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Jean-Philippe Boussemart
University of Lille 3 and IÉSEG School of Management (LEM-CNRS 9221)

Gary D. Ferrier
University of Arkansas

Hervé Leleu
CNRS-LEM 9221 and IÉSEG School of Management

Zhiyang SHEN
China Eximbank, Anhui University of Finance and Economics

IÉSEG School of Management
Lille Catholic University
3, rue de la Digue
F-59000 Lille
www.ieseg.fr

An Expanded Decomposition of the Luenberger Productivity Indicator with an Application to the Chinese Healthcare Sector

Jean-Philippe BOUSSEMART[a], Gary D. FERRIER[b], Hervé LELEU[c], and Zhiyang SHEN[d*]

[a] University of Lille, CNRS, IÉSEG School of Management, UMR 9221 - LEM, F-59000 Lille, France
email: jp.boussemart@ieseg.fr
[b] Department of Economics, University of Arkansas, Fayetteville AR 72701, USA
email: gferrier@walton.uark.edu
[c] CNRS, IÉSEG School of Management, University of Lille, UMR 9221 - LEM, F-59000 Lille, France
email: h.leleu@ieseg.fr
[d] China Eximbank, Anhui University of Finance and Economics, 233030 Bengbu, China
email: zhiyang86@163.com

Abstract: Productivity growth is an important determinant of the economic well-being of producers, consumers, and society overall. Given its importance, economists have long measured productivity growth, often decomposing the overall measure into constituent pieces to isolate and better understand the sources of productivity change. Typically, productivity change is analyzed at a single level of analysis—e.g., a firm or a country. The objective of this research is to combine productivity analysis at the "firm-level" and the "industry-level" so that a novel, fuller decomposition of the sources of productivity change can be undertaken. Specifically, our decomposition allows us to capture changes in productivity due to the reallocation of inputs or outputs across productive units. In practice, such reallocation might take place across plants operated by the same firm, across regions within a country, or via mergers and acquisitions. By shedding light on more dimensions of productivity growth, this expanded decomposition may facilitate policy development and other efforts to improve productivity. The expanded decomposition begins with a standard decomposition of the aggregate Luenberger productivity indicator into its technical progress and efficiency change components. The efficiency change component is then further decomposed into technical, mix, and scale efficiency effects. The decomposition yielding the mix and scale efficiency changes uses both aggregated and disaggregated data, which allows for productivity effects of reallocations of inputs and outputs across members of a group to be measured. The new decomposition of the aggregate Luenberger productivity indicator is illustrated using data at both the provincial and regional levels for China's healthcare sector over the period 2009-2014. Given the rapid growth in the Chinese healthcare sector in recent years and the various healthcare reforms initiated by the government, a deeper understanding of productivity in this traditionally low-productivity sector is warranted. Our results indicate that the growth of the aggregate Luenberger productivity indicator varied across both time and regions; the annual average growth rates were 0.73%, 0.53%, and 0.18% for China's Central, Eastern, and Western regions, respectively. We find that China's regional productivity growth in healthcare was primarily driven by technological progress; the contributions of the efficiency related elements of productivity change were smaller and more varied across regions.

Keywords: Luenberger Productivity Indicator; Chinese Healthcare; Structural Efficiency; Scale Efficiency; Mix Efficiency.

*Corresponding author: China Eximbank, Anhui University of Finance and Economics, No. 962 Caoshan Road, 233030 Bengbu, China Tel: +86 15210673725. Email: zhiyang86@163.com.

Highlights:

1. A novel decomposition of the aggregate Luenberger Productivity indicator is proposed;

2. The proposed decomposition captures the productivity effects of reallocations of inputs and outputs across productive units;

3. The efficiency change element can be decomposed to technical, mix and scale effects;

4. The model is applied to a provincial and regional data in China’s healthcare sector;

5. China’s healthcare productivity growth was driven by technological progress over 2009-2014.

An Expanded Decomposition of the Luenberger Productivity Indicator with an Application to the Chinese Healthcare Sector

1. Introduction

Productivity measures compare the quantity of output obtained relative to the quantity of input employed in a production process. Productivity growth—increases in the amount of output that can be generated per unit of input—is crucial for increasing economic well-being. Interest in identifying the sources of productivity growth has led to the development of a variety of productivity measures and different decompositions of productivity change into various subcomponents. The two most basic subcomponents of productivity change are technological change and efficiency change, underscoring that productivity depends upon the state of technology and the efficacy with which technology is applied. The objective of this research is to present a novel decomposition of an aggregate Luenberger productivity indicator. The proposed decomposition utilizes measures of productive performance at two levels—"individual" and "group" (e.g., firm and industry)—which allows the productivity effects of reallocations across productive units within a group to be captured. The expanded decomposition sheds light on more dimensions of productivity growth, which may facilitate policy development and assessment.

We illustrate the new decomposition using data from the healthcare sector of the Chinese economy over the period 2009-2014. Healthcare spending in China has grown rapidly but access to and the cost of healthcare remain issues. Numerous reforms, at both the provincial and regional levels, have sought to improve healthcare performance in China. The application of the proposed expanded decomposition of the aggregated Luenberger productivity indicator to the Chinese healthcare system will shed new light on the effectiveness of reforms.

The remainder of the paper is organized as follows. Section 2 reviews the aggregate Luenberger productivity indicator and then derives a novel decomposition of the indicator into four constituent sources of productivity change. Section 3 illustrates the decomposition using data on the healthcare sector of the Chinese economy, which has undergone considerable change in recent years. Section 4 concludes.

2. The aggregate Luenberger productivity indicator and its decomposition

To isolate the contributions of specific factors to changes in productivity, productivity measures are often decomposed into constituent pieces. For example, Färe et al. (1994) proposed the initial decomposition of the Malmquist productivity index into two mutually exclusive and exhaustive components—changes in technology and changes in efficiency. Since then, further decompositions of the Malmquist productivity index have appeared in the literature; e.g., the overall efficiency change can be decomposed into pure technical efficiency change, scale efficiency change, etc. Analogous to the decomposition of the Malmquist productivity index, Chambers et al. (1996) presented a decomposition of the Luenberger productivity indicator into technological change and efficiency change components. Like the Malmquist productivity index, the Luenberger productivity indicator has also been further decomposed. For example, Epure et al. (2011) decomposed the efficiency change component of the Luenberger productivity indicator into pure efficiency change, scale efficiency change, and congestion change subcomponents. Mussard and Peypoch (2006) "multi-decomposed" an aggregate Luenberger productivity indicator by attribute and firm. In their decomposition, the attributes were technical change and efficiency change; they then found each firm's contribution to these two components.

In this section, we present an expanded decomposition of the aggregate Luenberger productivity indicator. We begin with the basic decomposition of the aggregate Luenberger productivity indicator into its technical change and efficiency change components. The efficiency change component is then further decomposed into technical efficiency and structural efficiency components. Finally, the structural efficiency component is decomposed into mix efficiency and scale efficiency components. The decomposition is illustrated in Figure 1.

