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Do Smart Beta ETFs Capture Factor Premiums? A Bayesian Perspective

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Abstract

We investigate which factors matter to explain the returns of smart beta and conventional ETFs using a Bayesian approach. Smart beta ETFs are well explained by the market, size and the betting-against-beta factor, whereas conventional ETFs are well explained by the market, the quality-minus-junk factor, and a value factor. Smart beta ETFs benefit from their exposure to the betting-against-beta factor, however this is offset by their negative alphas, while the factor exposure of conventional ETFs is purely detrimental. Our results suggest investors should be skeptical about the ability of smart beta ETFs to capture factor premiums.

Keywords: Smart Beta, strategic beta, factor investing, factor selection, Bayesian variable selection

1. Introduction

The popularity of factor investing among institutional investors has spawned a range of financial products, most notably Exchange Traded Funds (ETFs), that aim to provide factor exposure in a cheap and transparent way[1]. "Smart" or "strategic" beta ETFs, which either explicitly target one or more factors, or make use of alternative weighting schemes using fundamental variables (*i.e. fundamental indexation*), have become a significant portion of the ETF market. A recent study published by MorningStar (Johnson, 2017) shows that, as of June 2017, there were 1,320 "strategic beta" exchange traded products, with global assets under management of over U$700 billion worldwide, the majority of which in the form of U.S. equity ETFs.

The factor exposure of smart beta ETFs is an important issue for investors, however it is not a straightforward one, due to the various ways in which these

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[1] A recent survey on factor investing among investment professionals (Amen *et al.* , 2017) report that 73% make use of a multi-factor framework, with another 18% planning to implement one. See Ang (2014) for a review of the factor investing approach.

products attempt to capture factor returns, potential time variation in factor loadings (Ang *et al.* , 2017), differences with respect to factor definitions in the asset pricing literature, and the inherent uncertainty regarding which factors are priced or robust sources of return (see Beck *et al.* , 2016). For example, even fundamental indexation strategies create unanticipated (typically value) factor tilts (see Asness, 2006; Blitz & Swinkels, 2008). Therefore, it is important to understand the factor exposures of these products not only to explicit factor bets, but also to other factors[2]. Additionally, the long-only restriction of smart beta ETFs suggests that smart beta ETFs may not be able to capture full factor premiums, and may also create unintended factor exposures (see Blitz, 2016).

In this study, we apply a Bayesian factor selection method to investigate which factors matter to explain the returns of smart beta as well as conventional ETFs. We consider 14 popular asset pricing factors, and apply the Bayesian variable selection method to search for the models with highest posterior probability for these two groups of ETFs. By comparing the best models obtained for each group of ETFs, we can investigate if the factors that explain smart beta ETFs are different from those for conventional ETFs, and how significant these factors are in terms of the contribution to the returns of smart beta ETFs.

We investigate all U.S. equity ETFs which are active as of December 2017 and have return data over the period from January 2013 to December 2017, comprising 200 smart beta ETFs and 168 conventional ETFs, totalling over $1.5 trillion in assets under management. We create automatic rules to classify ETFs as smart beta or conventional based on keyword searches in the ETFs names and descriptions obtained from Thomson Reuters DataStream, and then manually check the resulting classification to ensure it conforms to our definition of smart beta.

Due to the uncertainties regarding the (intended or unintended) factor exposures of smart beta ETFs, as well as to which factors are priced sources of return, we consider a comprehensive set of candidate factors. This set includes the factors popular in asset pricing such as those proposed by Fama & French (2015), Chen & Zhang (2010) and Hou *et al.* (2015), which comprise the market factor and factors related to the size, value, investment and profitability effects. Additionally, we consider factors related to the momentum (Jegadeesh & Titman, 1993), low volatility (Ang *et al.* , 2006), betting-against-beta (Frazzini & Pedersen, 2014), quality (Asness *et al.* , 2017), illiquidity (Amihud, 2002), and the alternative value factor of Asness & Frazzini (2013). Different versions of related factors are used to understand the relationship between factor implementation in smart beta ETFs and in academic studies. Multicollinearity is not an issue in our framework, as the variable selection methodology will focus on the most parsimonious sets of factors, and thus models which include highly correlated or redundant factors will naturally have low posterior probability[3].

[2]See for example Amenc *et al.* (2018) and Shirbini (2018).

[3]Some of these factors are highly correlated. For example, the correlation of the profitability factors based on ROE (return on equity) (Hou *et al.* , 2015) and ROA (return on assets) (Chen

We compare the performance of the best models selected using our methodology with that of a benchmark model that includes the largest number of factors possible by removing factors which cause extreme multicollinearity. The benchmark model includes nine factors such as the Fama & French (2015) factors, the momentum, quality-minus-junk, betting-against-beta, and volatility factors.

Our main results from applying our factor selection procedure to smart beta and conventional ETFs show that (i) parsimonious models with up to three factors are selected with high posterior probability to explain the returns on either group of ETFs; (ii) factors that are selected for smart beta ETFs are not the same as those for conventional ETFs; (iii) the performance of the highest posterior probability models to explain the returns on each group of ETFs is very similar to the performance of the full benchmark model with nine factors. Therefore, our Bayesian procedure finds parsimonious models that perform as well as models with many additional factors.

For smart beta ETFs, a two-factor model with the market and the size (small-minus-big, SMB) factors is selected with high posterior probability (0.67). The second best model includes the Frazzini & Pedersen (2014) betting-against-beta (BAB) factor, with a posterior probability of 0.29. The average R^2 of this three-factor model identified by our procedure is 0.84, compared to 0.90 using the benchmark model and 0.76 for the single-factor market model (*i.e.* the CAPM). The average absolute alpha from the three-factor model is 0.16% per month, whereas it is 0.14% for the full benchmark model and 0.24% for the CAPM. Therefore, adding the 2 factors (size and BAB) to the CAPM produces a parsimonious model that explains almost as much variability and average return as the full benchmark model in smart beta ETFs. This result raises an important question about the ability of smart beta ETFs to capture premiums related to other factors such as value, momentum, profitability, and investment, especially as the BAB factor is not significantly correlated to these factors.

For conventional ETFs, the highest probability model (with a posterior probability of 0.70) includes the market factor, the Asness & Frazzini (2013) HMLd factor, and the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017). The average R^2 (average absolute alpha) from this model is 0.66 (0.31%), compared to 0.73 (0.39%) for the benchmark model, and 0.57 (0.45%) for the CAPM. These results reveal that conventional ETFs also have significant factor exposures, although these factors are different from those of smart beta ETFs.

We find that, although smart beta ETFs, on average, benefit from their exposure to the BAB factor, they still underperform the market due to their negative alphas and a small negative contribution from the SMB factor. The factor exposures of conventional ETFs to non-market factors such as QMJ, CMA and HMLd, on the other hand, are purely detrimental, reducing the average

& Zhang, 2010) is close to 0.95, and the correlation between the Fama & French (2015) size factor and the Amihud (2002) illiquidity factor is 0.92. Other related factors such as the Fama & French (2015) HML (High Minus Low) value factor and the Asness & Frazzini (2013) HMLd (High Minus Low "Devil") factor have correlations close to 0.80.

ETF return by -0.17% per month. These results suggest that investors should be skeptical about the possibility of obtaining factor exposure through smart beta ETFs.

2. Methodology

Consider N assets and K predictor variables (factors) over T periods. We define a linear factor model, *i.e.* a multivariate linear regression with N equations:

$$\mathbf{r}_i = \mathbf{X}\boldsymbol{\beta}_i + \mathbf{e}_i, \qquad i = 1, \ldots, N \tag{1}$$

where, for each asset i (in this work, an ETF), $\mathbf{r}_i$ is the $T \times 1$ vector of excess returns, $\mathbf{X}$ is the matrix of factors with dimension $T \times K$, $\boldsymbol{\beta}_i = (\beta_{i,1}, \ldots, \beta_{i,K})'$ is a vector of unknown regression coefficients (factor sensitivities), and $\mathbf{e}_i$ is a $T \times 1$ vector of disturbances[4]. If the error terms are contemporaneously cross-correlated, the system of regressions above is a special case of the Seemingly Unrelated Regressions (SUR) model, where the predictor variables are the same for all equations[5].

