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The output-oriented plant capacity notion has been around since more than two decades. It has mainly been applied empirically in the fishery and the hospital sectors. A problem known since its introduction into the literature is that it may not be attainable, in that it presupposes potentially unlimited amounts of variable inputs to determine the maximum of outputs available. This issue of the lack of attainability has never been explored. This paper fills this void both theoretically and empirically. It finds that the attainability may be problematic, and that bounds on the amounts of variable inputs may well need to be imposed.

Key words: technology; plant capacity; attainability

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1. Introduction

In the economic literature a variety of capacity notions have been developed (see e.g. Johansen (1968) or Nelson (1989)). One useful taxonomy distinguishes between technical or engineering notions on the one hand and economic capacity concepts on the other hand, whereby the latter are mainly based or derived from some cost function. This paper focuses on the plant capacity notion that is part of the family of technical or engineering notions.

Johansen (1968, p. 362) defined the notion of plant capacity informally as "... the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." Färe et al. (1989a) and Färe et al. (1989b) translated this plant capacity notion into a single, respectively, multiple output nonparametric frontier framework in which plant capacity as well as a measure of capacity utilization can be determined from information on observed inputs and outputs using a pair of output-oriented efficiency measures.

For over two decades, empirical applications have occurred using this output-oriented plant capacity in mainly fisheries (e.g., Felthoven (2002), Pascoe et al. (2013), Tingley and Pascoe (2005) or Walden and Tomberlin (2010)) and hospital industries (e.g., Karagiannis (2015), Kerr et al. (1999), Valdmanis et al. (2010) or Valdmanis et al. (2015)). One study focuses on banking (e.g., Sahoo and Tone (2009)), and we are aware of one article describing a macro-economic application on trade barriers (e.g., Badau (2015)). But, no major methodological innovation has occurred related to this plant capacity concept. However, recently Cesaroni et al. (2017a) use the same nonparametric frontier framework to define a new input-oriented measure of plant capacity utilization based on a couple of input-oriented efficiency measures.

Already Johansen (1968, p. 362) pointed out that the plant capacity concept need not necessarily be attainable, in that the amounts of variable inputs needed to determine the maximum potential outputs may well be unavailable at either the firm level or the sector level. To the best of our knowledge, the literature has completely ignored this issue of attainability. This paper sets as a major goal to explore this attainability problem. At the theoretical level, we will argue that there is indeed such an issue for the output-oriented plant capacity notion, but we will also show that the new input-oriented plant capacity concept does not suffer from this problem. At the empirical level, we illustrate the extent to which the amounts of variable inputs needed to determine the plant capacity output are plausible or not using a secondary data set.

It is becoming known that the axiom of convexity has a potentially large impact on the empirical analysis based on technologies (for example, Tone and Sahoo (2003)). For instance, in the context of plant capacity utilization, Walden and Tomberlin (2010) empirically illustrate the effect of convexity on the output-oriented plant capacity notion. In a similar way, Cesaroni et al. (2017a) reveal the influence of convexity on the input-oriented plant capacity concept. Therefore, we also analyze the issue of attainability in terms of the potential effect of the convexity axiom.

The structure of this contribution is as follows. Section 2 provides the basic definitions of technology and efficiency measures representing these technologies. The next Section 3 starts out by defining both the traditional output-oriented and the new input-oriented plant capacity notions. Thereafter, we argue and illustrate that the output-oriented plant capacity notion may well fail attainability, while there is no such an issue for the input-oriented plant capacity concept. We end this section by defining an attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. Section 4 describes the secondary data set selected for the empirical illustration and summarizes the empirical results in great detail. A final Section 5 ends with some concluding remarks.

2. Technology: Basic Definitions

This section introduces some basic notation and defines the technology. Given an N-dimensional input vector $x \in \mathbb{R}_+^N$ and an M-dimensional output vector $y \in \mathbb{R}_+^M$, the production possibility set or technology T is defined as follows: $T = \{(x,y) | x \text{ can produce } y\}$. Associated with T, the input set denotes all input vectors x capable of producing a given output vector y: $L(y) = \{x | (x,y) \in T\}$. Analogously, the output set associated with T denotes all output vectors y that can be produced from a given input vector x: $P(x) = \{y | (x,y) \in T\}$.

Throughout this contribution, technology T satisfies some combination of the following standard assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0,0) \in T$ and if $(0,y) \in T$, then $y = 0$.

(T.2) T is a closed subset of $\mathbb{R}_+^N \times \mathbb{R}_+^M$.

(T.3) Strong input and output disposal, i.e., if $(x,y) \in T$ and $(x',y') \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, then $(x',-y') \geq (x,-y) \Rightarrow (x',y') \in T$.

(T.4) T is convex.

Briefly discussing these traditional axioms on technology, it is useful to recall the following (see, e.g., Hackman (2008) for details). Inaction is feasible, and there is no free lunch. Technology is closed. We assume free disposal of inputs and outputs in that inputs can be wasted and outputs can be discarded. Finally, technology is convex. In our empirical analysis not all these axioms are simultaneously maintained.1. In particular, key assumption distinguishing some of the technologies in the empirical analysis is convexity versus nonconvexity.

The radial input efficiency measure characterizes the input set $L(y)$ completely and can be defined as follows:

$$DF_i(x,y) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \tag{1}$$

This radial input efficiency measure has the main property that it is smaller than or equal to unity ($DF_i(x,y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity, and that it has a cost interpretation (see, e.g., Hackman (2008)).