Figure 1 about here

Farrell (1957, p. 254) noted that the efficiency measures he presented were "…intended to be quite general, applicable to any productive organization from a workshop to a whole economy." In addition to considering the efficiency of individual entities, Farrell (1957) also considered the efficiency of a group of productive units. He argued that "structural efficiency"—the efficiency of a group of firms producing the same product—was not necessarily a weighted average of the technical efficiencies of the firms in the group. For example, if firms use different input proportions, are not all at optimal scale, etc., then the technical efficiency of the industry would differ from a weighted average of the technical efficiency of its member firms. The decomposition of the aggregate Luenberger productivity indicator that we present uses efficiency measures based on both "individual-level" (e.g., firm) and "group-level" data (e.g., industry) as developed in Shen et al. (2017). The use of data at two different levels of observation allows us to measure the efficiency effects of reallocations of inputs or outputs across entities within a group, which captures their contribution to productivity change.

The Malmquist productivity index has commonly been used to measure productivity and to decompose overall productivity change into subcomponents. As an alternative to the Malmquist, we propose a novel, expanded decomposition of the aggregated Luenberger productivity indicator. The Malmquist index is based on the ratio of Shephard distance functions and its decomposition is multiplicative, while the Luenberger productivity indicator is given by the difference of directional distance functions and its decomposition is additive (Chambers, 1996). Both types of distance functions provide measures of the distance from a point to the boundary of a feasible set—i.e., they can be interpreted as measures of efficiency along the lines of the efficiency measure introduced by Farrell (1957). The Luenberger productivity indicator's use of directional distance functions allows for the simultaneous contraction of inputs and expansion of outputs as observations are projected onto the production frontier. In contrast, the Malmquist productivity index must have either an output orientation or an input orientation. Thus, the Luenberger productivity indicator is more general than the Malmquist productivity index.[1]

Typically, a measure of productivity and its decomposition is based on data from a single level of analysis; e.g., firms, countries, etc. Our goal is to develop a decomposition of a group level Luenberger productivity indicator into a number of different constituents, some of which reflect the effects of reallocating productive activity across members of the group. In our exposition, we'll refer to the two levels as the industry and the firm. The proposed decomposition requires the computation of directional distance functions at both the firm-level and the industry-level as well as the summation of firm-level directional distance functions. The computation of industry-level directional distance functions requires the specification of technology at the industry-level; the

[1] The Shephard output distance function ($g = (0, y)$) and Shephard input distance function ($g = (x, 0)$), the building blocks of the output-oriented and input-oriented Malmquist productivity index, are special cases of the directional distance function.

summation of firm-level directional distance functions requires that certain conditions regarding their aggregation be met.

Suppose that there are K firms in an industry using M inputs to produce N outputs. Further suppose that in time period t the k^{th} firm uses the input vector $x_k^t = \left(x_{1k}^t, x_{2k}^t, \ldots, x_{Mk}^t\right) \in \mathbb{R}_+^M$ to produce the output vector $y_k^t = \left(y_{1k}^t, y_{2k}^t, \ldots, y_{Nk}^t\right) \in \mathbb{R}_+^N$ while facing the technology set $T_k^t = \left\{\left(x_k^t, y_k^t\right):\ x_k^t \text{ can produce } y_k^t\right\}$. Let $X^t = \sum_{k=1}^{K} x_k^t$ and $Y^t = \sum_{k=1}^{K} y_k^t$ be the aggregate industry input and output vectors, respectively. When all firms face the same variable returns to scale technology,[2] Li and Ng (1995) show that the "group technology" of the K firms is the aggregate of the individual firm technologies:

$$T_{Industry}^t = \left\{\left(X^t, Y^t\right):\ \left(X^t, Y^t\right) = \left(\sum_{k=1}^{K} x_k^t, \sum_{k=1}^{K} y_k^t\right), \left(x_k^t, y_k^t\right) \in T_k^t, k = 1, \ldots, K\right\} = \sum_{k=1}^{K} T_k^t = \sum_{k=1}^{K} T^t, \tag{1}$$

where the last equality follows from the assumption of a homogeneous production technology across firms. Li and Ng (1995) further show that under a convex technology set the aggregate industry technology is given by $T_{Industry,VRS}^t = \sum_{k=1}^{K} T_{k,VRS}^t = \sum_{k=1}^{K} T_{VRS}^t = K \cdot T_{VRS}^t$. They further show that under constant returns to scale the aggregate industry technology is equivalent to the individual firms' common technology, $T_{Industry,CRS}^t = \sum_{k=1}^{K} T_{k,CRS}^t = T_{CRS}^t$.

We represent technology using the directional distance function, an implicit representation of a multi-input, multi-output production technology. For time period t, the directional distance function is given by:

[2] While this is a restrictive assumption, efficiency and productivity studies typically assume that all entities under analysis face a common best-practice frontier.

$$D^t\left(x^t, y^t; g_x, g_y\right) = sup\left\{\beta : \left(x^t - \beta g_x, y^t + \beta g_y\right) \in T^t\right\}. \quad (2)$$

The value of the directional distance function can be interpreted as an efficiency measure— β gives the maximal expansion of $\left(x^t, y^t\right)$ to the frontier in the direction $\left(g_x, g_y\right)$.[3] An observation is efficient if it lies on the boundary of the production technology (i.e., $D^t\left(x^t, y^t; g_x, g_y\right) = 0$) and it is inefficient if it lies on the interior of the production technology (i.e., $D^t\left(x^t, y^t; g_x, g_y\right) > 0$); the further an observation lies from the frontier, the greater the value of the distance function.

Now suppose the production unit's input-output combination in period t+1 is $\left(x^{t+1}, y^{t+1}\right)$. To assess a production unit's productivity change across periods t and t+1, the "efficiencies" of its two observed production plans across time are contrasted while holding technology constant. Using period t as the base, the Luenberger productivity indicator at the firm level is defined as:

$$LPI^t = D^t\left(x^t, y^t; g_x, g_y\right) - D^t\left(x^{t+1}, y^{t+1}; g_x, g_y\right). \quad (3)$$

As noted above, the Luenberger productivity indicator is based on differences in directional distance functions. Thus, in equation (3), the value of the Luenberger productivity indicator based period t is found by subtracting the value of the directional distance function for the period t+1 production plan based on the technology for period t from the value of the directional distance function for the period t production plan based on technology in time t. If, relative to T^t, $\left(x^{t+1}, y^{t+1}\right)$ is more efficient than $\left(x^t, y^t\right)$, then $D^t\left(x^t, y^t; g_x, g_y\right) > D^t\left(x^{t+1}, y^{t+1}; g_x, g_y\right)$ and $LPI^t > 0$, indicating that productivity improved between periods t and t+1. On the other hand, if $\left(x^{t+1}, y^{t+1}\right)$ is less

[3] The choice of the direction $\left(g_x, g_y\right)$ is an important issue that will be discussed below. For now, the direction will be treated as generally as possible.

efficient than $\left(x^t, y^t\right)$ relative to T^t, then $D^t\left(x^t, y^t; g_x, g_y\right) < D^t\left(x^{t+1}, y^{t+1}; g_x, g_y\right)$ and productivity declined, so $LPI^t < 0$.