The system can be stacked in a single equation $\tilde{\mathbf{r}} = \tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}} + \tilde{\mathbf{e}}$ in the following way:

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \end{bmatrix} = \begin{bmatrix} \mathbf{X} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_N \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_N \end{bmatrix}, \tag{2}$$

where $\tilde{\mathbf{e}} = (\mathbf{e}_1' \quad \mathbf{e}_2' \quad \ldots \quad \mathbf{e}_N')'$, and $\mathbb{E}(\tilde{\mathbf{e}}\tilde{\mathbf{e}}') = \boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \mathbf{I_T}$.

In order to carry out factor selection in model 2, we introduce a vector $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_K)'$ of dummy variables, where if $\gamma_j = 1$, the $j-th$ predictor is included in the model. We are interested in the posterior distribution of $\boldsymbol{\gamma}$, which will indicate which sets of factors have high posterior probability to explain the returns on the N assets. Let $\mathbf{X}_{\boldsymbol{\gamma}}$ represent the matrix $\mathbf{X}$ where each column has been multiplied by the corresponding γ_j. Then we can write the model with variable selection as $\mathbf{r}_i = \mathbf{X}_{\boldsymbol{\gamma}}\boldsymbol{\beta}_i + \mathbf{e}_i, i = 1, \ldots, N$, or stacking the N equations as before,

$$\tilde{\mathbf{r}} = \tilde{\mathbf{X}}_{\boldsymbol{\gamma}}\tilde{\boldsymbol{\beta}} + \tilde{\mathbf{e}},$$

where $\tilde{\mathbf{X}}_{\boldsymbol{\gamma}}$ is defined analogously as before.

Since the vector of K dummy variables indicates 2^K possible models, comparison of all possible models becomes computationally infeasible for even moderate numbers of regressors. In this case, Markov Chain Monte Carlo (MCMC)

[4]To avoid ambiguity, throughout this article we use the subscripts i and j for assets and predictor variables, respectively.

[5]The SUR model, introduced by Zellner (1962), consists of N regression equations, each with T observations, which are linked solely through the covariance structure of error terms at each observation, *i.e.* errors are contemporaneously correlated but not autocorrelated. Bayesian inference in the SUR model can be carried out in a relatively straightforward manner, see for example Giles (2003).

methods provide a fast way to obtain consistent estimates of model probabilities[6].

We apply the variable selection methodology for the SUR model introduced by Hwang & Rubesam (2018), which assumes independence between the factor sensitivities and the dummy variables γ_j, providing an efficient way to carry out factor selection with large panels of data. We review the main aspects in this section, provide more details in Appendix A, and refer the reader to the original paper for a detailed derivation of the conditional posterior distributions.

Suppose $\tilde{\mathbf{e}} \sim N(\mathbf{0}, \mathbf{\Sigma} \otimes \mathbf{I_T})$. Let $\tilde{\boldsymbol{\beta}}_{-i}$ denote the full vector $\tilde{\boldsymbol{\beta}}$ omitting $\boldsymbol{\beta}_i$ and assume the following prior distributions for $\boldsymbol{\beta}_i$, $\mathbf{\Sigma}$ and $\boldsymbol{\gamma}$:

$$\begin{aligned} \boldsymbol{\beta}_i|\tilde{\boldsymbol{\beta}}_{-i} &\sim N(\mathbf{b}_{0,i}, \mathbf{B}_{0,i}), \quad i = 1, \ldots, N \\ \mathbf{\Sigma} &\sim IW(\nu_0, \mathbf{\Phi}_0) \\ \gamma_j &\sim B(1, \pi_j), \quad j = 1, \ldots, K \end{aligned} \tag{3}$$

where $IW(\nu_0, \mathbf{\Phi}_0)$ denotes the inverted-Wishart distribution with ν_0 degrees of freedom and parameter matrix $\mathbf{\Phi}_0$, and $B(1, \pi_j)$ denotes the Bernoulli distribution with probability of success π_j. In the above, each γ_j is independent of the remaining ones, therefore the prior dor $\boldsymbol{\gamma}$ is given by $f(\boldsymbol{\gamma}) = \prod_{j=1}^{K} \pi_j^{\gamma_j}(1-\pi_j)^{1-\gamma_j}$.

With the priors above and given initial values for the variables, the estimation procedure using the Gibbs sampler is as follows:

1. Generate $\boldsymbol{\beta}_i|\tilde{\boldsymbol{\beta}}_{-i}, \boldsymbol{\gamma}, \mathbf{\Sigma}, \tilde{\mathbf{r}} \sim N(\mathbf{b}_{1,i}, \mathbf{B}_{1,i})$, where

$$\begin{aligned} \mathbf{b}_{1,i} &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}_{\boldsymbol{\gamma}}'\mathbf{X}_{\boldsymbol{\gamma}})^{-1}(\mathbf{B}_{0,i}\mathbf{b}_{0,i} + \sigma^{ii}\mathbf{X}_{\boldsymbol{\gamma}}'\mathbf{r}_i^*) \\ \mathbf{B}_1 &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}_{\boldsymbol{\gamma}}'\mathbf{X}_{\boldsymbol{\gamma}})^{-1}, \end{aligned}$$

   where σ^{ii} is the (i,i) element of $\mathbf{\Sigma}^{-1}$ and $\mathbf{A}_{-i}$ is a $T \times (N-1)T$ partition of $\mathbf{\Omega}^{-1}$ with the terms corresponding to the $i-th$ equation removed.

2. Generate $\mathbf{\Sigma}|\tilde{\boldsymbol{\beta}}, \mathbf{r} \sim IW(\nu_1, \mathbf{\Phi}_1)$, with $\nu_1 = \nu_0 + T$ and $\mathbf{\Phi}_1 = \mathbf{\Phi}_0 + \mathbf{S}$, where $\mathbf{S}$ is the matrix of cross-products of the residuals, that is, if $\mathbf{E} = [\mathbf{e}_1 \ldots \mathbf{e}_N]$, then $\mathbf{S} = \mathbf{E}'\mathbf{E}$.

3. Generate (in random order) γ_j conditional on the remaining $\gamma_k, k \neq j$, from the following conditional distribution:

$$P(\gamma_j = 1|\boldsymbol{\gamma}_{-j}, \tilde{\boldsymbol{\beta}}, \mathbf{\Sigma}, \tilde{\mathbf{r}}) = \left(1 + \frac{1-\pi_j}{\pi_j}\exp(-0.5\,\mathrm{Tr}(\mathbf{\Sigma}^{-1}(\mathbf{S}_{\boldsymbol{\gamma}}^1 - \mathbf{S}_{\boldsymbol{\gamma}}^0))\right)^{-1}, \tag{4}$$

   where $\mathbf{S}_{\boldsymbol{\gamma}}^1$ and $\mathbf{S}_{\boldsymbol{\gamma}}^0$ represent the matrices of residuals when $\gamma_j = 1$ and $\gamma_j = 0$, respectively.

[6]There is a vast literature focusing on Bayesian variable selection in linear models with a single response variable, see for example George & McCulloch (1993, 1997); Kuo & Mallick (1998); O'Hara & Sillanpää (2009). For the multivariate case, of which the SUR model is a special case, see Brown *et al.* (1998), Smith & Kohn (2000), Hall *et al.* (2002), Wang (2010), Ando (2011) and Puelz *et al.* (2017)

2.1. Prior Distributions

The most important prior distribution is the one for $\tilde{\boldsymbol{\beta}}$. As discussed by O'Hara & Sillanpää (2009), the MCMC algorithm might not mix well in the $\boldsymbol{\gamma}$ space if the prior for $\tilde{\boldsymbol{\beta}}$ is too vague. The reason for this is that, when $\gamma_j = 0$, $\beta_{ij}s, i = 1, \ldots, N$ are sampled from the full prior conditional distribution. In this case, it may be difficult for the model to transition between $\gamma_j = 0$ and $\gamma_j = 1$, since the generated β_{ij} will be unlikely to be in the region where θ_{ij} has higher posterior probability.