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and can be defined as:

$$DF_o(x,y) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}. \tag{2}$$

Its main properties are that it is larger than or equal to unity ($DF_o(x,y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that this radial output efficiency measure has a revenue interpretation (e.g., Hackman (2008)).

In the short run, we can partition the input vector into a fixed and variable part. In particular, we denote ($x = (x^f, x^v)$) with $x^f \in \mathbb{R}_+^{N_f}$ and $x^v \in \mathbb{R}_+^{N_v}$ such that $N = N_f + N_v$.

Similarly, a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}_+^{N_f} \times \mathbb{R}_+^M \mid x^f \text{ can produce } y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}_+^{N_f} \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$ can be defined. Note that technology T^f is in fact obtained by a projection of technology $T \in \mathbb{R}_+^{N+M}$ into the subspace $\mathbb{R}_+^{N_f+M}$ (i.e., by setting all variable inputs equal to zero). By analogy, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the output set $P^f(x^f)$ by $DF_o^f(x^f, y)$, this output-oriented efficiency measure can be defined as follows:

$$DF_o^f(x^f, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \tag{3}$$

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$DF_i^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \tag{4}$$

Next, we need the following particular definition of technology: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level. The sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is as follows:

$$DF_i^{SR}(x^f, x^v, 0) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \tag{5}$$

Given data on K observations $(k = 1, \ldots, K)$ consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under the flexible or variable returns to scale assumption is possible as follows:

$$T^\Lambda = \left\{ (x, y) \mid x \geq \sum_{k=1}^{K} z_k x_k, y \leq \sum_{k=1}^{K} z_k y_k, z \in \Lambda \right\}, \tag{6}$$

where

$$\text{(i) } \Lambda \equiv \Lambda^{\mathrm{C}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}; \quad \text{(ii) } \Lambda \equiv \Lambda^{\mathrm{NC}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0, 1\} \right\}.$$

The activity vector z of real numbers summing to unity represents the convexity axiom. This same sum constraint with each vector element being a binary integer is representing nonconvexity. The convex technology satisfies axioms (T.1) (except inaction) to (T.4), while the nonconvex technology adheres to axioms (T.1) to (T.3). It is now useful to condition the above notation of the efficiency measures relative to these nonparametric frontier technologies by distinguishing between convexity (convention C) and nonconvexity (convention NC).

Commonly, it is assumed that the input and output data satisfy a series of conditions (Färe et al. (1994, p. 44-45)): (i) each producer employs nonnegative amounts of each input to produce nonnegative amounts of each output; (ii) there is an aggregate production of positive amounts of every output as well as an aggregate utilization of positive amounts of every input; and (iii) each producer employs a positive amount of at least one input to produce a positive amount of at least one output.

3. Plant Capacity Concepts

3.1. Plant Capacity: Basic Definitions

Recall the informal definition of plant capacity by Johansen (1968, p. 362) as "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." This clearly output-oriented plant capacity notion has been admirably made operational by Färe et al. (1989a) and Färe et al. (1989b) using a pair of output-oriented efficiency measures. We now recall the definition of this output-oriented plant capacity utilization (PCU).

Definition 1. The output-oriented plant capacity utilization (PCU_o) is defined as:

$$PCU_o(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)},$$

where $DF_o(x, y)$ and $DF_o^f(x^f, y)$ are output efficiency measures including, respectively excluding, the variable inputs as defined before in (2) and (3). Since $1 \leq DF_o(x, y) \leq DF_o^f(x^f, y)$, notice that $0 < PCU_o(x, x^f, y) \leq 1$. Thus, output-oriented plant capacity utilization has an upper limit of unity. Following the terminology introduced by Färe et al. (1989a), Färe et al. (1989b) and Färe et al. (1994), one can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f, y)$ and an unbiased plant capacity measure $PCU_o(x, x^f, y)$. Taking the ratio of efficiency measures eliminates any existing inefficiency and yields in this sense a cleaned concept of output-oriented plant capacity.

To guarantee the existence of the efficiency measures, Färe et al. (1989a, p. 659-660) sharpen the conditions on the input and output data for nonparametric frontier technologies.2. In particular, each fixed input is used by some producer and each producer uses some fixed input.

In case of C, the efficiency measure $DF_o^f(x^f, y)$ is computed for observation (x_p, y_p) as follows:

$$
\begin{aligned}
DF_o^f(x_p^f, y_p) = \max_{\theta, z_k} \ & \theta \\
s.t \quad & \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\
& \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned} \tag{7}
$$

In case of NC, the variables z_k in this model need to be binary variables. In all LP models mentioned hereafter, a similar adaptation is required if NC is assumed. To save space, we will not mention this again, nor formulate the corresponding models.

Observe that there are no input constraints on the variable inputs in the model (7). Note that Färe et al. (1994) introduce an alternative linear program (LP) with a scalar for each variable input dimension. Also note that LP (7) is equivalent to the following LP obtained by making each variable input a decision variable:

$$
\begin{aligned}
DF_o^f(x_p^f, y_p) = \max_{\theta, z_k, x^v} \ & \theta \\
s.t \quad & \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\
& \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \sum_{k=1}^{K} z_k x_k^v \leq x^v, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, x^v \geq 0, \ k = 1, \ldots, K.
\end{aligned} \tag{8}
$$

Cesaroni et al. (2017a) define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures.