Alternatively, one could gauge productivity change based on technology T^{t+1}; in this case, the Luenberger productivity indicator measures the difference in the performances of $\left(x^t, y^t\right)$ and $\left(x^{t+1}, y^{t+1}\right)$ relative to T^{t+1}:

$$LPI^{t+1} = D^{t+1}\left(x^t, y^t; g_x, g_y\right) - D^{t+1}\left(x^{t+1}, y^{t+1}; g_x, g_y\right). \tag{4}$$

Rather than arbitrarily choosing the technology in either period *t* or period *t*+1 as the base, the Luenberger productivity indicator is typically calculated as the arithmetic mean of the productivity change relative to the technologies in periods *t* and *t* + 1:

$$\begin{aligned} LPI^{t,t+1} &= \frac{1}{2}\left(LPI^t + LPI^{t+1}\right) \\ &= \frac{1}{2}\left(\left[D^t\left(x^t, y^t; g_x, g_y\right) - D^t\left(x^{t+1}, y^{t+1}; g_x, g_y\right)\right] + \left[D^{t+1}\left(x^t, y^t; g_x, g_y\right) - D^{t+1}\left(x^{t+1}, y^{t+1}; g_x, g_y\right)\right]\right). \end{aligned} \tag{5}$$

Positive values of $LPI^{t,t+1}$ indicate productivity growth, while negative values of $LPI^{t,t+1}$ indicate productivity decline.

We now turn to the novel decomposition of the Luenberger productivity indicator, which uses both industry-level directional distance functions and the summation of firm-level directional distance functions. This follows Farrell (1957) who defined structural efficiency as a measure of efficiency of the industry as a whole and industrial efficiency is defined as the sum of all individual firm's efficiencies. Using directional distance functions, Briec et al. (2003) defined the structural technical efficiency (*STE*) index as the directional distance function evaluated at the aggregated input-output vectors of the group of firms:

$$STE = D\left(\sum_{k=1}^{K} x_k, \sum_{k=1}^{K} y_k; g_x, g_y\right) = D\left(X, Y; g_x, g_y\right). \tag{6}$$

The *STE* index measures the efficiency of the industry as a whole, allowing the reallocation of inputs and outputs among firms. In addition, Briec et al. (2003) defined the industrial technical efficiency (*ITE*) index as the sum of the efficiencies of the members of a group of firms:

$$ITE = \sum_{k=1}^{K} D\left(x_k, y_k; g_x, g_y\right). \tag{7}$$

The *ITE* index is the aggregation of individual firm's efficiency scores given their observed levels of inputs and outputs. Briec et al. (2003) prove that $STE \geq ITE$; any difference between these two indices was termed aggregation bias. Note that equality holds if all firms use inputs and produce outputs in the same proportions (i.e., they all have the same mix), suggesting that properly reallocating inputs and outputs among firms would remove the aggregation bias. This insight contributes to our novel decomposition of the Luenberger productivity indicator.

The pieces are now in place to present our group-level Luenberger productivity indicator and its decomposition into changes in productivity arising from technological change, technical efficiency change, mix efficiency change, and scale efficiency change. The group level Luenberger productivity indicator is given by:

$$\begin{aligned} LPI_{group}^{t,t+1} &= \frac{1}{2}\left(LPI_{group}^{t} + LPI_{group}^{t+1}\right) \\ &+ \frac{1}{2}\left(\left[D^t\left(X^t, Y^t; g_x, g_y\right) - D^t\left(X^{t+1}, Y^{t+1}; g_x, g_y\right)\right] + \left[D^{t+1}\left(X^t, Y^t; g_x, g_y\right) - D^{t+1}\left(X^{t+1}, Y^{t+1}; g_x, g_y\right)\right]\right). \end{aligned} \tag{8}$$

It is the simple average of productivity indicators based on technology in time *t* and technology in time *t*+1. Note that the group Luenberger productivity indicator is based on structural technical efficiency (*STE*) indices.

The initial step in the decomposition of the group Luenberger productivity index follows the original decomposition of Chambers et al. (1996). Following the standard decomposition of the Malmquist productivity index, Chambers et al. (1996) decomposed the Luenberger productivity indicator into productivity changes due to technological progress $\left(TP^{t,t+1}\right)$ and changes changes in efficiency $\left(OEC^{t,t+1}\right)$. In the following, we use the term overall efficiency to stand for the structural efficiency as defined by Briec and al. (2003): i.e., $OEC = STE$. This new terminology is well-suited since we propose a decomposition of the overall efficiency change into changes in technical, scale, and mix efficiencies. The first step in the decomposition of the Luenberger productivity indicator is thus:

$$LPI^{t,t+1} = TP^{t,t+1} + OEC^{t,t+1}. \tag{9}$$

Note that the decomposition is additive for the Luenberger productivity indicator. The technological progress component captures the distance between two technologies, while the overall efficiency component captures changes in how close observations are to the technologies. These two components of the Luenberger productivity indicator involve the structural technical efficiency (*STE*) index—all input vectors and output vectors used in the calculation of $\left(TP^{t,t+1}\right)$ and $\left(OEC^{t,t+1}\right)$ are aggregates of the inputs and outputs available to the firms in the group. The technological change component of the Luenberger productivity indicator is the arithmetic mean of the performances of two input-output bundles, $\left(X^t, Y^t\right)$ and $\left(X^{t+1}, Y^{t+1}\right)$, across two different technologies, T^t and T^{t+1}:

$$\begin{aligned} TP^{t,t+1} &= \frac{1}{2}\Big(\Big[D^{t+1}\left(X^t, Y^t; g_x, g_y\right) - D^t\left(X^t, Y^t; g_x, g_y\right)\Big] \\ &\quad + \Big[D^{t+1}\left(X^{t+1}, Y^{t+1}; g_x, g_y\right) - D^t\left(X^{t+1}, Y^{t+1}; g_x, g_y\right)\Big]\Big) \\ &= \frac{1}{2}\left(\left[D^{t+1}\left(\sum_{k=1}^{K} x^t, \sum_{k=1}^{K} y^t; g_x, g_y\right) - D^t\left(\sum_{k=1}^{K} x^t, \sum_{k=1}^{K} y^t; g_x, g_y\right)\right]\right. \\ &\quad \left. + \left[D^{t+1}\left(\sum_{k=1}^{K} x^{t+1}, \sum_{k=1}^{K} y^{t+1}; g_x, g_y\right) - D^t\left(\sum_{k=1}^{K} x^{t+1}, \sum_{k=1}^{K} y^{t+1}; g_x, g_y\right)\right]\right) \end{aligned} \tag{10}$$

Recall that the directional distance function can be interpreted as a measure of efficiency. Therefore, the change in overall efficiency between periods t and t+1 is given by:

$$
\begin{aligned}
OEC^{t,t+1} &= D^{t}\left(\sum_{k=1}^{K} x^{t}, \sum_{k=1}^{K} y^{t}; g_{x}, g_{y}\right) - D^{t+1}\left(\sum_{k=1}^{K} x^{t+1}, \sum_{k=1}^{K} y^{t+1}; g_{x}, g_{y}\right) \\
&= D^{t}\left(X^{t}, Y^{t}; g_{x}, g_{y}\right) - D^{t+1}\left(X^{t}, Y^{t}; g_{x}, g_{y}\right).
\end{aligned}
\tag{11}
$$

This decomposition of the group Luenberger productivity indicator demonstrates that productivity changes over time are the results of technological improvement (shifts of the frontier) and changes in efficiency (movements toward or away from the frontier).

