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The duality of Shephard's weakly disposable technology

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Abstract

A part of the recent literature on the treatment of undesirable outputs by weakly disposable models advocates for the use of the Kuosmanen approach to weak disposability. It defines the minimum extrapolation technology satisfying the disposability assumptions specific to weakly disposable models under three different assumptions on convexity: convexity of the technology set, convexity of output sets only and no convexity at all. We contribute to this taxonomy by adding the case of assuming both input and output sets convex, restoring the classical Shephard approach to weak disposability on the output correspondence. After defining the technology through the definition of a directional distance function, we also linearize the corresponding program and show how the application of duality results in a clear, intuitive, and economically relevant interpretation of the assumption of weak disposability on outputs. Finally, we provide an original proof of the existing relationship between weak disposability and returns to scale.

Keywords

Undesirable output; weak disposability; duality; convexity; data envelopment analysis.

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1. Introduction

The current literature with DEA proposes several treatments for detrimental variables or byproducts. Among the approaches that do not treat them as inputs and avoid data transformation[1], weakly disposable models treat them as weakly disposable undesirable outputs while assuming free disposability on inputs and desirable outputs. The complementary reviews provided by Chen (2014) and Piot-Lepetit (2014) on that part of the literature explain how two approaches have been proposed until now regarding the specifications of weak disposability. The classical approach of Shephard (1970) defines a non-convex technology that results in specifying a non-linear model, but we show later that it can be linearized in some cases. The recent approach of Kuosmanen (2005) defines a convex technology with multiple abatement factors that results in specifying a model that can be linearized. The relevance of defining a Kuosmanen technology rather than a Shephard technology has been strongly debated in the recent literature, and mainly rests on the arguments of convexity of the technology set and the possibility to linearize the model. The interest of the latter is to open the door to a dual interpretation of the assumption of weak disposability. Kuosmanen & Podinovski (2009, 2011) proved that the latter approach defines the minimal extrapolation technology satisfying the assumptions of strong disposability of inputs, strong disposability of desirable outputs and weak disposability of undesirable outputs. However, their proof relies on only three of the possible assumptions that can be made on convexity. After assuming in the first place a convex technology, they address the criticism of Färe & Grosskopf (2009) that convexity of the technology is no necessary assumption and consequently consider the case of assuming convex output sets only, and finally explore the case of no convexity at all.

The convexity assumption that remains omitted in this debate is in our opinion particularly relevant from an economic viewpoint: convexity of both input and output sets. Since Petersen (1990), Bogetoft (1996) and Bogetoft & al. (2000), the discussion on convexity of the

[1] Liu, Meng & Zhang (2010) propose a review of the various treatments applied to detrimental variables (inputs or outputs) in the literature with DEA.

technology $T(x,y)$, the output correspondence $P(x)$ and the input correspondence $L(y)$ is well-known. Petersen (1990) provides a very clear summary of this matter: "*Convexity of $T(x,y)$ obviously implies convex input and output possibility sets, $P(x)$ and $L(y)$, but convexity of $P(x)$ and $L(y)$ does not imply a convex production possibility set. Convexity of $P(x)$ and $L(y)$ is a typical neoclassical assumption justified by the law of diminishing marginal rates of substitution. In addition, $P(x)$ and $L(y)$ must be convex in order to develop the duality between the output set and the revenue function and between the input set and the cost function. These observations indicate that the specification of the reference technology should yield convex input and output sets; on the other hand, the assumption of a convex production possibility set should not be invoked*." In this paper, we propose to follow Petersen (1990) and use the approach of Shephard (1970, 1974) to define weak disposability.