We propose a few choices for the priors on $\tilde{\boldsymbol{\beta}}$. The first is $\tilde{\boldsymbol{\beta}} \sim N(\mathbf{0}, c\mathbf{I})$. This choice reflects a complete lack of knowledge about the predictors, both in terms of which predictors should enter the model as well as regarding the dependence structure of the regression coefficients. A second possibility is to use an empirical Bayes prior, *i.e.* center each $\boldsymbol{\beta}_i$ around their OLS or maximum likelihood estimate: $\boldsymbol{\beta}_i \sim N((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r}_i, c_i\sigma_i^2(\mathbf{X}'\mathbf{X})^{-1})$. Either choice can be made less informative by increasing c. Note that the first component of each β_i is for the alpha of each regression. The intercept is included as a factor because there is no guarantee that the factors we test in this study can fully explain individual ETF returns.

The standard choice for the prior of $\boldsymbol{\Sigma}$ is to set $\nu_0 = N$ and $\boldsymbol{\Phi}_0 = \mathbf{I}$. For the prior probability of π_j that predictor j is included in the model, we choose an equal probability of $\frac{1}{2}$ for all factors. This prior reflects the lack of knowledge about the inclusion of the predictors, and implies that any model, regardless of its possible number of combinations, has an equal prior probability of $\frac{1}{2^K}$.

3. Data

3.1. ETFs

We obtain all U.S. equity ETFs that are active as of the end of 2017 from Thomson Reuters. These 799 ETFs have approximately \$1.6 trillion of collective assets under management (AUM). Since we are interested in equity factor exposure, we remove leveraged and inverse ETFs, as well as ETFs which make use of derivatives. We further require 60 months of available returns, which leads to a sample of 368 ETFs with aggregate AUM of \$1.54 trillion[7].

3.1.1. Classification of ETFs

There is no universally accepted definition of smart beta. In this study, we employ an automatic procedure to search each ETF's name and description for certain keywords. We then manually review the list and the descriptions of smart beta and conventional ETFs to ensure the classification is consistent, consulting the fact sheet or other ETF documentation in case of doubt[8].

[7]Most of ETFs that are excluded from our sample are due to their shorter history. If we were to require a much longer history, the number of smart beta ETFs would decrease significantly.

[8]The details from this procedure are available upon request.

Smart beta ETFs in this study are those that have at least one of the following characteristics:

- Attempt to increase returns relative to a market capitalization-weighted index by providing exposure to one or more factors thought to be sources of return (*e.g.* ETFs focused on value, size, quality, or momentum factors);
- Attempt to reduce risk or increase diversification (*e.g.* low volatility and minimum variance ETFs);
- Alternative weighting schemes (*e.g.* ETFs weighted by fundamentals; equally-weighted ETFs);
- Deviation from market capitalization-weighted schemes in a systematic, rules-based way (*e.g.* ETFs based on dividend or shareholder yield screens).

The ETFs that do not have any of the above characteristics are grouped as "conventional ETFs". This includes all passive ETFs which track common indices, as well as sector-specific ETFs, as long as they do not employ any of the strategies above.

Using the procedure outlined above, we classify 200 ETFs in the smart beta category, and 168 ETFs in the conventional category. Smart beta ETFs as a group manage $515 billion in assets, while the combined AUM of conventional ETFs is over $1 trillion.

3.2. Factors

We use a total of 14 factors in this study. These are the five Fama & French (2015) factors, as well as the momentum (MOM) factor, from Professor Kenneth French's data library[9]. The five Fama & French (2015) factors are the market (MKT), size or Small-Minus-Big (SMB), value or High-Minus-Low (HML), profitability or Robust-Minus-Weak (RMW), and Investment or Conservative-Minus-Aggressive (CMA). We also include the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014), and the alternative value factor HML "devil" (HMLd) of Asness & Frazzini (2013), which we download from the AQR data library[10]. Finally, we add five self-constructed factors related to illiquidity (ILL, Amihud, 2002), volatility (VOL, Ang *et al.* , 2006), and investment (INV) and profitability based on return on assets (ROA, Chen & Zhang, 2010) and return on equity (ROE, Hou *et al.* , 2015).

The self-constructed factors are based on all available U.S. common stocks from the CRSP and Compustat databases, excluding micro-cap stocks, defined as those with market capitalization lower than the 20% percentile of all NYSE stocks. The factors are constructed as hedge portfolio returns based on double

[9] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`
[10] `https://www.aqr.com/Insights/Datasets`

sorts and using value-weighted returns. The illiquidity factor based on a two-by-three sort on volatility and illiquidity as in Amihud *et al.* (2015) because of the high correlationship between illiquidity and volatility. For each month, we calculate the median return volatility using the NYSE breakpoint, and use it to assign all stocks into low or high volatility groups. We then calculate the Amihud (2002) illiquidity measure for all stocks, and use the NYSE low 30%, middle 40% and high 30% breakpoints to assign stocks into three illiquidity groups. The illiquidity factor is calculated as the difference between the average return on the two high illiquidity portfolios and the average return on the two low illiquidity portfolios. The volatility, investment, and profitability factors based on ROE and ROA are constructed using double sorts on size (two portfolios) and the variables in question (three portfolios). Stocks are first sorted into small and large groups based on the median NYSE market capitalization, and then three groups (low, medium and high) are created based on the second variable. The volatility factor is the difference between the average return on the two low volatility portfolios and the average return on the two high volatility portfolios. The investment factor is constructed following Chen & Zhang (2010), *i.e.* it is the difference between the average return on the two low investment portfolios and the average return on the two high investment portfolios. Finally, the ROA (ROE) profitability factors are the differences between the average return on the two high ROA (ROE) portfolios and the average return on the two low ROA (ROE) portfolios.

Table 1 reports descriptive statistics on the 14 factors for the period from January 2013 to December 2017. The average returns on most long-short factors are relatively small during this period, with the exception of the QMJ (0.95% per month) and BAB (1.25% per month) factors. In fact, from the factors in the Fama & French (2015) model, the size (SMB), value (HML), and investment (CMA) factors all present negative returns. Interestingly, the average return on the Chen & Zhang (2010) investment factor is positive at 0.30% per month, which could reflect differences in the definition of investment, and highlights the importance of considering alternative factors when studying the factor exposure which may differ significantly in terms of implementation[11]. The only factors with t-statistics above 2 are MKT (t-stat=3.44) and BAB (t-stat=4.94).

Many of these factors are highly correlated. The most extreme correlations are between ROE and ROA (0.95), SMB and ILL (0.92), VOL and ROA (0.86), QMJ and ROA (0.84), and VOL and ROE (0.82). As mentioned, this is not an issue for our variable selection methodology, but multicollinearity may be problematic in the conventional regression when all these factors are included as explanatory variables. Therefore, using all 14 factors as the benchmark, we calculate variance inflation factors (VIFs) and remove the factors with the high-

[11]Fama & French (2015) define investment as "the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1, divided by t-2 total assets", while Chen & Zhang (2010) definition is "annual change in gross property, plant, and equipment plus annual change in inventories divided by lagged book assets ".

est VIFs one at a time, recalculating the VIFs each time. Using this approach, we remove the following factors: ROA, ROE, ILL, INV and HMLd. The resulting model has 9 factors, the highest VIF (corresponding to VOL) is 7.01 and the highest correlation is 0.81, between VOL and QMJ.

[Table 1 about here.]