The initial decomposition of the Luenberger productivity indicator given above captures technological and overall efficiency changes at the group level, but it doesn't shed light on productivity gains that may arise from structural change within the group of observations. To achieve insight into possible structural gains, we further decompose the overall efficiency change component of the Luenberger productivity indicator in a fashion analogous to Ferrier et al.'s (2010) decomposition of Debreu's (1951) aggregate measure of inefficiency (the coefficient of resource utilization) and Farrell's firm level measures of efficiency. To proceed, we employ the aggregate values of inputs and outputs, $\left(g_{x}, g_{y}\right) = \left(\sum_{k=1}^{K} x_{k}, \sum_{k=1}^{K} y_{k}\right)$, as the direction for projecting observations onto the frontier of technology. By using a common direction to evaluate each observation, efficiency scores will have the same units of measure so we can aggregate efficiency scores across observations. At least three natural choices are possible for choosing the common direction: the first period, $\left(\sum_{k=1}^{K} x_{k}^{t}, \sum_{k=1}^{K} y_{k}^{t}\right)$, the last period, $\left(\sum_{k=1}^{K} x_{k}^{t+1}, \sum_{k=1}^{K} y_{k}^{t+1}\right)$, or the average of the two. In the exposition of the decomposition of the Luenberger productivity indicator, and in the empirical application that follows, we use the initial period data as the common direction of projection.

The next step is to show how to decompose $OEC^{t,t+1}$ into changes in technical efficiency $TEC^{t,t+1}$ and changes in structural efficiency $SEC^{t,t+1}$:

$$OEC^{t,t+1} = TEC^{t,t+1} + SEC^{t,t+1}. \tag{12}$$

The change in technical efficiency is an aggregate measure. It is the sum over all firms of their changes in efficiency between periods t and t+1 relative to a variable returns to scale (VRS) technology:

$$TEC^{t,t+1} = \sum_{k=1}^{K}\left[D^{t}_{VRS}\left(x^{t}_{k}, y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{VRS}\left(x^{t+1}_{k}, y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right]. \tag{13}$$

Note that *TEC* is based industrial technical efficiency (*ITE*) indices. Under VRS, *TEC* measures whether the firms in a group have moved closed to or farther away from the frontier over time. Remember that $D^{k} \geq 0$, so if *TEC* > 0 then on average D^{t} is greater than D^{t+1} indicating that aggregate efficiency increased.

The structural efficiency component of the Luenberger productivity indicator captures differences between industry technical efficiency (*ITE*) and structural technical efficiency (*STE*); i.e., the aggregation bias of Briec et al. (2003):

$$\begin{aligned} SEC^{t,t+1} &= OEC^{t,t+1} - TEC^{t,t+1} \\ &= \left[D^{t}_{VRS}\left(X^{t}, Y^{t}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{VRS}\left(X^{t+1}, Y^{t+1}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &\quad - \sum_{k=1}^{K}\left[D^{t}_{VRS}\left(x^{t}_{k}, y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{VRS}\left(x^{t+1}_{k}, y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &= \left[D^{t}_{VRS}\left(\sum_{k=1}^{K} x^{t}_{k}, \sum_{k=1}^{K} y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{VRS}\left(\sum_{k=1}^{K} x^{t+1}_{k}, \sum_{k=1}^{K} y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &\quad - \sum_{k=1}^{K}\left[D^{t}_{VRS}\left(x^{t}_{k}, y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{VRS}\left(x^{t+1}_{k}, y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right]. \end{aligned} \tag{14}$$

Note that the structural efficiency component (*SEC*) of the Luenberger productivity indicator is what remains after the technical efficiency component (*TEC*) has been stripped from the overall efficiency component (*OEC*). *SEC* measures any difference between *STE* and *ITE*. If *STE* = *ITE*, then *SEC* = 0; i.e., the aggregation bias of Briec et al. (2003) is zero and industry efficiency is simply the sum of member firms' efficiencies.

Farrell (1957, p. 262) argued that the structural efficiency of industry captures "…the extent to which its firms are of optimum size, to which its high-cost firms are squeezed out or reformed, to which production is optimally allocated between firms in the short run." That is, even if every firm in an industry were operating on the production frontier, there might still be efficiency gains available by reallocating inputs and outputs among the industry's firms. This reallocation could improve productivity by improving the mix and scale of its firms' operations. To determine the contributions of these factors to productivity change, the structural efficiency component of Luenberger productivity indicator is further decomposed into mix and scale effects:

$$SEC^{t,t+1} = MIX^{t,t+1} + SCALE^{t,t+1}, \tag{15}$$

where

$$\begin{aligned} MIX^{t,t+1} &= \left[D^{t}_{CRS}\left(X^{t}, Y^{t}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{CRS}\left(X^{t+1}, Y^{t+1}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &\quad - \sum_{k=1}^{K}\left[D^{t}_{CRS}\left(x^{t}_{k}, y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{CRS}\left(x^{t+1}_{k}, y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &= \left[D^{t}_{CRS}\left(\sum_{k=1}^{K} x^{t}_{k}, \sum_{k=1}^{K} y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{CRS}\left(\sum_{k=1}^{K} x^{t+1}_{k}, \sum_{k=1}^{K} y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right] \\ &\quad - \sum_{k=1}^{K}\left[D^{t}_{CRS}\left(x^{t}_{k}, y^{t}_{k}; g^{t}_{x}, g^{t}_{y}\right) - D^{t+1}_{CRS}\left(x^{t+1}_{k}, y^{t+1}_{k}; g^{t}_{x}, g^{t}_{y}\right)\right] \end{aligned} \tag{16}$$

and

$$
\begin{aligned}
SCALE^{t,t+1} = &\left(\sum_{k=1}^{K} \left[D_{CRS}^{t}\left(x_k^t, y_k^t; g_x, g_y\right) - D_{CRS}^{t+1}\left(x_k^{t+1}, y_k^{t+1}; g_x, g_y\right) \right] \right. \\
&\left. - \left[D_{CRS}^{t}\left(X^t, Y^t; g_x, g_y\right) - D_{CRS}^{t+1}\left(X^{t+1}, Y^{t+1}; g_x, g_y\right) \right] \right) \\
&- \left(\sum_{k=1}^{K} \left[D_{VRS}^{t}\left(x_k^t, y_k^t; g_x, g_y\right) - D_{VRS}^{t+1}\left(x_k^{t+1}, y_k^{t+1}; g_x, g_y\right) \right] \right. \\
&\left. - \left[D_{VRS}^{t}\left(X^t, Y^t; g_x, g_y\right) - D_{VRS}^{t+1}\left(X^{t+1}, Y^{t+1}; g_x, g_y\right) \right] \right) \\
= &\left(\sum_{k=1}^{K} \left[D_{CRS}^{t}\left(x_k^t, y_k^t; g_x, g_y\right) - D_{CRS}^{t+1}\left(x_k^{t+1}, y_k^{t+1}; g_x, g_y\right) \right] \right. \\
&\left. - \left[D_{CRS}^{t}\left(\sum_{k=1}^{K} x_k^t, \sum_{k=1}^{K} y_k^t; g_x, g_y \right) - D_{CRS}^{t+1}\left(\sum_{k=1}^{K} x_k^{t+1}, \sum_{k=1}^{K} y_k^{t+1}; g_x, g_y \right) \right] \right) \\
&- \left(\sum_{k=1}^{K} \left[D_{VRS}^{t}\left(x_k^t, y_k^t; g_x^t, g_y^t\right) - D_{VRS}^{t+1}\left(x_k^{t+1}, y_k^{t+1}; g_x^t, g_y^t\right) \right] \right. \\
&\left. - \left[D_{VRS}^{t}\left(\sum_{k=1}^{K} x_k^t, \sum_{k=1}^{K} y_k^t; g_x^t, g_y^t \right) - D_{VRS}^{t+1}\left(\sum_{k=1}^{K} x_k^{t+1}, \sum_{k=1}^{K} y_k^{t+1}; g_x^t, g_y^t \right) \right] \right).
\end{aligned} \tag{17}
$$

The change in mix efficiency is measured under CRS, thus removing any scale effects. It compares the efficiency at the group level with the sum of the efficiencies of individual firms. Taking inputs and outputs in aggregate, the *STE* can reallocate them to find the optimal mix of inputs and outputs. If firms operate with different proportions of inputs and outputs, then on average they can't be as efficient as the industry.