Regarding the dual interpretation of the models, Kuosmanen & Kazemi Matin (2011) develop the dual formulation of the model corresponding to Kuosmanen's technology and identify a "*limited liability condition*" that would be the economic interpretation of weak disposability. This condition would correspond to a situation where "*the economic loss of the benchmark should not exceed the sunk cost of the inputs*". By referring to the inputs, this interpretation can however lead to confusion as primal interpretations of weak disposability are based on the jointness between desirable and undesirable outputs. We show in this paper that the model corresponding to the Shephard technology can also be linearized and result in a clear, intuitive, and economically relevant interpretation of weak disposability. Unveiling our result, assuming weak disposability on the outputs of a Shephard technology can be interpreted from the dual perspective as imposing the following condition: revenues generated by the desirable outputs must at least compensate for the costs incurred by the undesirable outputs. This interpretation is in line with the natural economic translation of the primal notion of undesirable outputs to the dual notion of costs, revenues and profit. It implies that any firm is better be inactive when the costs incurred by the deleterious share of the production exceed the revenues from the desirable outputs.

Thanks to our duality results we ultimately provide a dual proof to the existing relationships between weak disposability and returns to scale. Färe & Grosskopf (2009) show that outputs are weakly disposable in cases where the technology exhibits constant or non-increasing returns to scale, provided weak disposability on inputs. While their proof is based on a set of operations on the primal specification of the model and returns to scale, we propose an original proof based on the dual specification that can deal with shadow revenue and profit.

2. Linearization of Shephard's weakly disposable technology

We denote inputs by $x = (x_1, \dots, x_N) \in \mathbb{R}_+^N$, desirable or desirable outputs by $v = (v_1, \dots, v_M) \in \mathbb{R}_+^M$, and desirable outputs by $w = (w_1, \dots, w_J) \in \mathbb{R}_+^J$. The production technology transforming inputs into outputs can be defined on the output correspondence $P: \mathbb{R}_+^{M+J} \to 2^{\mathbb{R}_+^N}$ on which any $P(x)$ is the set of all outputs vectors that can be produced from an input vector x such that:

(1) $P(x) = \{(v, w) : (v, w) \text{ can be produced from } x\}$

Throughout the paper, we assume the output correspondence satisfies the following axioms:

(P1) $\forall (v, w) \in P(x), 0 \leq v' \leq v \Rightarrow (v', w) \in P(x)$

(P2) $\forall (v, w) \in P(x), 0 \leq \theta \leq 1 \Rightarrow (\theta v, \theta w) \in P(x)$

(P3) $P(x)$ is convex for all $x \in \mathbb{R}_+^N$

Following Petersen (1990), we also define the following axioms on the input correspondence:

(L1) $\forall x \in L(v, w), x' \geq x \Rightarrow x' \in L(v, w)$

(L2) $L(v, w)$ is a convex set for all $(v, w) \in \mathbb{R}_+^{+J}$

(P1) and (L1) define strong disposability on inputs and desirable outputs, respectively; (P2) defines weak disposability on undesirable outputs with jointness between desirable and undesirable outputs. (L2) and (P3) define convexity on the input and output correspondences, respectively. Following Petersen (1990), we do not invoke the convexity of $T(x, y)$ here, but rather the convexity of $P(x)$ and $L(y)$, which are the typical neoclassical assumptions justified by the law of diminishing marginal rates of substitution. In addition, the convexity on the input and output correspondences are the minimal assumptions for developing duality between the output set and the revenue function on one hand, and the input set and the cost function on the other hand. As emphasized in Petersen (1990), "*these observations indicate that the specification of the reference technology should yield convex input and output sets; on the* *other hand, the assumption of a convex production possibility set should not be invoked*". A condition of null-jointness is sometimes imposed as well (see Färe & Grosskopf (2004) for instance) and can be expressed as follows:

(P4) $\forall (v, w) \in P(x), w = 0 \Rightarrow v = 0$

The property of null-jointness characterizes the pollution problem, namely that the desirable output cannot be produced without producing some undesirable output as well. We express this separately since this latter axiom is mostly data driven. It is violated if any observed production plan shows a strictly positive quantity of desirable output while producing no undesirable output (see for instance Ramli & al., 2013). If no such production plan is observed, then the condition of null-jointness is automatically verified through the axiom of weak disposability (P2). Let's also mention that no specific returns to scale assumption is defined in the above set of axioms (P1)–(P3). Variable returns to scale will therefore prevail. In the final section we show how weak disposability and other returns to scale assumptions are related.