4. Empirical Results

4.1. Exploratory Analysis of ETF Factor Exposure

We start by conducting an exploratory analysis of ETFs using OLS regression for individual ETFs. Table 2 reports statistics for three groups of ETFs: all ETFs (Panel A), smart beta ETFs (Panel B) and conventional ETFs (Panel C). We report the average sensitivity to each factor, as well as the corresponding t-statistic, the 5% and 95% percentiles of factor sensitivities, the percentage of ETFs for which the factor is significant, either with a positive or negative sign at the 95% confidence level, the average R^2 when only the market factor is considered and with the benchmark model[12]. Additionally, we report the number of ETFs and total AUM for each group of ETFs.

Panel A of the table shows that the aggregate factor sensitivities are close to zero for most factors and are not significant except for the excess market return and QMJ, as evidenced by their t-statistics. This is not surprising, as there are many ETFs with conflicting factor sensitivities (i.e., positive and negative coefficients). For individual ETFs, however, many factors are still significant. For example, the SMB factor is significantly positive (negative) for 30% (19%) of the ETFs. It is also evident that factor exposure is skewed; the aggregate exposure to most factors is not different from zero, although the 5% and 95% percentiles do not appear equidistant from their means. The average R^2 using the full model is 0.80, compared to 0.64 using only the market factor.

Panels B and C reveal similarities as well as differences in the factor sensitivities of smart beta and conventional ETFs. In aggregate, smart beta ETFs are tilted towards smaller firms, as evidenced by the average significant exposure to the SMB factor (average sensitivity=0.26, t-stat=11.07), while conventional ETFs have an insignificant exposure to SMB on average. Another difference is the exposure to the BAB and VOL factors, which shows positive (and significant) aggregate sensitivity for smart beta ETFs but are insignificant for conventional ETFs. Since the BAB and VOL factors take long positions in low (short positions in high) beta an volatility stocks, respectively, this suggests that, in aggregate, smart beta ETFs are tilted towards low volatility and low beta stocks. Exposure to other factors is similar in aggregate (except for the HML factor, which is positive for smart beta ETFs and negative for conventional ETFs), although the percentage of significantly positive or negative factor sensitivities seems to be higher for smart beta ETFs. Interestingly, a higher proportion of

[12] All R^2 values used in this study are adjusted R^2.

the variance of the returns on smart beta ETFs is explained by the market factor (0.74 compared to 0.56 for conventional ETFs). Also, the increase in R^2 from adding the additional eight factors is more pronounced for conventional ETFs (from 0.56 to 0.73, increase of 30%) compared to smart beta ETFs (0.74 to 0.89, increase of 21%).

Summarizing, smart beta ETFs track the market portfolio more than conventional ETFs, but appear to have larger percentages of significantly positive or negative factor sensitivities, and are, in aggregate, tilted towards small, low beta and low volatility stocks. Therefore, the trading strategies of smart beta ETFs would satisfy investors who pursue the overall market performance but at the same time seek for higher returns or lower risk by attempting to exploit various trading strategies, in particular, the size and low volatility/beta effects.

[Table 2 about here.]

The range of factor sensitivities as well as the differences in factor sensitivities between the two groups of ETFs are visualized in Figure 1, which plots nonparametric kernel density estimates of the factor sensitivities of ETFs in each group. The estimated densities of smart beta ETFs are shown in blue, while those of conventional ETFs are shown in red. In general, despite the fact that both groups of ETFs have similar mean factor sensitivities for many factors, conventional ETFs have a much wider range of factor sensitivities compared to smart beta ETFs, which is apparent from the longer tails of the estimated densities. The wider range of factor sensitivity in the conventional ETFs may reflect other factors such as sector returns which we have not considered in this study. These results suggest that we need other factors to explain the group of conventional ETFs compared to smart beta ETFs. Interestingly, some of the densities are multimodal. For example, the distribution of the sensitivity of smart beta ETFs to the SMB factor has a prominent mode close to 0, and a second mode close to 1, reflecting the fact that many smart beta ETFs focus on small caps.

[Figure 1 about here.]

4.2. Bayesian Factor Selection

The main results of applying our Bayesian factor selection method to the groups of conventional and smart beta ETFs are obtained using an empirical Bayes prior for the factor sensitivities, *i.e.* we center each $\boldsymbol{\beta}_i$ around their OLS estimate by setting $\boldsymbol{\beta}_i \sim N((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r}_i, c\sigma_i^2(\mathbf{X}'\mathbf{X})^{-1})$, with $c = 1$. We consider an equal prior probability for each factor: $\pi_j = \pi = 0.5$. The results are based on 50,000 iterations of the MCMC algorithm. In section 4.3 we test the robustness of our results regarding the choice of the prior.

We focus on the posterior distribution of $\boldsymbol{\gamma}$, which reveals which factors matter for each group of ETFs. Table 3 reports the results for the two groups of ETFs. The marginal posterior factor probabilities are reported in Panel A of the table. For smart beta ETFs, there is strong evidence that the market factor (MKT) and the size factor (SMB) are included in the model, as the posterior probability of these factors is equal to 1. There is weaker evidence for the inclusion of the betting-against-beta (BAB) factor, with a marginal posterior probability of 0.30. All other factors have negligible marginal posterior probabilities. For conventional ETFs, different factors appear to be significant. The market (MKT), quality-minus-junk (QMJ) and the alternative value factor HMLd have marginal posterior probabilities equal to 1, while the Fama & French (2015) investment factor CMA has a lower marginal posterior of 0.30. Note that in all cases, the intercept is not selected. This means that the few selected factors from the set of 14 factors are enough to explain the returns of ETFs, and that other factors are not required.

[Table 3 about here.]

Panel B reports the highest posterior probability models for each group of ETFs. With 14 factors plus the intercept, there are $2^{15} = 32768$ possible models. Nevertheless, the results reveal that only a handful of models have high posterior probabilities. The highest posterior probability model for the group smart beta ETFs includes the market (MKT) and size (SMB) factors (posterior probability = 0.67). The second best model (posterior probability = 0.29) also adds the BAB factor. The other models have very low posterior probabilities. For conventional ETFs, there is even less model uncertainty, as only two models have relevant posterior probabilities. The best model (posterior probability = 0.70) includes the market factor (MKT), the quality-minus-junk (QMJ) and the HMLd factors. The second best model (posterior probability = 0.30) adds the CMA factor. Interestingly, the factors selected for the group of conventional ETFs includes many of the factors typically targeted by smart beta ETFs.

The results obtained with smart beta ETFs are somewhat surprising, considering many of these products explicitly attempt to capture premiums related to other factors such as value, momentum and volatility. In order to better understand these results, and assess to what degree smart beta ETFs capture any factor premiums, we estimate (using OLS) the three-factor model suggested by our Bayesian procedure for this group of ETFs, which includes the MKT, SMB and BAB factors. The results are reported on Panel A of Table 4.

As expected, we find that the SMB and BAB factors tend to be significant for many smart beta ETFs, as evidenced by the high t-statistics and the percentage of significant factor sensitivities. On average, although smart beta ETFs benefit from their positive exposure to the BAB factor, which in our sample generated a monthly premium of 1.25%, this is completely offset by their negative average alpha and their positive average exposure to SMB, which in our sample produced a small negative return of -0.04% per month. The average excess return of smart beta ETFs is 1.20% per month, of which 1.21% per month on average is due to their market exposure, with their non-market factor exposure generating a small average loss of -0.01% per month. Thus, although as a group smart beta ETFs appear to benefit from the BAB factor, this is offset by their negative alphas.

The three-factor model for smart beta ETFs obtained using the Bayesian approach performs very similarly to the benchmark nine-factor model in terms of average absolute alpha and R^2. The average absolute alpha from the three-factor model for the group of smart beta ETFs is 0.16% per month, and the average R^2 is 0.84. For the full nine-factor benchmark model, the numbers are 0.14% and 0.90. Therefore, it is unlikely that these smart beta ETFs are exploiting other factors; if they were, adding these factors to the model would significantly reduce alphas. This result severely questions the ability of smart beta ETFs to capture factor premiums, which may be related to their long-only restriction, or to other differences related to how factors are constructed in the asset pricing literature.