Definition 2. The input-oriented plant capacity utilization (PCU_i) is defined as:

$$
PCU_i(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{DF_i^{SR}(x^f, x^v, 0)},
$$

where $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$ are both sub-vector input efficiency measures reducing only the variable inputs relative to the technology, the latter efficiency measure being evaluated at a zero output level. Since $0 < DF_i^{SR}(x^f, x^v, 0) \leq DF_i^{SR}(x^f, x^v, y)$, notice that $PCU_i(x, x^f, y) \geq 1$. Thus, input-oriented plant capacity utilization has a lower limit of unity. Similar to the previous case, one can distinguish between a so-called biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i^{SR}(x, x^f, y)$, the latter being cleaned of any prevailing inefficiency.3.

To guarantee the existence of the efficiency measures, we also need to sharpen the conditions on the input and output data: each variable input is used by some producer and each producer uses some variable input.

Both these Definitions 1 and 2 are graphically illustrated with the help of a Figure in the e-companion in Subsection EC.1. We now turn to the issue of attainability of both these plant capacity concepts.

3.2. Plant Capacity: The Question About Attainability

While these definitions in itself are sufficiently clear, it may be useful to underscore that these concepts differ with respect to the property of attainability. As stressed by Johansen (1968, p. 362) the output-oriented plant capacity notion is not attainable in that the extra variable inputs necessary to reach the maximal plant capacity output may not be available. While the axiom of strong disposability in the inputs in principle allows for wasting infinitely many inputs to determine the maximal plant capacity outputs, in practice there may well be restrictions of various kinds that limit the availability of variable inputs.4.

First, at the firm level there may be quasi-fixed factors like labor where firms have to invest in hiring and training activities that limit the amounts of labor that can be recruited at once. By definition, quasi-fixed factors are characterized by the fact that their supply cannot be expanded rapidly. Furthermore, depending on the nature of the labour market and the size of the firm (e.g., it may have some monopsony power), recruiting a large amount of people may well have an impact on their salaries. While this does not show up in the analytical framework of the output-oriented plant capacity notion that ignores input prices, firms may well in fact take account of these general equilibrium effects and constrain their recruitment of the quasi-fixed factor. In brief, the quasi-fixity of labor as well as other production factors may seriously impede the expansion of variable inputs and may thus prevent reaching the maximal plant capacity outputs (e.g., Oi (1962) for the seminal article in economics and Barney (2001) for the resource-based view of the firm).

Second, even if these extra variable inputs are available at the firm level, as stressed by Johansen (1968) there may be restrictions on the available extra variable inputs at the sector level that may prevent that all firms simultaneously can reach their maximal plant capacity output. For instance, quasi-fixed factors may operate at the industry level and prevent the rapid expansion of their supply in amounts needed to allow for the realization of the maximal plant capacity outputs for all firms. At the sectoral level, it is obvious that general equilibrium effects may play a role: if all firms simultaneously increase their demand for a production factor, then the price of that production factor may well increase. Again, while this does not show up in the framework of the output-oriented plant capacity notion which ignores factor prices, firms may take these general equilibrium effects into account and constrain their expansion of the production factor.

By contrast, the input-oriented plant capacity notion is always attainable in that one can always reduce the amount of existing variable inputs such that one reaches an input set with zero output level. Reducing variable inputs to reach zero production levels is normally possible because of the axiom of inaction. Inaction implies that one can stop producing at all: but, in modern production facilities producing a zero output need not imply that no inputs are used.5. Examples of zero production with positive amounts of variable inputs include critical maintenance activities at a large industrial plant impeding production, making inventories in a retailer while temporarily suspending sales, or temporarily closing a mine while keeping it exploitable with the option of reopening it as part of a real options strategy. Closing down production is therefore possible at the firm level, but it can be done as well at the sectoral level.

Therefore, attainability is a potential issue for the output-oriented plant capacity notion, while it is a priori not an issue for the new input-oriented plant capacity concept. We now turn to the modeling of constraints on the availability of variable inputs in the output-oriented plant capacity notion.

A somewhat related issue is the economic relevance of these plant capacity notions. Starting again with the output-oriented plant capacity concept, even if the firm would have sufficiently variable inputs at its disposal and the attainability issue would not exist, it is clear that it rarely will be cost minimizing or profit maximizing to produce the output-oriented plant capacity outputs. This technical or engineering capacity concept just serves as a generalization of other popular capacity concepts (e.g., in the hotel industry room occupancy rates are very popular) for multiple output production processes. For the case of the input-oriented plant capacity concept, for which the attainability issue does not exist, the question as to the pertinence of the optimal variable inputs at the level of the initialization of production is also relevant. As earlier stated, maintenance activities may lead to temporarily suspend production, as may be temporary mothballing operations. However, for most firms also these optimal variable inputs at zero output levels may not follow from a cost minimizing or profit maximizing strategy. Again, this technical or engineering capacity concept just serves as framework to summarize capacity measurement for multiple input and multiple output production processes.

Cesaroni et al. (2017b) also recently defined new long-run output- and input-oriented plant capacity concepts that allow for changes in all input dimensions simultaneously rather than changes in the variable inputs only. The above plant capacity concepts focusing on changes in the variable inputs alone can then be interpreted as short-run concepts. Obviously, the whole issue of attainability also transposes to the output- and input-oriented long-run plant capacity concepts.

3.3. Attainable Output-Oriented Plant Capacity: Proposals

We now first turn to the specification of attainability constraints at the firm level. Thereafter, we explore how to model attainability constraints at the industry level.

