequently use both ratios and raw data: measures that account for each output variables and possibly additional input variables (additional costs) will be expressed relative to the initial investment, and this initial investment that account for the main input variable will be expressed as raw data, without its value implying any change in output variables.

Our treatment of the initial investment is similar to the one of Basso & Funari (2005) who use as input variables a fixed invested amount for all funds and subscription and redemption costs. However, they point out that the invested amount can be dropped (as it is the same for every fund) as long as there is at least one other positive measure of input. Keeping a constant invested amount as an input is somehow a guarantee that the DEA problem can be solved in case additional costs are null.

We will consequently avoid the use of traditional performance ratios that relate risks measures to return measures in our approach; still, we will keep most of our data under the form of performance measures (return on investment, deviation of rates of returns) and mix it with some activity level (the initial investment). Even additional costs (commissions, loads) can be expressed as a percentage of the initial amount invested. Still, it implies dealing with an implicit assumption of constant returns to scale regarding all our input and output measures that are already performance measures.

6. Conclusion

As financial assets differ in their nature from traditional DMUs studied in production theory or operations research, performance of such assets cannot consequently be assessed by applying a strictly identical methodology, even if the tool (DEA) is the same. By way of example, the linear dependence between financial assets (the fact that their returns co-vary) can lead to reconsider the technical axiom of convexity of the set. From the definition of the underlying technology to the choice of the appropriate model orientation, the whole methodology that has been developed for the use of DEA and almost strictly transposed to the analysis of portfolios must consequently be questioned and adapted.

Under a new approach that we call ‘DEA-financial’ by contrast to the traditional DEA-production and DEA-benchmarking approaches, the technology is defined through the preliminary identification of a ‘financial production process’ (and consequently inputs and outputs) that account for the ‘real’ production process and characterized by a set of regularity conditions. This article focuses on the definition of this financial production process and defines it as the generation of a distribution of returns over time by an initial investment. The set of axioms that characterize a financial technology will then have to be discussed in order to complete this first step. For instance, as a consequence of our definition of a financial production process, any risk-return analysis will imply working on the output correspondence, which ensures that the set is closed and that performance measurement relative to the frontier can be made no matter which preference is assumed (projections to any direction are allowed for the risks of second or higher orders, which allows taking into consideration both risk-lovers and risk-averse decision-makers). As a consequence of the linear correlation between the assets' distributions, convexity cannot be imposed as regularity condition for a financial technology. Free disposability will only be assumed on desirable outputs (as return measures) that can be wasted but weak disposability on any constraining risk measures (that can only be reduced at the cost or diversifying or hedging) will have to be accepted instead. The possibility of short selling assets might also be taken into consideration, which implies getting rid of the convexity constraint. Decision-making units with negative output might as well be included to the set, as well as risk-free assets, either return-generating risk-free asset or cash.

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