We repeat this exercise for conventional ETFs, estimating a four-factor model with the MKT, CMA, QMJ, and HMLd factors identified by our Bayesian procedure. The results are shown in Panel B of Table 4. We find that, on average, the only factor other than the market return which has a significant sensitivity is the QMJ factor, although CMA and HMLd are significant for many ETFs. Contrary to the group of smart beta ETFs, the non-market factor exposure of conventional ETFs in this sample period is purely detrimental, reducing the average ETF return by -0.17% per month. These ETFs had average positive exposures to CMA and HMLd, both of which had negative returns during the period, and average negative exposure to QMJ, which had a high positive return of 0.95%.

The average absolute alpha and average R^2 for the model identified with the Bayesian method for conventional ETFs are 0.31% and 0.66, respectively, while for the benchmark model the numbers are 0.39% and 0.73, respectively. Again, we find that the Bayesian method finds a parsimonious model which performs quite well compared to the benchmark model.

[Table 4 about here.]

4.3. Robustness Analysis

Our main results were obtained using an empirical Bayes prior, centering each $\boldsymbol{\beta}_i$ around their OLS estimate by setting $\boldsymbol{\beta}_i \sim N((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r}_i, c\sigma_i^2(\mathbf{X}'\mathbf{X})^{-1})$, with $c = 1$. In this subsection we analyze the robustness of our results relative to these choices, by varying both the type of prior and the value of c. Concerning the prior for the $\boldsymbol{\beta}_i$, we obtain results with a different prior by setting $\tilde{\boldsymbol{\beta}} \sim N(\mathbf{0}, c\mathbf{I})$. This choice reflects a complete lack of knowledge about the predictors. We also vary the value of c and obtain results using $c = 1, 2, 5$. A larger c reflects a less informative prior regarding the range of possible values for the regression coefficients (*i.e.* factor sensitivities).

The results for $\boldsymbol{\beta}_i \sim N((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{r}_i, c\sigma_i^2(\mathbf{X}'\mathbf{X})^{-1})$ using the empirical Bayes prior with $c = 2$ are reported on Table B.5, and are essentially similar to our main results with $c = 1$. The best models remain the same for both smart beta as well as conventional ETFs. This is also the case with $c = 5$, and we omit the results.

Table B.6 reports results using the prior centered on a vector of zeros with $c = 1$ ($\tilde{\boldsymbol{\beta}} \sim N(\mathbf{0}, c\mathbf{I})$). The only difference compared to our previous results is that the QMJ factor is not selected in the best model for conventional ETFs, although both the QMJ and CMA factors are present in the second and third best models, as before. The results with $c = 2$ are not significantly different. The results with $c = 5$ (omitted) show that the best model for both smart beta ETFs (posterior probability=0.90) as well as conventional ETFs (posterior probability = 0.80) is the model with only the MKT factor. Thus, when coefficient priors are centered on zero and the prior variance is large, the model selection procedure can only find posterior evidence for the market beta. This did not occur when used the empirical prior, as the point of departure is in the neighborhood where factor sensitivities are different from zero.

Overall, we interpret that our results are robust to the prior specification for the regression coefficients, except in cases when the prior variance is too large and the prior is centered on zeros.

5. Conclusion

Smart beta ETFs have grown enormously over the last years. These products promise to increase returns or lower risk relative to market capitalization-weighted indices by attempting to capture premiums on well-known factors such as size, value, quality, momentum and volatility.

In this paper, we employ a Bayesian variable selection methodology to investigate the factor exposure of smart beta and conventional ETFs, using a large group of factors covering all the most commonly used factors. Our methodology allows us to select which factors matter to explain each category of ETFs, using all individual ETFs in each category simultaneously. Our results reveal that the market and the Fama & French (2015) size (SMB) factors are relevant to explain the returns of smart beta ETFs, with weaker evidence for the inclusion of the Frazzini & Pedersen (2014) betting-against-beta (BAB) factor. For conventional ETFs, the best model includes the quality-minus-junk (QMJ) factor of Asness *et al.* (2017) and the alternative value factor (HML "devil") of Asness & Frazzini (2013), with weaker evidence for the inclusion of the Fama & French (2015) investment (CMA) factor.

Although on average smart beta ETFs benefit from their exposure to the BAB factor, they still underperform the market due to their negative alphas and a small negative contribution from the SMB factor. The factor exposures of conventional ETFs to non-market factors such as QMJ, CMA and HMLd, on the other hand, are purely detrimental, reducing the average ETF return by -0.17% per month.

The best models selected by the Bayesian method perform very similarly to a benchmark model including nine factors in terms of their ability to explain the average returns and the return variation on each set of ETFs, as measured by the average absolute alpha and the average R^2. Therefore, it is unlikely that smart beta ETFs are exploiting other factors. Overall, our results suggest investors should be skeptical about the ability of smart beta ETFs to capture factor premiums. This may be related to their long-only restriction, as mentioned by Blitz (2016), or to differences in how ETFs implement factor exposure compared to asset pricing studies.

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Figure 1: **Kernel density estimation of ETF factor sensitivities**

The figure plots the sensitivities of smart beta and conventional ETFs to a set of seven factors. Our sample includes all U.S. equity ETFs which were active as of December 2017, and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. We regress ETF returns on the returns of seven tradable factors and an intercept term. The factors include the five Fama & French (2015) factors: the market (MKT), size or Small-Minus-Big (SMB), value or High-Minus-Low (HML), profitability or Robust-Minus-Weak (RMW), and Investment or Conservative-Minus-Aggressive (CMA); the momentum (MOM) factor and a low volatility (VOL) factor. The first six factors are obtained from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`), while the volatility factor is self-constructed using a two-by-three sort on size and volatility using all CRSP stocks.

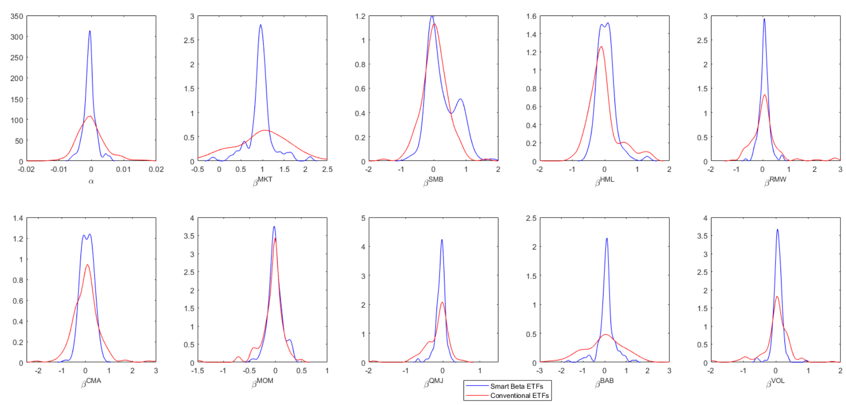

Table 1: Descriptive statistics of factors, Jan/2013-Dec/2017

The table shows descriptive statistics for the 14 factors used in the study, considering monthly returns over the period from January 2013 to December 2017. The market excess return (MKT), the size (Small-Minus-Big, SMB), value (High-Minus-Low, HML), profitability (Robust-Minus-Weak, RMW), investment (Conservative-Minus-Aggresive, CMA), and momentum (MOM) factors are obtained from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The Quality-Minus-Junk (QMJ), Betting-Against-Beta, and High-Minus-Low "devil" (HMLd) factors are obtained from the AQR Data Library (`https://www.aqr.com/Insights/Datasets`). The illiquidity (ILL) factor is based on a two-by-three sort on volatility and the Amihud (2002) illiquidity measure, and is calculated as the difference between the average return on the two high illiquidity portfolios and the average return on the two low illiquidity portfolios. The volatility, investment, and profitability factors based on ROE and ROA are constructed using double sorts on size (2 portfolios) and the variable in question (3 portfolios). The volatility factor is the difference between the average return on the two low volatility portfolios and the average return on the two high volatility portfolios. The investment factor is the difference between the average return on the two low investment portfolios and the average return on the two high investment portfolios. The ROA (ROE) profitability factors are the differences between the average return on the two high ROA (ROE) portfolios and the average return on the two low ROA (ROE) portfolios.