3.3.1. Attainability Constraints at the Firm Level. The standard assumption (T.3) of strong input and output disposability implies that variable inputs can be increased without limitation in the absence of price information. However, from the nature of reality with limited resources, we know that this possibility of allowing unlimited increase of inputs creates a potential issue. This issue also affects all notions built upon this possibility, especially the output-oriented plant capacity notion. As a possible remedy, we first define an attainability level $\bar{\lambda}$ of an observation (firm) as follows:

Definition 3. An attainability level $\bar{\lambda}$ of observation (x_p, y_p) (abbreviated to level $\bar{\lambda}$) is any value $\bar{\lambda} \in \mathbb{R}_+$ satisfying

$$\exists \lambda \in \mathbb{R}_+ \text{ with } \lambda \leq \bar{\lambda} \text{ and } \exists \theta \in \mathbb{R}_+ \text{ such that } (x^f, \lambda x^v, \theta y) \in T.$$

It follows from this definition that every value $\bar{\lambda} \geq 1$ can act as attainability level for all observations (e.g., set $\lambda = 1$ and $\theta = 1$). An attainability level $\bar{\lambda} < 1$ might not be possible for some observations (as can be observed in the empirical illustration in Section 4). However, this level should be chosen to reflect a realistic achievable up-scaling of variable inputs for a particular observation. To give an example, a value $\bar{\lambda} = 3$ means that one considers tripling variable inputs as being realistic (or achievable).

Note that Definition 3 differs from the rather well-known axiom of attainability as developed by Shephard in his work (see, e.g., Färe and Mitchell (1987) for a critical discussion).

With an attainability level set to some realistic value, the following attainable output-oriented efficiency measure can be defined:

Definition 4. The attainable output-oriented efficiency measure (ADF_o) at level $\bar{\lambda} \in \mathbb{R}_+$ is defined as:

$$ADF_o^f(x^f, y, \bar{\lambda}) = \max\{\theta \mid \theta \geq 0, 0 \leq \lambda \leq \bar{\lambda}, \theta y \in P(x^f, \lambda x^v)\}.$$

The amount of variable inputs is now bounded to be at most a scalar-wise multiple smaller than $\bar{\lambda}$. Obviously, $ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$. Note that Definition 4 is written in absolute terms.

For instance, $\bar{\lambda} = 3$ corresponds with the impossibility of variable inputs to exceed three times the current amount of variable inputs. Alternatively, one could focus on relative comparisons to the sector aggregates ($\sum_{p=1}^{K} x_p^v$). Then, one could impose that variable inputs at the firm level cannot exceed a certain share of the total amount of variable inputs available in the sector. We opt for the first approach.

Using the attainable output-oriented efficiency measure introduced in Definition 4, it is natural to come up with a new attainable output-oriented plant capacity concept at the firm level.

Definition 5. An attainable output-oriented plant capacity utilization ($APCU_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ is defined as:

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x,y)}{ADF_o^f(x^f, y, \bar{\lambda})},$$

with $DF_o(x,y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$ as defined before.

By analogy with the plant capacity utilization measures introduced in Definitions 1 and 2, one can distinguish between the biased attainable plant capacity measure $ADF_o^f(x^f, y, \bar{\lambda})$ and the unbiased attainable plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures eliminating any existing inefficiency.

Since $ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$, clearly $APCU_o(x, x^f, y, \bar{\lambda}) \geq PCU_o(x, x^f, y)$. Thus, the attainable output-oriented measure of plant capacity utilization is always larger or equal to the traditional measure of output-oriented plant capacity utilization.

Proposition 1. *The attainable output-oriented plant capacity utilization $APCU_o(x, x^f, y, \bar{\lambda})$ converges to the output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ as $\bar{\lambda} \longrightarrow \infty$, i.e.,* $\lim_{\bar{\lambda} \to \infty} APCU_o(x, x^f, y, \bar{\lambda}) = PCU_o(x, x^f, y)$.

Note first that the proofs of all propositions are in the e-companion in Subsection EC.2.

Note furthermore that the output-oriented plant capacity utilization $PCU_o(x, x^f, y, \bar{\lambda})$ might be unrealistic since the amounts of variable inputs needed to reach the maximum capacity outputs may simply not be available. This can be observed in the empirical illustration in Section 4.

Hence, $APCU_o(x, x^f, y, \bar{\lambda})$ should be a more realistic alternative plant capacity utilization measure provided an achievable level $\bar{\lambda}$ is chosen.

Modeling attainability constraints at the firm level can now be done as follows:

$$
\begin{aligned}
ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \max_{\theta, z_k, x^v} \quad & \theta \\
s.t \quad & \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\
& \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \sum_{k=1}^{K} z_k x_k^v \leq x^v, \\
& \sum_{k=1}^{K} z_k = 1, \\
& x^v \leq \bar{\lambda} x_p^v, \\
& \theta \geq 0, z_k \geq 0, x^v \geq 0, \ k = 1, \ldots, K.
\end{aligned}
\tag{9}
$$

The constraint $x^v \leq \bar{\lambda} x_p^v$ establishes a link between the decision variable x^v and the value x_p^v of the firm under observation. In the empirical analysis of Section 4, we choose $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. Thus, we consider an increase of the variable inputs with a factor more than five or less than 0.5 (i.e., halving these variable inputs) as implausible.

In model (9), the scalar $\bar{\lambda}$ can be varied over some part of the interval $(0, \infty)$. To determine the complete feasible interval for $\bar{\lambda}$ and to classify $ADF_o^f(x^f, y, \lambda)$ and $APCU_o(x, x^f, y, \lambda)$ further on, we need the following definition of critical points.