For inefficiency measurement, we define a directional distance function that simultaneously increases desirable outputs and decreases undesirable outputs on the output correspondence [2], with $(g^v, g^w) \in \mathbb{R}_+^{M+J}$:

(2) $\vec{D}(x, v, w; g^v, g^w) = \max\, \{\delta \in \mathbb{R} : \ (v + \delta g^v, w - \delta g^w) \in P(x)\}$

Considering a sample of K observed decision-making units (DMUs), the specification of the DEA model satisfying (P1)–(P3) and (L1)–(L2) using the directional distance function defined in **(2)** is given below. NLP1 denotes a nonlinear program for Shephard's weakly disposable technology under variable returns to scale (VRS). The evaluated DMU is denoted by k'. [3]

(NLP1) $\max_{\delta,\boldsymbol{\mu},\theta} \{\delta\}$ s.t.

- $\theta \sum_{k=1}^{K} \mu_k v_{m,k} \geq v_m^{k'} + \delta g_m^v \qquad m = 1, \dots, M$
- $\theta \sum_{k=1}^{K} \mu_k w_{j,k} = w_j^{k'} - \delta g_j^w \qquad j = 1, \dots, J$
- $\sum_{k=1}^{K} \mu_k x_{n,k} \leq x_n^{k'} \qquad n = 1, \dots, N$

[2] For a similar model that uses a directional distance function that simultaneously increases desirable outputs and decreases undesirable outputs as well as inputs, see Bilsel & Davutyan (2014).

[3] Throughout the article, we use superscripts for inactive indices (variable labels) and subscripts for active ones.

- $\sum_{k=1}^{K} \mu_k = 1$
- $\mu_k \geq 0$ $\quad k = 1, \dots, K$
- $0 \leq \theta \leq 1$

Program (NLP1) is a desirable way to model weak disposability, but as such, it is nonlinear due to the variables θ and μ that appear multiplicatively in the output constraints. As discussed in Leleu (2013), many linearizations of (NLP1) have been proposed in the literature, among which many were erroneous. Correct linearizations of (NLP1) were proposed by Zhou & al. (2008), Sahoo & al. (2011), Lozano & Gutierrez (2011) and Kuntz & Sülz (2011). A correct linearization that opens the door to duality is presented in (LP2).

(LP2) $\max_{\delta,\lambda,\theta} \{\delta\}$ s.t.

- $\sum_{k=1}^{K} \lambda_k v_{m,k} \geq v_m^{k'} + \delta g_m^v$ $\quad m = 1, \dots, M$
- $\sum_{k=1}^{K} \lambda_k w_{j,k} = w_j^{k'} - \delta g_j^w$ $\quad j = 1, \dots, J$
- $\sum_{k=1}^{K} \lambda_k x_{n,k} \leq \theta x_n^{k'}$ $\quad n = 1, \dots, N$
- $\sum_{k=1}^{K} \lambda_k = \theta$
- $\lambda_k \geq 0$ $\quad k = 1, \dots, K$
- $0 \leq \theta \leq 1$

3. The duality of Shephard's weak disposability

The main contribution of this paper is to propose an alternative method of linearizing Shephard's weakly disposable technology under the VRS assumption that prove useful for the dual interpretation of weak disposability.

A first step consists in rewriting (NLP1) as the linear program (LP3) presented below:

(LP3) $\max_{\delta,\lambda} \{\delta\}$ s.t.