Panel A. Average and standard deviations

	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	HMLd	ILL	VOL	INV	ROE	ROA
Average monthly return	1.27%	-0.04%	-0.04%	0.11%	-0.18%	0.23%	0.95%	1.25%	-0.23%	0.04%	0.18%	0.30%	0.23%	0.20%
Standard deviation	2.85%	2.43%	2.30%	1.56%	1.42%	3.05%	4.53%	1.96%	2.67%	2.61%	3.15%	1.14%	1.79%	2.09%
Standard error	0.37%	0.31%	0.30%	0.20%	0.18%	0.39%	0.58%	0.25%	0.35%	0.34%	0.41%	0.15%	0.23%	0.27%
t-statistic	3.44	-0.12	-0.13	0.56	-1.00	0.59	1.63	4.94	-0.68	0.11	0.43	2.03	1.00	0.76

Panel B: Correlation matrix

	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	HMLd	ILL	VOL	INV	ROE	ROA
MKT	1													
SMB	0.27	1												
HML	0.05	0.28	1											
RMW	-0.13	-0.48	-0.04	1										
CMA	-0.02	0.16	0.64	0.13	1									
MOM	-0.25	-0.17	-0.50	-0.08	-0.42	1								
QMJ	-0.31	-0.61	-0.19	0.58	-0.12	0.30	1							
BAB	0.67	0.24	-0.10	0.06	0.01	0.22	0.10	1						
HMLd	0.19	0.22	0.77	-0.06	0.62	-0.77	-0.44	-0.18	1					
ILL	0.24	0.92	0.38	-0.42	0.18	-0.22	-0.46	0.24	0.27	1				
VOL	-0.49	-0.51	-0.01	0.55	0.14	0.37	0.81	0.04	-0.32	-0.35	1			
INV	0.30	0.30	0.18	-0.41	0.37	0.00	-0.16	0.32	0.05	0.23	-0.15	1		
ROE	-0.32	-0.44	0.08	0.68	0.14	0.22	0.77	0.12	-0.19	-0.32	0.82	-0.16	1	
ROA	-0.39	-0.51	0.00	0.70	0.04	0.26	0.84	0.06	-0.28	-0.39	0.86	-0.25	0.95	1

Table 2: Ordinary Least Square Analysis of ETFs, Jan/2013-Dec/2017

The table reports results from regressing ETF returns on the returns of nine tradable factors and an intercept term. Our sample includes all U.S. equity ETFs which were active as of December 2017 and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. The set of factors include the five Fama & French (2015) factors *i.e.* the market (MKT), size (SMB), value (HML), profitability (RMW), and investment (CMA) factors, the momentum (MOM) factor, the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014), and the volatility (VOL) factor. The Fama & French (2015) factors and the momentum factors are obtained from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The QMJ and BAB factors are obtained from the AQR data library (`https://www.aqr.com/Insights/Datasets`). The VOL factor is self-constructed using a two-by-three sort on size and volatility, using all available CRSP non-micro cap stocks, *i.e.* stocks with a market capitalization larger than the 20-th percentile of NYSE stocks. The VOL factor is calculated as the difference between the average value-weighted return on the two low volatility portfolios, and the average value-weighted return on the two high volatility portfolios.

Panel A: All ETFs	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	VOL
Average sensitivity	-0.03%	1.00	0.13	-0.02	0.07	0.07	-0.03	-0.11	0.04	0.03
t-statistic	-0.52	27.33	4.56	-0.88	1.76	1.49	-1.36	-5.19	0.84	0.83
Percentile 5%	-0.53%	0.01	-0.55	-0.59	-0.56	-0.62	-0.35	-0.58	-1.30	-0.55
Percentile 95%	0.59%	1.94	0.90	0.75	0.65	0.73	0.25	0.18	1.28	0.39
% significantly positive	2%	85%	30%	21%	12%	17%	8%	5%	24%	5%
% significantly negative	6%	0%	19%	23%	8%	13%	7%	20%	15%	5%
Average R^2 (MKT)	0.64									
Average R^2 (Benchmark)	0.80									
# ETFs	368									
AUM (US$ Billions)	1542									
Panel B: Smart Beta ETFs	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	VOL
Average sensitivity	-0.06%	0.96	0.26	0.03	0.06	0.07	0.00	-0.06	0.09	0.06
t-statistic	-1.34	32.86	11.07	1.42	1.78	1.83	-0.22	-3.64	2.05	2.09
Percentile 5%	-0.36%	0.42	-0.27	-0.32	-0.26	-0.32	-0.22	-0.28	-0.69	-0.13
Percentile 95%	0.23%	1.55	1.01	0.49	0.35	0.50	0.27	0.13	0.72	0.25
% significantly positive	0%	95%	42%	30%	14%	28%	15%	4%	23%	10%
% significantly negative	7%	0%	21%	21%	7%	12%	11%	22%	10%	1%
Average R^2 (MKT)	0.74									
Average R^2 (Benchmark)	0.90									
# ETFs	168									
AUM (US$ Billions)	515									
Panel C: Conventional ETFs	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	VOL
Average sensitivity	0.00%	1.03	0.03	-0.07	0.08	0.07	-0.06	-0.15	0.01	0.00
t-statistic	-0.04	21.34	0.72	-1.94	1.55	1.14	-1.78	-5.38	0.12	0.09
Percentile 5%	-0.60%	-0.10	-0.60	-0.70	-0.70	-0.77	-0.44	-0.71	-1.64	-0.92
Percentile 95%	0.90%	2.13	0.70	1.01	0.79	0.96	0.22	0.22	1.43	0.49
% significantly positive	3%	77%	20%	14%	11%	9%	2%	6%	26%	2%
% significantly negative	6%	0%	18%	18%	6%	10%	10%	19%	8%	1%
Average R^2 (MKT)	0.56									
Average R^2 (Benchmark)	0.73									
# ETFs	200									
AUM (US$ Billions)	1027									

Table 3: Posterior factor and model probabilities, Jan/2013-Dec/2017

We apply a Bayesian variable selection methodology to groups of ETFs and a set of 14 factors. Our sample includes all U.S. equity ETFs which were active as of December 2017 and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. The set of factors include the five Fama & French (2015) factors as well as the momentum (MOM) factor from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The five Fama & French (2015) factors are the market (MKT), size or Small-Minus-Big (SMB), value or High-Minus-Low (HML), profitability or Robust-Minus-Weak (RMW), and Investment or Conservative-Minus-Aggressive (CMA). We then add the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014) and the alternative value factor HML "devil" (HMLd) of Asness & Frazzini (2013), which we download from the AQR data library (`https://www.aqr.com/Insights/Datasets`). The illiquidity (ILL), volatility (VOL), investment (INV), return on assets (ROA) and return on equity (ROE) factors are self-constructed using double sorts and all available CRSP non-micro cap stocks, *i.e.* stocks with a market capitalization larger than the 20-th percentile of NYSE stocks. Panel A reports marginal factor posterior probabilities, while Panel B reports the highest posterior probability models. The results are based on 50,000 iterations of the MCMC algorithm using $c = 1$, where c is a scale parameter for the prior covariance matrix of factor sensitivities.