Definition 6. For a given observation (x_p, y_p), we can define the following three critical points L_p, M_p and U_p as follows:

$$L_p = DF_i^{SR}(x_p^f, x_p^v, 0), \tag{10}$$

$$M_p = DF_i^{SR}(x_p^f, x_p^v, y_p), \tag{11}$$

and

$$U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x_p^f, y_p) y_p). \tag{12}$$

Note that the critical points L_p and M_p make up the components of the input-oriented plant capacity measure $PCU_i(x, x^f, y)$ in Definition 2. To our knowledge, U_p has not been described earlier in the literature. It can be interpreted as the minimal expansion of variable inputs needed to produce the maximum plant capacity outputs and can be computed as follows:

$$\begin{aligned} U_p = \min_{\theta, z_k} \ & \theta \\ s.t \ & \sum_{k=1}^{K} z_k y_k \geq DF_o^f(x_p^f, y_p) y_p, \\ & \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \sum_{k=1}^{K} z_k x_k^v \leq \theta x_p^v, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \theta \geq 0, z_k \geq 0, k = 1, \ldots, K. \end{aligned} \tag{13}$$

These three critical points can be briefly illustrated with the help of Figure EC.1. First, the point L_p relates to the distance from point a to point e'': it indicates the minimal amount of variable inputs compatible with zero outputs. Second, the point M_p relates to the distance from point e'''' to point e: it indicates the minimal amount of variable inputs compatible with current levels of outputs. Third, the point U_p relates to the distance from point e to point e': it indicates the minimal amount with which variable inputs need to be expanded to be compatible with the maximal level of plant capacity outputs at point d.

We are now in a position to classify $ADF_o^f(x^f, y, \bar{\lambda})$ and $APCU_o(x, x^f, y, \bar{\lambda})$ in terms of these three critical points. In particular, we establish two propositions.

Proposition 2. *For the biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, for every observation (x_p, y_p) we have:*

(i) If $\bar{\lambda} < L_p$, then model (9) is infeasible.

(ii) If $L_p \leq \bar{\lambda} < M_p$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < 1$ and $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > 1$.

(iii) If $M_p \leq \bar{\lambda}$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ and $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) \leq 1$.

Proposition 3. *For the biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, for every observation (x_p, y_p), we have:*

(i) If $L_p \leq \bar{\lambda} < U_p$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ and $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$.

(ii) If $\bar{\lambda} \geq U_p$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$*.*

3.3.2. Attainability Constraints at the Industry Level. Similar to the firm level version, it is natural to come up with new industry attainable output-oriented plant capacity concepts. First, we introduce the industry attainable output-oriented efficiency measure as follows:

DEFINITION 7. The industry attainable output-oriented efficiency measure ($IADF_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation (x_p, y_p) is defined as

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) = \theta_p^*,$$

with θ_p^* the optimum value of θ_p in the following model:

$$\begin{aligned}
\max_{\theta_p, z_k^p, x_p^v} \quad & \sum_{p=1}^{K} \theta_p \\
s.t \quad & \sum_{k=1}^{K} z_k^p y_k \geq \theta_p y_p, \qquad && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} x_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v, \\
& \theta_p \geq 0, z_k^p \geq 0, x_p^v \geq 0, \ k, p = 1, \ldots, K.
\end{aligned} \tag{14}$$

Note that model (14) is a kind of central resource allocation model with K LPs (one for each observation) and a bogus objective function and with a common constraint on the total amount of variable inputs available in the sector. In particular, its aim is to simultaneously determine the maximum plant capacity outputs for all observations while reallocating variable inputs among units such that a global constraint on the industry amount of variable inputs is respected. Central resource reallocation models cover a heterogeneous variety of models reallocating some inputs and/or outputs across space and/or time while eventually accounting for multiple objectives (e.g., efficiency, effectiveness, equality, etc.) simultaneously. Examples include Athanassopoulos (1998), Färe et al. (1992), Golany and Tamir (1995), Korhonen and Syrjänen (2004), Lozano and Villa

(2004), and Ylvinger (2000), among others. One type of central resource reallocation model which also makes use of the notion of plant capacity is the so-called short-run Johansen industry model (see, e.g., Färe et al. (1992) for a single output version and Kerstens et al. (2006) for a multiple outputs version).

Second, using the industry attainable output-oriented efficiency measure of Definition 7, the industry attainable output-oriented plant capacity utilization is defined as follows:

DEFINITION 8. The industry attainable output-oriented plant capacity utilization ($IAPCU_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation (x_p, y_p) is defined as

$$IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = \frac{DF_o(x_p, y_p)}{IADF_o^f(x_p^f, y_p, \bar{\lambda})}.$$

Since $IADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$, clearly $IAPCU_o(x, x^f, y, \bar{\lambda}) \geq PCU_o(x, x^f, y)$. Thus, the industry attainable output-oriented measure of plant capacity utilization is always larger or equal to the traditional measure of output-oriented plant capacity utilization. By analogy, one can distinguish between the biased industry attainable plant capacity measure $IADF_o^f(x^f, y, \bar{\lambda})$ and the unbiased industry attainable plant capacity measure $IAPCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures eliminating any existing inefficiency.

Note that the industry attainable output-oriented measure of plant capacity utilization may be smaller or larger than the attainable output-oriented measure of plant capacity utilization. This holds true for both the biased and unbiased versions. Therefore, we have $IADF_o^f(x^f, y, \bar{\lambda}) \gtreqless ADF_o^f(x_p^f, y_p, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda}) \gtreqless APCU_o(x, x^f, y, \bar{\lambda})$.

By analogy to the firm level modeling, the scalar $\bar{\lambda}$ in model (14) can be varied over some part of the interval $(0, \infty)$. To determine this feasible interval for $\bar{\lambda}$ we can define the following two critical points L^I and U^I.