The change in the scale efficiency component measures the productivity gains realized by movements toward optimal scale (i.e., most productive scale size). The calculation of scale efficiency typically involves a contrast of efficiency measured relative to variable returns to scale and constant returns to scale technologies. Note that our measure of change in scale efficiency contrasts the sum of the firm efficiency changes and industry efficiency changes across time and across variable and constant returns for scale technologies.

To summarize, the Luenberger productivity indicator's $\left(LPI^{t,t+1}\right)$ full decomposition into changes in technology $\left(TP^{t,t+1}\right)$, technical efficiency $\left(TEC^{t,t+1}\right)$, mix $\left(MIX^{t,t+1}\right)$, and scale $\left(SCALE^{t,t+1}\right)$ is given by:

$$\begin{aligned} LPI^{t,t+1} &= TP^{t,t+1} + OEC^{t,t+1} \\ &= TP^{t,t+1} + TEC^{t,t+1} + SEC^{t,t+1} \\ &= TP^{t,t+1} + TEC^{t,t+1} + MIX^{t,t+1} + SCALE^{t,t+1}. \end{aligned} \tag{18}$$

4. An Illustration of the Proposed Decomposition

We illustrate the decomposition of the Luenberger productivity indicator presented in the previous section using data on the healthcare sector of China. With a growing, aging, and increasingly wealthy population, the demand for healthcare services in China is increasing. However, despite advances, China's healthcare system faces issues of access, quality, and affordability. In 2009, China announced a wide array of healthcare reforms that affected health insurance, primary care, hospital management, pharmaceutical use, and other elements of healthcare delivery. Reviewing the literature on healthcare delivery in China, Eggleston et al. (2008, p. 149) concluded, "…while there is broad agreement that the system needs reform, there is less agreement on the causes of the system's failure and the reforms necessary to improve it." Using data on China's healthcare sector over the period 2009-2014 to illustrate our expanded decomposition of the aggregate Luenberger productivity indicator may shed new light on the effects of reforms and suggest where further reforms are needed.

China's healthcare delivery system is largely hospital-based; in addition to inpatient care, hospitals account for a large portion of outpatient care, including primary care. Therefore, it is not surprising that a number of earlier studies have examined the efficiency and or productivity of hospitals in China. Ng (2011) used data envelopment analysis (DEA) and the Malmquist productivity analysis to examine the efficiency and productivity of a sample of hospitals in Guangdong province between 2004 and 2008. He found that hospitals were very inefficient and that efficiency change did not contribute to productivity growth. Instead, Ng (2011) found strong technological change, which fueled high productivity growth. Li et al. (2014) used DEA and the Malmquist productivity index to examine the performance of 12 large public hospitals in Beijing over the period 2006-2009. They also found that technological change drove productivity change over their sample period, though the average rate of change declined over time. Li et al. (2014) concluded that efficiency change had a negligible effect on productivity; they noted that their finding was consistent with other research and suggested that low efficiency change was a problem among Chinese public hospitals. Using a Bayesian stochastic frontier model and data from 2002 to 2011 for the hospital sectors of China's 31 provinces, Chen et al. (2016) found that public subsidies and health insurance reform contributed to increases in cost efficiency. For a recent survey of the research on the efficiency of Chinese hospitals, see Dong et al. (2017).

Given its size, China has long used regional policy to guide its economy. Regional policy has provided China with several decades of enjoyed sustained, rapid economic growth. While there are multiple definitions of China's regions, three regions—Eastern, Central, and Western—are commonly used to partition the country for policy purposes. We use these three groups to assess the "industry-level" productivity of healthcare in China, treating the provinces within each region as the regions' "firms" when calculating aggregate firm-level measures.[4] To the best of our knowledge, this application of our novel decomposition of the Luenberger productivity indicator

[4]The Eastern region includes twelve provinces—Beijing, Fujian, Guangdong, Guangxi, Hainan, Hebei, Jiangsu, Liaoning, Shandong, Shanghai, Tianjin, and Zhejiang; the Central region includes nine provinces—Anhui, Heilongjiang, Henan, Hubei, Hunan, Inner Mongolia, Jiangxi, Jilin, and Shanxi; and the Western region includes ten provinces—Chongqing, Gansu, Guizhou, Ningxia, Qinghai, Shaanxi, Sichuan, Tibet, Xinjiang, and Yunnan.

is the first study to assess the regional productivity of China's healthcare sector in a manner that addresses the possibility of reallocation of resources across provinces to improve productivity.

Provincial level annual data for inputs and outputs of the healthcare sector are selected from the China Statistical Yearbook (National Bureau of Statistics of China, 2010-2015) and the China Statistical Yearbook for Regional Economy (National Bureau of Statistics of China, 2010-2014). Our analysis included five inputs—the number of beds in medical institutions, the number of licensed doctors, the number of registered nurses, the number of pharmacists, and the number of other personnel—and three outputs—the number of visits to health institutions (outpatients), the number of inpatients, and the number of inpatient surgical operations. Descriptive statistics for the inputs and outputs appear in Table 1.

Table 1 about here

In this section we illustrate our decomposition of the Luenberger Productivity Indicator using data on the healthcare sectors of three Chinese regions over the period 2009-2014. The choice of the direction vector is important for computations of homogenous productivity indicators and comparisons of the results across the three regions. As the direction vector, we choose the aggregated output production plan for China for the initial year of our sample: $\left(g_x^t, g_y^t\right) = \left(0, \sum_{k=1}^{K} y_k^{2009}\right)$. Therefore the Luenberger productivity indicator and its components (*LPI*, *TP*, *TEC*, *MIX* and *SCALE*) are interpreted in terms of the total output of China. For example, a score of 0.01 means that a region can increase its output by an amount equal to 1% of the total Chinese healthcare output. Moving in the output direction only is consistent with China's goal to improve the availability of healthcare. The choice of direction affects the value of the directional distance function; in fact, Afsharian and Ahn (2014) include the effect of a change in the choice of the direction in a decomposition of the Luenberger productivity indicator into technical change, efficiency change, and direction change components. Choosing a common direction for all entities across all time periods assures that the Luenberger productivity indicator and all of it subcomponents are commensurable; therefore, direct comparisons of Luenberger productivity indicator or any decomposed indicators across regions are meaningful as are the addition, subtraction, or averaging of results. The commensurability of measures based on a common direction is an important property that is often neglected in empirical work.[5]

Table 2 reports the mean values of the cumulative Luenberger productivity indicator and its components (changes in technical progress, technical efficiency, mix efficiency, and scale efficiency) over the sample period for each of the three regions analyzed. Two things stand out. First, quantitatively, the values are relatively small; second, qualitatively, the drivers of productivity generally differ across regions. For all three regions, technological progress is positive and is the largest component of the Luenberger productivity indicator. The Eastern region also benefited from gains in technical efficiency, but suffered from decreases in mix and scale efficiency. The Central region enjoyed gains from technical, mix, and scale efficiency. The Western region benefitted from gains in mix efficiency, but lost due to declines in technical and scale efficiency. The individual measures will be discussed in turn.