- $\sum_{k=1}^{K} \lambda_k v_{m,k} \geq v_m^{k'} + \delta g_m^v$ $\quad m = 1, \dots, M$
- $\sum_{k=1}^{K} \lambda_k w_{j,k} = w_j^{k'} - \delta g_j^w$ $\quad j = 1, \dots, J$
- $\sum_{k=1}^{K} \lambda_k \left(x_{n,k} - x_n^{k'}\right) \leq 0$ $\quad n = 1, \dots, N$

- $\sum_{k=1}^{K} \lambda_k \leq 1$
- $\lambda_k \geq 0$ $\qquad k = 1, \dots, K$

The path from (NLP1) to (LP3) is quite straightforward. From the constraint $\sum_{k=1}^{K} \mu_k x_{n,k} \leq x_n^{k'}$ $\forall n = 1, \dots, N$ in (NLP1) and using $\sum_{k=1}^{K} \mu_k = 1$ the following constraint can be deduced: $\theta \sum_{k=1}^{K} \mu_k x_{n,k} \leq \theta x_n^{k'} \sum_{k=1}^{K} \mu_k$. It implies assuming the positivity of at least one desirable output and one desirable output direction ($\exists m, v_m$; $\exists m, g_m$), which ensures that θ is strictly positive. Simplifying this new constraint, we obtain $\sum_{k=1}^{K} \theta \mu_k (x_{n,k} - x_n^{k'}) \leq 0$ $\forall n = 1, \dots, N$. A change in variables such that $\lambda_k = \theta \mu_k$ then results in the first three constraints of (LP3).

Finally, from $\sum_{k=1}^{K} \mu_k = 1$ and $\mu_k = \lambda_k / \theta$ we obtain $\sum_{k=1}^{K} \lambda_k = \theta$. Since $0 \leq \theta \leq 1$ we obtain the last constraint $\sum_{k=1}^{K} \lambda_k \leq 1$. (NLP1) and (LP3) are equivalent because the original variables can be uniquely recovered by $\theta = \sum_{k=1}^{K} \lambda_k$ and $\mu_k = \lambda_k / \theta$.

A second step consists in rewriting (LP3) as program (LP4) below, which should greatly ease the interpretation of the dual model:

(LP4) $\max_{\delta, \lambda, \sigma} \{\delta\}$ s.t.

- $-\sum_{k=1}^{K} \lambda_k (v_{m,k} - v_m^{k'}) + \sigma v_m^{k'} + \delta g_m^v \leq 0$ $\qquad m = 1, \dots, M$
- $-\sum_{k=1}^{K} \lambda_k (w_{j,k} - w_j^{k'}) + \sigma w_j^{k'} - \delta g_j^w = 0$ $\qquad j = 1, \dots, J$
- $\sum_{k=1}^{K} \lambda_k (x_{n,k} - x_n^{k'}) \leq 0$ $\qquad n = 1, \dots, N$
- $\sum_{k=1}^{K} \lambda_k + \sigma = 1$
- $\lambda_k \geq 0$ $\qquad k = 1, \dots, K$
- $\sigma \geq 0$

The path from (LP3) to (LP4) is the following: from the constraint $\sum_{k=1}^{K} \lambda_k \leq 1$ in (LP3) and by defining a new variable $\sigma = 1 - \sum_{k=1}^{K} \lambda_k$ such that $\sigma \geq 0$ we obtain the fourth constraint of (LP4). From $\sum_{k=1}^{K} \lambda_k + \sigma = 1$ we obtain $v_m^{k'} = \sum_{k=1}^{K} \lambda_k v_m^{k'} + \sigma v_m^{k'}$ and similarly $w_j^{k'} = \sum_{k=1}^{K} \lambda_k w_j^{k'} + \sigma w_j^{k'}$; we can then deduce the first two constraints of (LP4). The third constraint on inputs remains unchanged.

The main interest in writing (LP4) compared to (LP3) is the emphasis put on the link between desirable and undesirable outputs through the variable σ that plays the same role as the variable θ in the traditional Shephard's axiom of weak disposability. It contrasts with Kuosmanen & Kazemi Matin (2011): in their model, the assumption of weak disposability appears in the input constraints under the form it takes in (LP3), leading to a dual interpretation of weak disposability as the "*limited liability condition*". As mentioned earlier, the economic interpretation of this condition is not straightforward in their definition. (LP4) then offers a relevant way to model jointness between the desirable and undesirable outputs and allows for a very intuitive interpretation of the dual model derived in (LP5).