Panel A: Marginal factor posterior probabilities

	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	HMLd	ILL	VOL	INV	ROE	ROA
Smart beta ETFs	0.00	1.00	1.00	0.03	0.00	0.00	0.00	0.00	0.30	0.01	0.00	0.00	0.00	0.00	0.00
Conventional ETFs	0.00	1.00	0.00	0.00	0.00	0.30	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00

Panel B: Posterior model probabilities

B.1: Smart Beta ETFs

Model	#Factors	Prob
MKT,SMB	2	0.67
MKT,SMB,BAB	3	0.29
MKT,SMB,HML	3	0.03
MKT,SMB,BAB,HMLd	4	0.01

B.2: Conventional ETFs

Model	#Factors	Prob
MKT,QMJ,HMLd	3	0.70
MKT,CMA,QMJ,HMLd	4	0.30

Table 4: Ordinary Least Square Analysis of High Posterior Probability Models, Jan/2013-Dec/2017

The table reports results from regressing the returns of ETFs on the best models selected using a Bayesian variable selection methodology. Our sample includes all U.S. equity ETFs which were active as of December 2017 and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. For the set of smart beta ETFs, the factors include the market excess return (MKT), the Fama & French (2015) size factor (SMB), and the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014). For conventional ETFs, the factors used are MKT, the Fama & French (2015) investment (CMA) factor, the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), and the HML "devil" (HMLd) factor of Asness & Frazzini (2013). The Fama & French (2015) factors factors are obtained from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The QMJ, BAB and HMLd factors are obtained from the AQR data library (`https://www.aqr.com/Insights/Datasets`).

Panel A: Smart Beta ETFs	Intercept	MKT	SMB	BAB	
Average sensitivity	-0.08%	0.95	0.29	0.07	
t-statistic	-1.48	42.28	14.33	2.11	
Percentile 5%	-0.43%	0.44	-0.28	-0.40	
Percentile 95%	0.21%	1.33	1.08	0.76	
% significantly positive	0%	98%	53%	30%	
% significantly negative	4%	0%	20%	9%	
Average excess return	1.20%				
Average return due to market factor	1.21%				
Average return not due to market factor	-0.01%				
Average absolute alpha (Bayesian model)	0.16%				
Average absolute alpha (Benchmark model)	0.14%				
Average R^2 (Bayesian model)	0.84				
Average R^2 (Benchmark model)	0.90				
Panel B: Conventional ETFs	Intercept	MKT	CMA	QMJ	HMLd
Average sensitivity	-0.03%	1.04	0.02	-0.13	0.04
t-statistic	-0.39	47.41	0.42	-8.55	1.41
Percentile 5%	-0.77%	0.46	-0.99	-0.58	-0.70
Percentile 95%	0.73%	2.06	0.74	0.22	1.47
% significantly positive	2%	96%	11%	16%	19%
% significantly negative	3%	0%	9%	36%	23%
Average excess return	1.15%				
Average return due to market factor	1.32%				
Average return not due to market factor	-0.17%				
Average absolute alpha (Bayesian model)	0.31%				
Average absolute alpha (Benchmark model)	0.39%				
Average R^2 (Bayesian model)	0.66				
Average R^2 (Benchmark model)	0.73				

Appendix A. Bayesian Variable Selection in the SUR Model

We start by reviewing the estimation of the SUR model (without variable selection) using the Gibbs sampler. Suppose $\tilde{\mathbf{e}} \sim N(\mathbf{0}, \mathbf{\Sigma} \otimes \mathbf{I_T})$ and the following prior distributions for $\tilde{\boldsymbol{\beta}}$ and $\mathbf{\Sigma}$:

$$\begin{aligned} \tilde{\boldsymbol{\beta}} &\sim N(\mathbf{b}_0, \mathbf{B}_0) \\ \mathbf{\Sigma} &\sim IW(\nu_0, \mathbf{\Phi}_0), \end{aligned} \quad (A.1)$$

where $IW(\nu_0, \mathbf{\Phi}_0)$ denotes the inverted-Wishart distribution with ν_0 degrees of freedom and parameter matrix $\mathbf{\Phi}_0$. With these choices, it can be shown that $\tilde{\boldsymbol{\beta}}|\mathbf{\Sigma}, \mathbf{r} \sim N(\mathbf{b}_1, \mathbf{B}_1)$ and $\mathbf{\Sigma}|\tilde{\boldsymbol{\beta}}, \mathbf{r} \sim IW(\nu_1, \mathbf{\Phi}_1)$, where

$$\begin{aligned} \mathbf{b}_1 &= (\mathbf{B}_0^{-1} + \tilde{\mathbf{X}}'\mathbf{\Omega}^{-1}\tilde{\mathbf{X}})^{-1}(\mathbf{B}_0\mathbf{b}_0 + \tilde{\mathbf{X}}'\mathbf{\Omega}^{-1}\tilde{\mathbf{r}}) \\ \mathbf{B}_1 &= (\mathbf{B}_0^{-1} + \tilde{\mathbf{X}}'\mathbf{\Omega}^{-1}\tilde{\mathbf{X}})^{-1} \\ \nu_1 &= \nu_0 + T, \quad \mathbf{\Phi}_1 = \mathbf{\Phi}_0 + \mathbf{S}. \end{aligned} \quad (A.2)$$

In the above, $\mathbf{S}$ is the matrix of cross-products of the residuals, that is, if $\mathbf{E} = [\mathbf{e}_1 \ldots \mathbf{e}_N]$, then $\mathbf{S} = \mathbf{E}'\mathbf{E}$. We also note that $\mathbf{\Omega}^{-1} = \mathbf{\Sigma}^{-1} \otimes \mathbf{I_T}$. The approach above may be computationally prohibitive if the number of assets (N) is large, since it requires multiplication and inversion of large matrices. For example, $\tilde{\mathbf{X}}$ has dimension $NT \times NK$ and $\mathbf{\Omega}^{-1}$ has dimension $NT \times NT$. A more efficient approach in this case is to sample each $\boldsymbol{\beta}_i$ conditionally on the remaining $\boldsymbol{\beta}_j, j \neq i$ and $\mathbf{\Sigma}$. Let $\tilde{\boldsymbol{\beta}}_{-i}$ denote the full vector $\tilde{\boldsymbol{\beta}}$ omitting $\boldsymbol{\beta}_i$ and assume that $\boldsymbol{\beta}_i|\tilde{\boldsymbol{\beta}}_{-i}, \mathbf{\Sigma} \sim N(\mathbf{b}_{0,i}, \mathbf{B}_{0,i})$. Then, $\boldsymbol{\beta}_i|\tilde{\boldsymbol{\beta}}_{-i}, \mathbf{\Sigma}, \mathbf{r} \sim N(\mathbf{b}_{1,i}, \mathbf{B}_{1,i})$, with

$$\begin{aligned} \mathbf{b}_{1,i} &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}'\mathbf{X})^{-1}(\mathbf{B}_{0,i}\mathbf{b}_{0,i} + \sigma^{ii}\mathbf{X}'\mathbf{r}_i^*) \\ \mathbf{B}_1 &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}'\mathbf{X})^{-1}, \end{aligned} \quad (A.3)$$

where σ^{ii} denotes the (i,i) element of $\mathbf{\Sigma}^{-1}$ and $\mathbf{r}_i^*$ is suitably defined based on a partition of the systems of equations (see Hwang & Rubesam (2018), Appendix A.2). Note that the expressions above depend only on the smaller matrices $\mathbf{X}$ and $\mathbf{\Sigma}$. In the Gibbs sampler, each $\boldsymbol{\beta}_i$ can be generated in random order.

The SUR model with variable selection can be written in the following way. Let $\mathbf{X}_{\boldsymbol{\gamma}}$ represent the matrix $\mathbf{X}$ where each column has been multiplied by the corresponding γ_j. Then we can write the model with variable selection as $\mathbf{r}_i = \mathbf{X}_{\boldsymbol{\gamma}}\boldsymbol{\beta}_i + \mathbf{e}_i, i = 1, \ldots, N$, or stacking the N equations as before,

$$\tilde{\mathbf{r}} = \tilde{\mathbf{X}}_{\boldsymbol{\gamma}}\tilde{\boldsymbol{\beta}} + \tilde{\mathbf{e}},$$

where $\tilde{\mathbf{X}}_{\boldsymbol{\gamma}}$ is defined analogously as before.

To derive the conditional distributions required for the Gibbs sampler, we need to specify the prior distribution for $\boldsymbol{\gamma}$. We set independent priors as $\gamma_j \sim B(1, \pi_j), j = 1, \ldots, K$, where B represents the Bernoulli distribution. Therefore, the prior distribution of $\boldsymbol{\gamma}$ is $f(\boldsymbol{\gamma}) = \prod_{j=1}^{K} \pi_j^{\gamma_j}(1 - \pi_j)^{1-\gamma_j}$.