Table 2 about here

[5] Daraio and Simar (2016) propose a data-driven approach for selecting the direction along which to gauge efficiency. The direction we've chosen is consistent with previous work and facilitates the interpretation of findings.

Figure 2 plots the cumulative aggregate Luenberger productivity indicator of the regional healthcare sectors and their annual rates of growth. For ease of interpretations, the Luenberger productivity indicator is normalized at 0 for the first period and is presented cumulatively over time. It is clear in Figure 2 that the Eastern and Central regions share similar patterns of the evolution for their Luenberger productivity indicators; the evolution of the Western region's Luenberger productivity indicator stands in contrast, both quantitatively and qualitatively. The Eastern and Western regions have relatively larger annual productivity growth rates (0.53% and 0.73%, respectively) compared to the Western region (0.18%) and the Eastern and Central regions had productivity gains in four of the five intervals analyzed while the Western region was found to have experienced productivity declines in four of the five intervals analyzed.

Figure 2 about here

The decomposition of the evolution of LPI growth into its four components is shown in Table 2. The primary driver of productivity is technological progress for all three regions. The effects of the other components of productivity change vary across the three regions. For the Central region all components are positive, but for the Eastern and Western regions some components are positive and while others are negative. For example, for Eastern region, the average annual total productivity gain of 0.53% is mainly due to an increase in technical progress (0.58%) and an increase in technical efficiency (0.19%) but a decrease in mix and scale efficiencies.

As illustrated in Figure 3, all three regions have qualitatively similar patterns of technological progress, though the quantitative changes are consistently greatest for the Eastern region and lowest for the Western region. This is perhaps indicative of a diffusion process where the effects of innovation are felt first in the Eastern region before spreading west. Technological progress was greatest for all three regions between 2011 and 2012, possibly due to the financing of public health infrastructure and primary healthcare that occurred under the healthcare reforms of 2009.

Figure 3 about here

Figure 4 presents the changes in technical efficiency over time for the three regions. Again, except for the last interval, the Eastern and Central regions had qualitatively similar experiences—small gains or declines initially, then relatively big gains from 2012-2013; from 2013-2014, the Eastern region enjoyed further gains, but the Central region declined. Over the sample period, the gains for the Eastern and Central regions averaged 0.23% and 0.19%, respectively. The Western region, on the other hand, experienced minimal changes in technical efficiency which averaged –0.03%.

Figure 4 about here

The mix efficiency component of the Luenberger productivity indicator is exhibited in Figure 5. Qualitatively, the patterns of changes in mix efficiency differ across the three regions. The Eastern region began the period with a relatively large gain, then experienced three years of decline before finishing with another gain; overall its annual average change in mix efficiency was –0.13%. The Central region alternated between declines and gains over the period, but averaged

0.06% over the period. The Western region enjoyed slow but steady gains in mix efficiency, which averaged 0.12% over the sample period.

Figure 5 about here

Figure 6 reveals the evolution of scale efficiency changes for regional healthcare sectors over the sample period. Quantitatively, the Eastern and Western regions experienced similar declines in scale efficiency, averaging –0.11% and –0.07%, respectively, suggested that both regions moved away from the most productive scale size. The Central region benefited from an annual rate of growth of 0.12% in scale efficiency, moving closer to optimal scale on average.

Figure 6 about here

5. Discussion and conclusion

This paper develops a novel decomposition of the aggregate Luenberger productivity indicator under the variable returns to scale technology that reveals the contributions of technological change, technical efficiency change, mix efficiency change, and scale efficiency change to productivity growth. Each of the components captures the effect of a different driver of productivity change, shedding light on the sources of productivity growth and their relative importance.

We illustrate our proposed decomposition of the Luenberger productivity indicator using data on the healthcare sector of three regions of China. Our results indicate that the growth of Luenberger productivity indicator vary in regions and the annual growth rates are 0.73%, 0.53%, and 0.18% in the Central, Eastern, and Western areas respectively. The primary driver of productivity for all regions is technological progress, suggesting that investment in new technologies, the construction of new hospitals, and other investments have improved technology in the healthcare sector. On the other hand, the novel components of our decomposition representing structural inefficiencies had smaller, more varied impacts on productivity change. For the Central region, reallocations improved both scale and mix efficiency, while in the Eastern region both scale and mix efficiency changes had negative effects on productivity; in the Western region, mix had a positive impact and scale had a negative impact on productivity.

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Table 1: Descriptive Statistics of Inputs and Outputs by Region

Variable	Region	Mean	Std. Dev.	Minimum	Maximum	Trend
Input 1	Eastern	23.43	14.73	3.45	50.06	7.29%
Beds	Central	24.66	11.04	12.90	45.93	8.28%
	Western	15.70	12.99	1.19	45.96	9.54%
Input 2	Eastern	9.74	5.67	1.41	19.58	5.39%
Licensed	Central	8.15	2.67	5.26	13.44	3.55%
doctors	Western	4.72	4.01	0.40	14.50	4.28%
Input 3	Eastern	12.04	7.16	2.24	24.57	9.29%
Registered	Central	10.42	4.53	5.67	19.11	9.13%
nurses	Western	6.21	5.05	0.27	17.55	11.61%
Input 4	Eastern	1.70	1.04	0.27	3.65	4.05%
Pharmacists	Central	1.42	0.54	0.78	2.40	1.54%
	Western	0.78	0.65	0.06	2.26	5.47%
Input 5	Eastern	9.56	6.38	1.60	23.47	2.64%
Other	Central	10.06	6.06	4.86	25.03	2.36%
personnel	Western	5.92	4.85	1.09	17.50	5.20%
Output 1	Eastern	182.20	125.24	16.06	469.92	10.36%
Surgical	Central	140.40	70.76	58.29	268.89	10.50%
operation	Western	93.30	81.42	3.78	277.92	12.26%
Output 2	Eastern	718.28	472.20	97.10	1501.82	8.71%
Inpatients	Central	751.34	415.86	300.47	1447.10	9.73%
	Western	505.98	433.15	22.93	1508.79	10.14%
Output 3	Eastern	3.44	2.25	0.45	7.81	7.73%
Outpatients	Central	2.31	1.47	1.00	5.50	6.97%
	Western	1.40	1.26	0.13	4.45	5.86%

Table 2: Growth Rates of the Luenberger productivity indicator and its Components by Region

Indicator	Eastern	Central	Western
LPI	0.53%	0.73%	0.18%
TP	0.58%	0.32%	0.15%
TEC	0.19%	0.23%	-0.03%
MIX	-0.13%	0.06%	0.12%
SCALE	-0.11%	0.12%	-0.07%

Figure1: Proposed Decomposition of the Luenberger Productivity Indicator

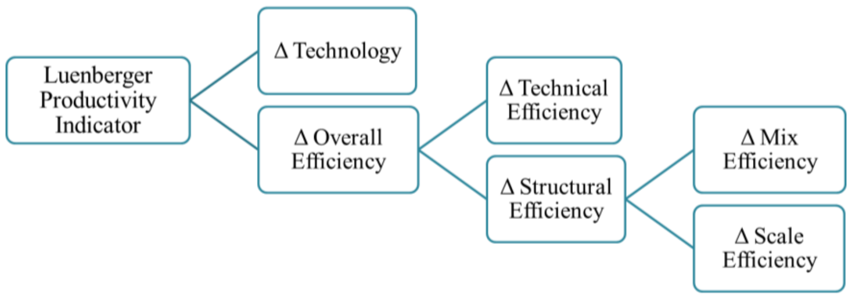


Figure 2: Regional Cumulative Luenberger Productivity Indicators for the Chinese Healthcare Sector