(LP5) $\min_{\pi^v,\pi^w,\pi^x,\gamma} \{\gamma\}$ s.t.

- $\left(\sum_{m=1}^{M} \pi_m^v v_{m,k} + \sum_{j=1}^{J} \pi_j^w w_{j,k} - \sum_{n=1}^{N} \pi_n^x x_{n,k}\right) - \left(\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'}\right) \leq \gamma \qquad k = 1, \dots, K$
- $\sum_{m=1}^{M} \pi_m^v g_m^v - \sum_{j=1}^{J} \pi_j^w g_j^w = 1$
- $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} + \gamma \geq 0$
- $\pi_m^v \geq 0$
- π_j^w unconstrained
- $\pi_n^x \geq 0$

(LP5) is a traditional dual DEA program apart from two main respects. It has the usual form of a DEA model in that it minimizes the shadow profit inefficiency of the evaluated DMU. However, the first difference is the unconstrained shadow price of undesirable outputs. This means that undesirable outputs can be viewed as a cost whenever their shadow prices are negative. Second, the following new constraint appears in (LP5) compared to the usual DEA model: $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} + \gamma \geq 0$. The Left Hand Side (LHS) of this constraint is the sum of three components, namely the shadow revenue of the desirable outputs, the shadow revenue of the undesirable outputs, and shadow profit inefficiency. Therefore, it can be interpreted as efficient shadow revenue, meaning the shadow revenue computed on the frontier. The economic interpretation of this constraint is straightforward: the efficient shadow revenue from desirable and undesirable outputs must be non-negative. Since shadow prices of undesirable outputs are free in (LP5), two cases have to be considered, as described below.

First, if shadow prices of undesirable outputs were non-positive ($\pi_j^w \leq 0\ \forall j = 1, \ldots, J$) as expected for undesirable outputs, they would be considered as costs (as proposed in Hailu & Veeman, 2001). In this case the dual interpretation of the assumption of weak disposability is illuminating: the revenues from the desirable outputs must at least compensate for the costs incurred by the undesirable outputs. Second, if the shadow prices were non-negative ($\pi_j^w \geq 0\ \forall j = 1, \ldots, J$), undesirable outputs would be positively valued and generate revenues in addition to those of desirable outputs. The approaches of Färe & Grosskopf (2003) and Kuosmanen & Kazemi Matin (2011) allow for this possibility, while Hailu & Veeman (2001) and Hailu (2003) criticize this consequence – quite rightly in our view. Following the latter view, we can easily impose negative shadow prices for undesirable outputs in model (LP5).

4. Weak disposability and returns to scale: a dual perspective

The dual approach developed in (LP5) also leads to an insightful relationship between the assumptions of weak disposability and returns to scale, as developed in Färe & Grosskopf (2009). We developed first our model under the assumption of VRS, but obviously any type of returns to scale could be considered by simply adding one of the constraints on the shadow profit as defined in (LP6) below. In order to show that some assumptions of returns to scale and the assumption of weak disposability are redundant, we start by expressing in (LP6) a Shephard's strongly disposable technology from a dual perspective (a "multiplier" DEA model) in which various returns to scale are included, namely constant returns to scale (CRS), non-increasing returns to scale (NIRS), non-decreasing returns to scale (NDRS), and VRS. As discussed in Leleu (2009), a straightforward and standard economic interpretation of the nature of returns to scale is given by defining them as constraints on the shadow profit in a dual DEA program.

(LP6) $\min_{\pi^v, \pi^w, \pi^x, \gamma} \{\gamma\}$ s.t.