Conditional on a known value of $\boldsymbol{\gamma}$, the model reduces to a SUR with the corresponding predictors for which $\gamma_j = 1$. Therefore, using the same prior distributions for $\tilde{\boldsymbol{\beta}}$ and $\boldsymbol{\Sigma}$ as before, the conditional distributions for $\tilde{\boldsymbol{\beta}}$ and $\boldsymbol{\Sigma}$ are those given in equation (A.2), with $\tilde{\mathbf{X}}$ replaced by $\tilde{\mathbf{X}}_{\boldsymbol{\gamma}}$. We can also use the sequential approach, sampling each $\boldsymbol{\beta}_i, i = 1, \ldots, N$ in turn from $\boldsymbol{\beta}_i|\tilde{\boldsymbol{\beta}}_{-i}, \boldsymbol{\gamma}, \boldsymbol{\Sigma}, \tilde{\mathbf{r}} \sim N(\mathbf{b}_{1,i}, \mathbf{B}_{1,i})$, where

$$\begin{aligned}\mathbf{b}_{1,i} &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}'_{\boldsymbol{\gamma}}\mathbf{X}_{\boldsymbol{\gamma}})^{-1}(\mathbf{B}_{0,i}\mathbf{b}_{0,i} + \sigma^{ii}\mathbf{X}'_{\boldsymbol{\gamma}}\mathbf{r}_i^*) \\ \mathbf{B}_1 &= (\mathbf{B}_{0,i}^{-1} + \sigma^{ii}\mathbf{X}'_{\boldsymbol{\gamma}}\mathbf{X}_{\boldsymbol{\gamma}})^{-1}.\end{aligned}$$

To generate $\boldsymbol{\gamma}$, we sample each γ_j conditional on the remaining $\gamma_k, k \neq j$. Let $\boldsymbol{\gamma}_{-j}$ denote the vector $\boldsymbol{\gamma}$, with the $j - th$ entry removed. The relevant conditional posterior probability of $\gamma_j = 1$ for the SUR model is given by

$$P(\gamma_j = 1|\boldsymbol{\gamma}_{-j}, \tilde{\boldsymbol{\beta}}, \boldsymbol{\Sigma}, \tilde{\mathbf{r}}) = \left(1 + \frac{1-\pi_j}{\pi_j}\exp(-0.5\,\mathrm{Tr}(\boldsymbol{\Sigma}^{-1}(\mathbf{S}^1_{\boldsymbol{\gamma}} - \mathbf{S}^0_{\boldsymbol{\gamma}}))\right)^{-1}, \quad \text{(A.4)}$$

where $\mathbf{S}^1_{\boldsymbol{\gamma}}$ and $\mathbf{S}^0_{\boldsymbol{\gamma}}$ represent the matrices of residuals when $\gamma_j = 1$ and $\gamma_j = 0$, respectively. Each γ_j can be generated, preferably in random order, using the expression above.

Appendix B. Results from Robustness Tests

[Table 5 about here.]

[Table 6 about here.]

Table B.5: Posterior factor and model probabilities, $c = 2$, Jan/2013-Dec/2017

We apply a Bayesian variable selection methodology to groups of ETFs and a set of 14 factors. Our sample includes all U.S. equity ETFs which were active as of December 2017 and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. The set of factors include the five Fama & French (2015) factors as well as the momentum (MOM) factor from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The five Fama & French (2015) factors are the market (MKT), size or Small-Minus-Big (SMB), value or High-Minus-Low (HML), profitability or Robust-Minus-Weak (RMW), and Investment or Conservative-Minus-Aggressive (CMA). We then add the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014) and the alternative value factor HML "devil" (HMLd) of Asness & Frazzini (2013), which we download from the AQR data library (`https://www.aqr.com/Insights/Datasets`). The illiquidity (ILL), volatility (VOL), investment (INV), return on assets (ROA) and return on equity (ROE) factors are self-constructed using double sorts and all available CRSP non-micro cap stocks, *i.e.* stocks with a market capitalization larger than the 20-th percentile of NYSE stocks. Panel A reports marginal factor posterior probabilities, while Panel B reports the highest posterior probability models. The results are based on 50,000 iterations of the MCMC algorithm using $c = 2$, where c is a scale parameter for the prior covariance matrix of factor sensitivities.

Panel A: Marginal factor posterior probabilities

	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	HMLd	ILL	VOL	INV	ROE	ROA
Smart beta ETFs	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.10	0.01	0.00	0.00	0.00	0.00	0.00
Conventional ETFs	0.00	1.00	0.00	0.00	0.00	0.02	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00

Panel B: Posterior model probabilities

B.1: Smart Beta ETFs

Model	#Factors	Prob
MKT,SMB	2	0.90
MKT,SMB,BAB	3	0.09
MKT,SMB,BAB,HMLd	4	0.01

B.2: Conventional ETFs

Model	#Factors	Prob
MKT,QMJ,HMLd	3	0.98
MKT,CMA,QMJ,HMLd	4	0.02

Table B.6: Posterior factor and model probabilities, $c = 1$, $\tilde{\boldsymbol{\beta}} \sim N(\mathbf{0}, c\mathbf{I})$, Jan/2013-Dec/2017

We apply a Bayesian variable selection methodology to groups of ETFs and a set of 14 factors. Our sample includes all U.S. equity ETFs which were active as of December 2017 and which had return data for the period from January 2013 to December 2017. We classify ETFs into the smart beta or conventional category according to their characteristics. The set of factors include the five Fama & French (2015) factors as well as the momentum (MOM) factor from Professor Kenneth French's data library (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The five Fama & French (2015) factors are the market (MKT), size or Small-Minus-Big (SMB), value or High-Minus-Low (HML), profitability or Robust-Minus-Weak (RMW), and Investment or Conservative-Minus-Aggressive (CMA). We then add the Quality-Minus-Junk (QMJ) factor of Asness *et al.* (2017), the Betting-Against-Beta (BAB) factor of Frazzini & Pedersen (2014) and the alternative value factor HML "devil" (HMLd) of Asness & Frazzini (2013), which we download from the AQR data library (`https://www.aqr.com/Insights/Datasets`). The illiquidity (ILL), volatility (VOL), investment (INV), return on assets (ROA) and return on equity (ROE) factors are self-constructed using double sorts and all available CRSP non-micro cap stocks, *i.e.* stocks with a market capitalization larger than the 20-th percentile of NYSE stocks. Panel A reports marginal factor posterior probabilities, while Panel B reports the highest posterior probability models. The results are based on 50,000 iterations of the MCMC algorithm using $c = 1$, where c is a scale parameter for the prior covariance matrix of factor sensitivities.

Panel A: Marginal factor posterior probabilities

	Intercept	MKT	SMB	HML	RMW	CMA	MOM	QMJ	BAB	HMLd	ILL	VOL	INV	ROE	ROA
Smart beta ETFs	0	1	1	0.02766	0	0	0.00874	0.0121	0.1	0.02782	0	0	0	0	0
Conventional ETFs	0	1	0	0	0	0.01028	0	0.2	0	1	0	0	0	0	0

Panel B: Posterior model probabilities

B.1: Smart Beta ETFs

Model	#Factors	Prob
MKT,SMB	2	0.9
MKT,SMB,BAB	3	0.07218
MKT,SMB,HML,BAB,HMLd	5	0.01556
MKT,SMB,HML,MOM,QMJ,BAB,HMLd	7	0.00874
MKT,SMB,HML,QMJ,BAB,HMLd	6	0.00336
MKT,SMB,BAB,HMLd	4	0.00016

B.2: Conventional ETFs

Model	#Factors	Prob
MKT,HMLd	2	0.8
MKT,QMJ,HMLd	3	0.18972
MKT,CMA,QMJ,HMLd	4	0.01028