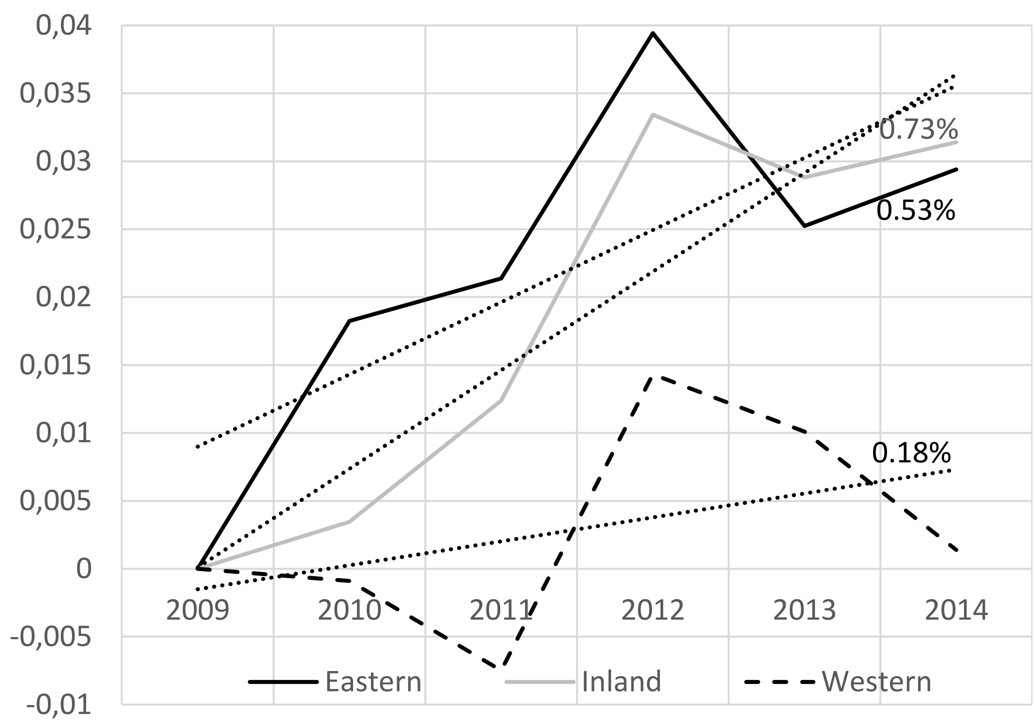


Figure 3: Regional Technological Progress for the Chinese Healthcare Sector

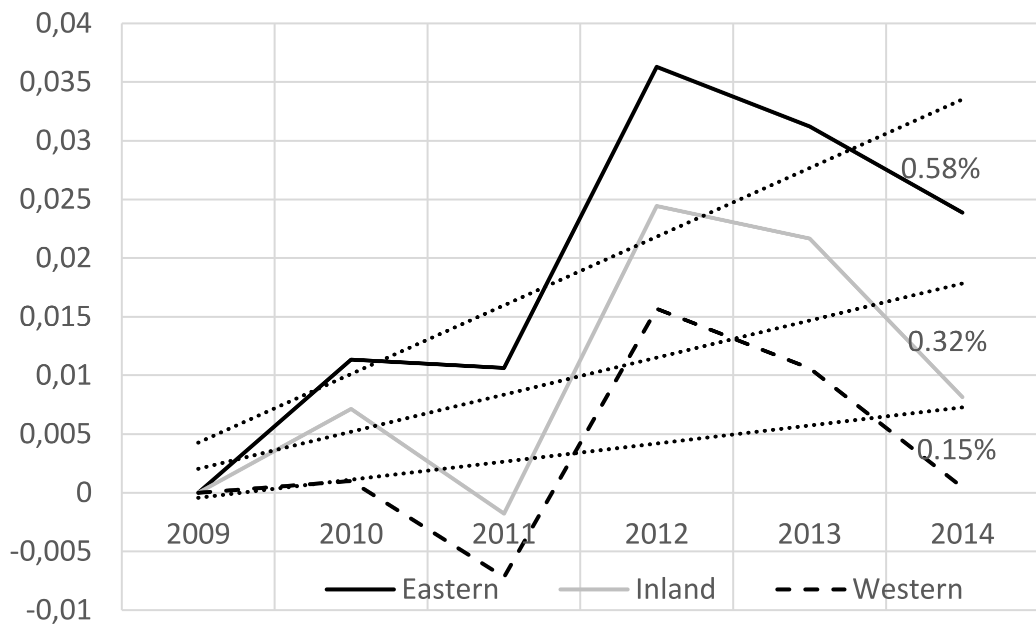


Figure 4: Regional Technical Efficiency Change for the Chinese Healthcare Sector

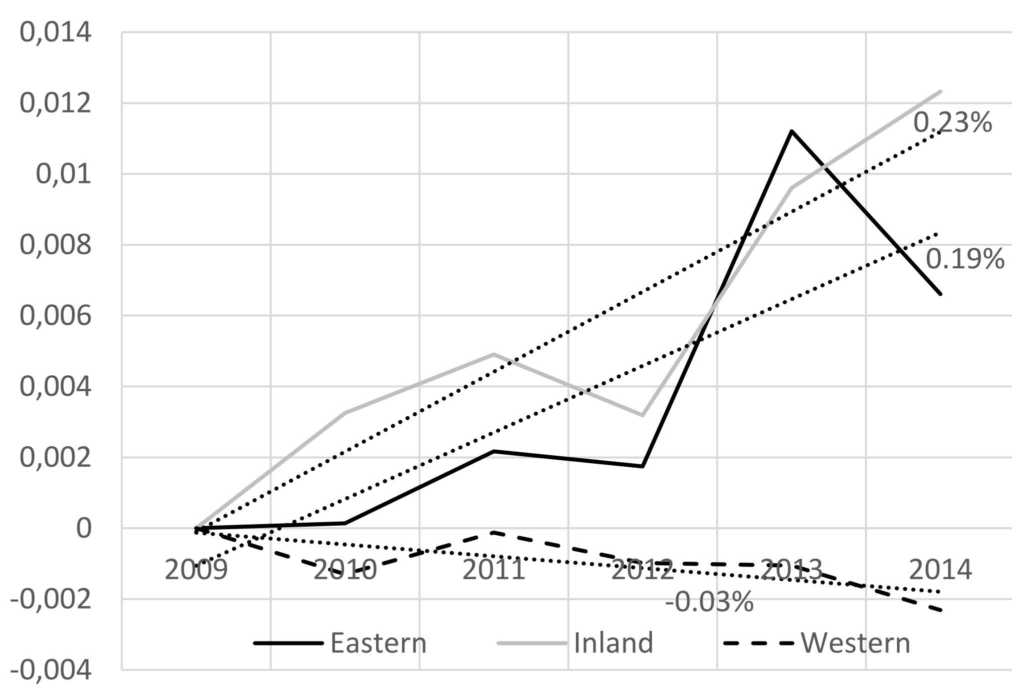


Figure 5: Regional Mix Efficiency Change for the Chinese Healthcare Sector

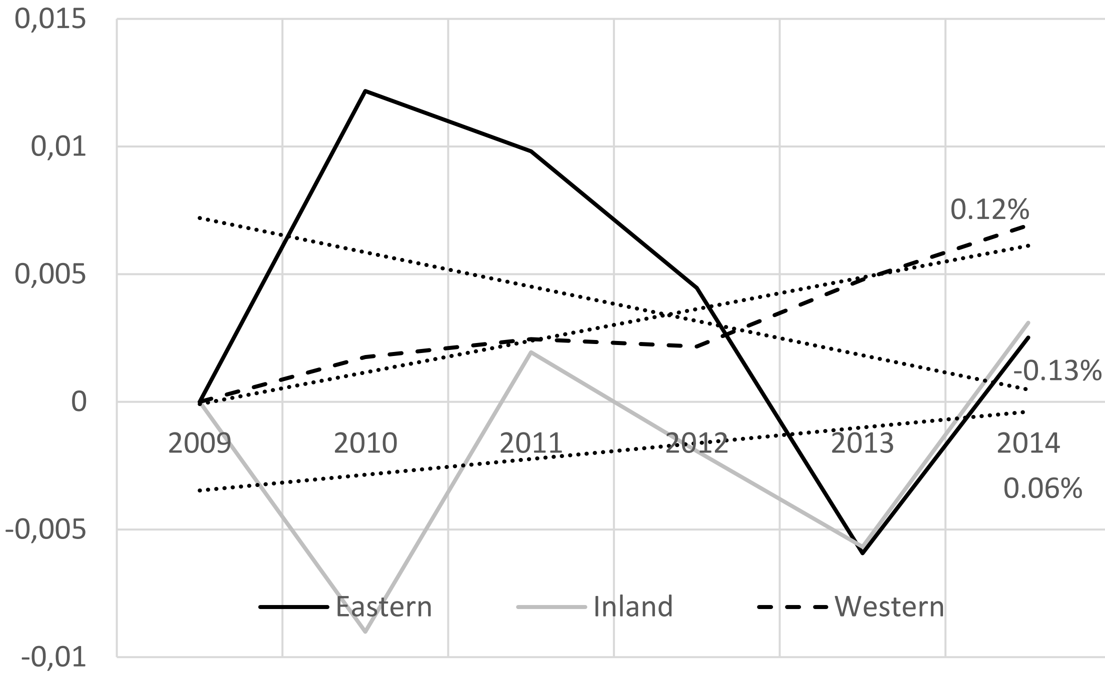
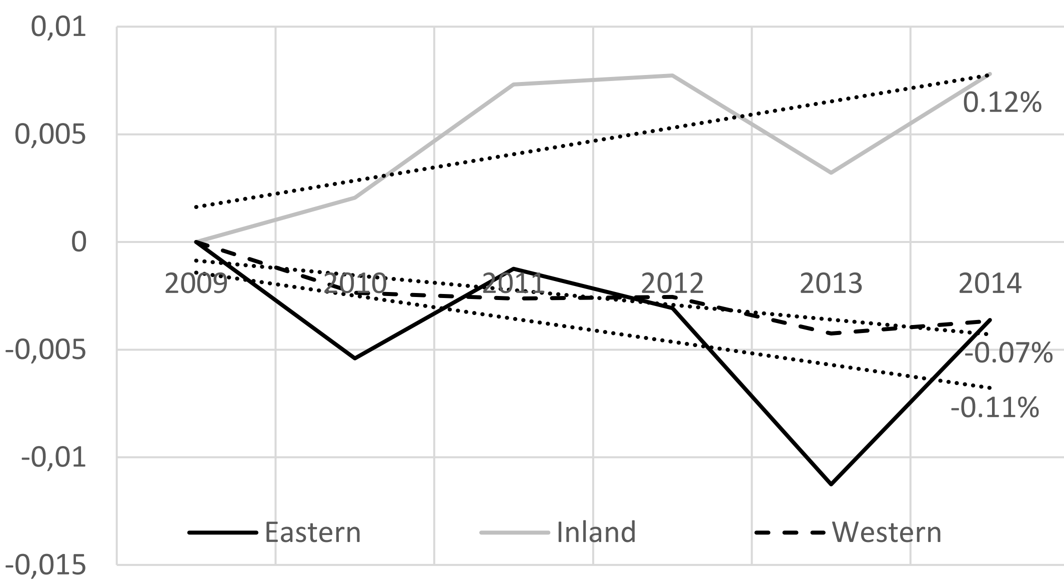


Figure 6: Regional Scale Efficiency Change for the Chinese Healthcare Sector



Appendix

This appendix presents the linear programs used to calculate the firm-level and industry-level (i.e., aggregate) distance functions needed to compute the Luenberger productivity indicator and its components.

Suppose the firms in an industry use *N* inputs to produce *M* outputs. Let $x_k^t \in \mathbb{R}_+^N$ be the input vector and $y_k^t \in \mathbb{R}_+^M$ be the output vector of the k^{th} firm in time period *t*. In our empirical illustration, the aggregate Chinese output production plan for the initial year in our sample, 2009, was used as the direction vector in all calculations; i.e., $\left(g_x^t, g_y^t\right) = \left(0, \sum_{k=1}^{K} y_k^{2009}\right)$. A common direction was chosen to assure conformability of distance functions, the initial year of data was used as a baseline for changes in productivity, and the output direction vector was employed based on the assumption that China is seeking to ensure access to healthcare.

A firm- (province-) level (output) directional distance function is evaluated by solving the following linear program problem:

$$