Definition 9. L^I can be determined from the following LP:

$$\begin{aligned} L^I = \min_{\theta, z_k^p, x_p^v} \ & \theta \\ s.t \ & \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\ & \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\ & \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\ & \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\ & \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, \ k, p = 1, \ldots, K. \end{aligned} \tag{15}$$

U^I is obtained solving the following LP:

$$\begin{aligned} U^I = \min_{\theta, z_k^p, x_p^v} \ & \theta \\ s.t \ & \sum_{k=1}^{K} z_k^p y_k \geq DF_o^f(x_p^f, y_p) y_p, && p = 1, \ldots, K, \\ & \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\ & \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\ & \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\ & \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\ & \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, && k, p = 1, \ldots, K. \end{aligned} \tag{16}$$

Note that U^I can be interpreted as the minimal expansion of overall variable inputs needed to produce the plant capacity outputs for all units for the industry model (14).

We are now in a position to classify $IADF_o^f(x^f, y, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda})$ in terms of these two critical points in the following proposition:

Proposition 4. *For the industry biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, we have:*

(i) If $\bar{\lambda} < L^I$, then model (14) *is infeasible.*

(ii) If $L^I \leq \bar{\lambda} < U^I$, then at least for one observed observation (x_p, y_p) we have

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p) \text{ and } IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p).$$

(iii) If $U^I \leq \bar{\lambda}$*, then for every observation* (x_p, y_p) *we have* $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ *and* $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$*.*

4. Empirical Illustration

4.1. Description of the Sample

For the empirical illustration of the attainability notions introduced in previous section, we use a secondary data set from Atkinson and Dorfman (2009). The sample is based on 16 Chilean hydro-electric power generation plants observed on a monthly basis. We limit ourselves to the observations for the year 1997 and, assuming that there is no technical change, we specify an inter-temporal frontier across all twelve months resulting in a total of 192 units. It is well-known that Chile was one of the first countries deregulating its electricity market and that hydro-power was a dominant source of energy during the 90's. These hydro-power plants generate one output (electricity) using three inputs: labor, capital, and water. Except for the fixed input capital, the remaining flow variables are expressed in physical units. Table 1 presents basic descriptive statistics for the inputs and the single output. One can observe a large heterogeneity in terms of size among the different inputs as well as the single output.

Table 1 Descriptive Statistics for Hydro-Power Plants (1997)

Variable	Trimmed mean[a]	Minimum	Maximum
Billions of m^3 of water (variable input)	126.80	0.49	1347.47
# workers (variable input)	15.62	2.00	52.86
Billions of capital (fixed input)	0.47	0.04	5.98
Thousands of kWh (output)	46.79	0.40	353.70

Note: [a]10% trimming level.

4.2. Empirical Results for Firm Level

Tables 2 and 3 are structured in a similar way. While Table 2 reports on the biased plant capacity utilization measures $DF_o^f(x^f, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$, Table 3 focuses on the unbiased plant capacity utilization measures $PCU_o(x, x^f, y)$ and $APCU_o(x, x^f, y, \bar{\lambda})$. In each table, the second column reports the standard plant capacity utilization measures, while the next ten columns describe the attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5 (thus, $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$). Hence, we somewhat arbitrary assume that variable inputs can be magnified at most fivefold. Obviously, we could have selected a wider range of values to experiment with $\bar{\lambda}$. Based on Proposition 2, note that for 37 observations under C and 41 observations under NC $\bar{\lambda} = 0.5$ is too small for model (9) to be feasible. Hence, these observations are not included in the corresponding descriptive statistics computations.

Table 2 Descriptive Statistics of Biased Plant Capacity Utilization

		$ADF_o^f(x^f, y, \bar{\lambda})$									
Convex	$DF_o^f(x^f, y)$	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Average	13.655	1.017	1.663	2.191	2.594	2.912	3.153	3.358	3.547	3.721	3.877
Stand. Dev.	77.137	1.027	1.721	2.421	3.107	3.770	4.349	4.927	5.502	6.077	6.645
Minimum	1.000	0.252	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	7.732	15.465	21.695	27.937	34.322	38.807	43.290	47.775	52.259	56.743
		$ADF_o^f(x^f, y, \bar{\lambda})$									
Nonconvex	$DF_o^f(x^f, y)$	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Average	12.541	0.600	1.275	1.508	1.746	1.942	2.166	2.367	2.547	2.701	2.762
Stand. Dev.	77.226	0.907	1.511	1.818	2.157	2.830	2.856	3.501	4.303	4.725	5.079
Minimum	1.000	0.118	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	7.714	13.500	19.000	21.000	33.250	33.250	33.250	40.286	43.714	45.500

The fact of our data set containing a single output implies that $DF_o^f(x_p^f, y_p)$ is homogeneous of degree -1 in the output. From (12), it then follows that $U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x_p^f, 1))$. Obviously, U_p now only depends on the variable and fixed inputs. However, in the multi-output case, this observation no longer holds true. For an example with multiple outputs, we refer to Subsection EC.3 in the e-companion.

Analyzing the results in Table 2, one can draw the following conclusions. First, on average the biased plant capacity utilization measure $DF_o^f(x^f, y)$ indicates that outputs can be magnified by at least 13.65 times under C and 12.54 times under NC. Second, there is a lot of variation in $DF_o^f(x^f, y)$ as indicated by the standard deviation and the range is even huge: the maximum increase in outputs amounts to 884.25 times under both C and NC. Third, the biased attainable plant capacity utilization measure $ADF_o^f(x^f, y, \bar{\lambda})$ increases monotonically in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda}=5$), we obtain on average a 3.87 output magnification under C and a 2.76 output magnification under NC. This is ways below the average output magnification computed by the biased plant capacity utilization measure $DF_o^f(x^f,y)$.