- $\left(\sum_{m=1}^{M} \pi_m^v v_{m,k} + \sum_{j=1}^{J} \pi_j^w w_{j,k} - \sum_{n=1}^{N} \pi_n^x x_{n,k}\right) - \left(\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'}\right) \leq \gamma \qquad k = 1, \ldots, K$
- $\sum_{m=1}^{M} \pi_m^v g_m^v - \sum_{j=1}^{J} \pi_j^w g_j^w = 1$
- $\pi_m^v \geq 0$

- $\pi_j^w \leq 0$
- $\pi_n^x \geq 0$
- Case 1 : $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'} + \gamma = 0$ under CRS
- Case 2 : $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'} + \gamma \geq 0$ under NIRS
- Case 3 : $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'} + \gamma \leq 0$ under NDRS
- Case 4 : $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'} + \gamma$ unconstrained under VRS

We see that the returns to scale assumptions are expressed in (LP6) by the constraints on the efficient shadow profit. In a traditional multiplier model such as the BCC model proposed in Banker, Charnes & Cooper (1984), various returns to scale can be imposed by introducing a fixed cost (*revenue*) component to the objective function of the cost minimization (*revenue maximization*) problem and by imposing constraints on the sign of this variable. Yet, this is not the most relevant economic interpretation of return to scale assumptions. (LP6) allows to retrieve the usual economic interpretation: CRS imposes a zero shadow profit condition, while shadow profit is imposed to be non-negative under NDRS and non-positive under NIRS. Under VRS, it is not constrained at all.

We then use the result of Färe & Grosskopf (2009) who analyzed general relations between returns to scale and disposability. In particular they proved the following:

1) If $P(\lambda x) \supseteq P(x), \lambda \geq 1$ and $P(\delta x) \supseteq \delta P(x), \delta \geq 0, x \in \mathbb{R}_+^N$, then $\theta P(x) \subseteq P(x), 0 \leq \theta \leq 1$ (proposition 2, p. 535);

2) If $P(\lambda x) \supseteq P(x), \lambda \geq 1$ and $P(x) \supseteq P(\gamma x), 0 \leq \gamma \leq 1, x \in \mathbb{R}_+^N$, then $\theta P(x) \subseteq P(x), 0 \leq \theta \leq 1$ (proposition 3, p. 536).

In other words, if the technology exhibits CRS (case 1) or NIRS (case 2), then, given the weak disposability of inputs, outputs are weakly disposable. While Färe & Grosskopf (2009) give a mathematical proof of these propositions based on set operations, we provide here a more economical proof based from the dual perspective.

The proof of Färe & Grosskopf (2009) follows by inspecting the two constraints that define the CRS and NIRS assumptions in (LP6) and the constraints on weak disposability in (LP5). Considering first the CRS case, the constraint $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} - \sum_{n=1}^{N} \pi_n^x x_n^{k'} +$

$\gamma = 0$ necessarily implies $\sum_{m=1}^{M} \pi_m^v v_m^{k'} + \sum_{j=1}^{J} \pi_j^w w_j^{k'} + \gamma \geq 0$, since $\sum_{n=1}^{N} \pi_n^x x_n^{k'} \geq 0$. The constraint on weak disposability is therefore redundant under CRS. In other words, the CRS technology imposes a zero-profit condition on the shadow profit. Since input costs are non-negative, this implies that under CRS, the total revenue is necessarily non-negative, which is precisely our condition for weak disposability. The same is true for NIRS, which imposes a positive profit.

5. Conclusion

The treatment of undesirable outputs modelled by weak disposability has been recently studied under alternative assumptions of convexity. While the convexity of the technology, the convexity of the output set only, and the case where no convexity is imposed at all were treated, the usual economic case of convexity of both output and input correspondences related to Shephard technology were omitted. Following Petersen (1990), Bogetoft (1996) and Bogetoft & al. (2000), we explicitly consider this last case. We show that the Shephard weakly disposable technology can be linearized in such a way that its dual linear counterpart has a relevant economic interpretation: the revenues from the desirable outputs must at least compensate for the costs incurred by the undesirable outputs.

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