\begin{aligned}
D^t\left(x_k^t, y_k^t; 0, \sum_{k=1}^{K} y_k^{2009}\right) = \max_{\delta,\lambda} \ & \beta \\
s.t. \ & \sum_{k=1}^{K} \lambda_k y_{k(m)}^t \geq y_m^t + \beta \cdot \sum_{k=1}^{K} y_{k(m)}^{2009}, \ \forall m = 1, \cdots, M \\
& \sum_{k=1}^{K} \lambda_k x_{k(n)}^t \leq x_n^t, \ \forall n = 1, \cdots, N \\
& \lambda_k \geq 0 \ \ \forall k = 1, ..., K \\
\text{and} \ & \sum_{k=1}^{K} \lambda_k = 1.
\end{aligned} \tag{19}
$$

The variable β estimates the radial distance between the evaluated plan and production frontier in the output direction. The last constraint allows for variables returns to scale; dropping it imposed constant returns to scale on the technology underlying the distance function.

To evaluate an industry- (regional-) level distance function, the following linear program must be solved:

$$D^t\left(\sum_{k\in R_i} x_k^t, \sum_{k\in R_i} y_k^t; 0, \sum_{k=1}^{K} y_k^{2009}\right) = \max_{\delta,\lambda} \ \beta$$

$$\begin{aligned} s.t.\ & \sum_{k=1}^{K}\lambda_k y_{k(m)}^t \geq \sum_{k\in R_i} y_{k(m)}^t + \beta\cdot\sum_{k=1}^{K} y_{k(m)}^{2009},\ \forall m=1,\cdots,M \\ & \sum_{k=1}^{K}\lambda_k x_{k(n)}^t \leq \sum_{k\in R_i} x_{k(n)}^t,\ \forall n=1,\cdots,N \\ & \lambda_k \geq 0\ \ \forall k=1,...,K \\ \text{and}\ \ & \sum_{k=1}^{K}\lambda_k = 1. \end{aligned} \tag{20}$$

As with the firm-level linear program, β measures the radial distance between the evaluated plan and production frontier in the output direction and the last constraint allows for variables returns to scale; dropping it imposed constant returns to scale on the technology underlying the distance function.