Table 3 Descriptive Statistics of Unbiased Plant Capacity Utilization

		$APCU_o(x,x^f,y,\lambda)$									
Convex	$PCU_o(x,x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	0.522	1.952	1.000	0.778	0.687	0.637	0.611	0.594	0.581	0.572	0.564
Stand. Dev.	0.269	0.705	0.000	0.113	0.156	0.179	0.193	0.203	0.211	0.218	0.223
Minimum	0.016	1.000	1.000	0.495	0.331	0.272	0.231	0.200	0.177	0.159	0.155
Maximum	1.000	4.916	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$APCU_o(x,x^f,y,\bar{\lambda})$									
Nonconvex	$PCU_o(x,x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	0.553	2.964	1.000	0.868	0.782	0.733	0.677	0.658	0.647	0.634	0.631
Stand. Dev.	0.304	1.424	0.000	0.146	0.187	0.205	0.236	0.244	0.251	0.258	0.260
Minimum	0.015	1.000	1.000	0.459	0.392	0.387	0.259	0.099	0.099	0.099	0.099
Maximum	1.000	8.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Turning to the analysis of Table 3, we can infer the following conclusions. First, on average the unbiased plant capacity utilization measure $PCU_o(x^f,y)$ indicates that current outputs make up 52% from maximal plant capacity outputs under C and 55% under NC. Second, the heterogeneity in $PCU_o(x^f,y)$ is large as indicated by the standard deviation and the range is again huge: the minimum of about 1.5% under both C and NC is simply extremely low. Third, the unbiased attainable plant capacity utilization measure $APCU_o(x,x^f,y,\bar{\lambda})$ decreases monotonically in $\bar{\lambda}$ and on average $APCU_o(x,x^f,y,\bar{\lambda})$ is always smaller under C than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda}=5$), $APCU_o(x,x^f,y,\bar{\lambda})$ is getting close to $PCU_o(x,x^f,y)$ in the C case (a difference of only about 4%), while this gap is larger in the NC case (a difference of about 8%).

Table 4 reports descriptive statistics on the three critical points L_p, M_p and U_p as defined in Definition 6. The following conclusions can be inferred. First, the average values for L_p and M_p are rather moderate, whereby the values are each time lower under C than under NC. This leads to rather plausible results for the input-oriented plant capacity measure $PCU_i(x,x^f,y)$. Under C one needs on average 4.39 more variable inputs with current outputs than with zero outputs, while under NC one employs 6.21 more variable inputs with current outputs than with zero outputs.

Table 4 Descriptive Statistics for Three Critical Points

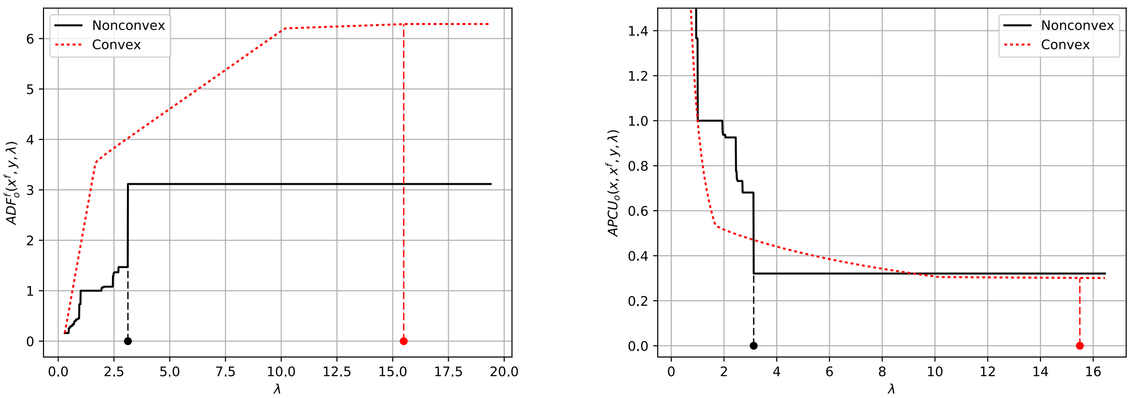
	Convex				Nonconvex				
	L_p^C	M_p^C	U_p^C	$PCU_i(.)$	L_p^{NC}	M_p^{NC}	U_p^{NC}	$PCU_i(.)$	$U_p^C - U_p^{NC}$
Average	0.338	0.715	31.585	4.397	0.352	0.944	28.753	6.214	2.832
Stand. Dev.	0.301	0.256	106.385	4.877	0.314	0.164	106.372	6.335	4.895
Minimum	0.038	0.132	1.000	1.000	0.038	0.267	0.904	1.000	0.000
1st Quartile	0.113	0.557	2.628	1.272	0.121	1.000	1.283	1.995	0.000
Median	0.200	0.754	4.031	2.485	0.245	1.000	2.627	3.566	0.571
3rd Quartile	0.451	0.952	12.295	5.732	0.451	1.000	5.444	7.359	2.692
Maximum	1.000	1.000	648.998	26.070	1.000	1.000	643.500	26.428	25.759

Second, on average the critical point U_p is very high: one needs 31.58 times more variable inputs than currently in use to reach maximum plant capacity outputs under C, while one can magnify variable inputs by just a factor 28.75 under NC. These amounts are huge in comparison to our prior value of allowing for only a fivefold increase in variable inputs. Third, the variation in this factor U_p is huge. For instance, at the third quartile we obtain a 12.29 magnification factor under C and only a 5.44 magnification factor under NC. The maximal magnification factor of 648.99 and 643.50 under C respectively NC are very similar in magnitude and both are clearly impossible in reality. These extreme requirements on the availability of variable inputs clearly cast doubts on the plausibility of the traditional output-oriented plant capacity measure. Fourth, the last column reporting the difference $U_p^C - U_p^{NC}$ reveals that on average the variable inputs under C should be increasing at least 2.83 times more than under NC. Furthermore, there is quite a bit of heterogeneity in this difference $U_p^C - U_p^{NC}$. Thus, in short, while these magnification factors for the variable inputs are clearly implausible, it seems that the non-convex results are the least implausible.

We end this analysis with some results for certain individual observations. Each figure has two parts: the left-hand side (LHS) displays the biased attainable plant capacity in function of the value of $\bar{\lambda}$; the right-hand side (RHS) shows the unbiased attainable plant capacity in function of the value of $\bar{\lambda}$. Both figures are drawn under both the C and NC assumption. Furthermore, the same critical point U_p is drawn for both C and NC in both figures.

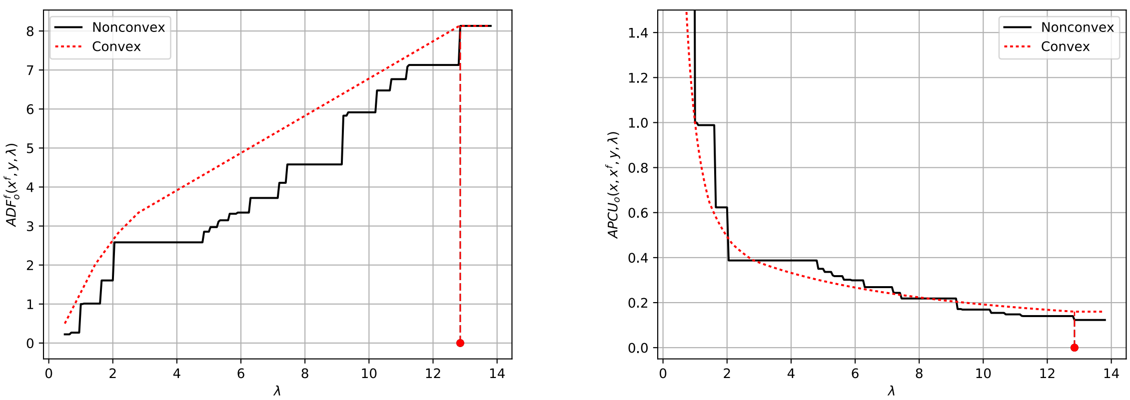
Figure 1 shows results for plant number 9. One can make the following series of observations on the LHS figure. First, the biased attainable plant capacity increases monotonically with $\bar{\lambda}$ under C and in a stepwise fashion under NC: these steps reveal the pervasive problem of slacks that

Figure 1 Biased and Unbiased Attainable Plant Capacity for Plant 9



is well-known under NC. Second, the maximum increase in outputs (i.e., the vertical distance between both lines) for the biased attainable plant capacity is almost double under C compared to NC. Third, the value of U_p is almost four times bigger under C (15.48) compared to NC (3.11). The following observations apply to the RHS figure. First, the unbiased attainable plant capacity decreases again monotonically with $\bar{\lambda}$ under C and in a stepwise fashion under NC. Second, the unbiased attainable plant capacity under C compared to NC cross one another: only for very high values of $\bar{\lambda}$ both estimates are close to one another. Overall, this again confirms that the NC results are less implausible.

Figure 2 Biased and Unbiased Attainable Plant Capacity for Plant 105



Finally, Figure 2 depicts the results for plant number 105. Now, the value of U_p under C and NC is identical (12.82). In this case, the differences between C and NC biased attainable plant capacity are rather pronounced, while these differences are mainly visible for the low range values of $\bar{\lambda}$ for the unbiased attainable plant capacity.

4.3. Empirical Results for Industry Level

Tables 5 and 6 are structured in a way similar to the corresponding firm level tables. While Table 5 reports on the industry biased plant capacity utilization measure $IADF_o^f(x^f, y, \bar{\lambda})$, Table 6 focuses on the industry unbiased plant capacity utilization measures $IAPCU_o(x, x^f, y, \bar{\lambda})$. Again, we have ten columns describing the industry attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5. New is that the three last rows of Tables 5 and 6 show the number of observed units that have the amounts $ADF_o^f(.) < IADF_o^f(.)$, $ADF_o^f(.) = IADF_o^f(.)$ and $ADF_o^f(.) > IADF_o^f(.)$, respectively. Thus, these lines focus on comparing firm level and industry level results.

Table 5 Descriptive Statistics of Biased Industry Plant Capacity Utilization

	$IADF_o^f(x^f, y, \lambda)$									
Convex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	12.092	12.973	13.366	13.576	13.644	13.655	13.655	13.655	13.655	13.655
Stand. Dev.	77.335	77.236	77.181	77.148	77.139	77.137	77.137	77.137	77.137	77.137
Minimum	0.010	0.010	0.318	0.318	0.918	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250
$ADF_o^f(.) < IADF_o^f(.)$	73	112	128	145	130	119	110	99	89	82
$ADF_o^f(.) = IADF_o^f(.)$	0	2	7	20	38	73	82	93	103	110
$ADF_o^f(.) > IADF_o^f(.)$	82	78	57	27	24	0	0	0	0	0
	$IADF_o^f(x^f, y, \lambda)$									
Nonconvex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	11.547	12.225	12.450	12.541	12.541	12.541	12.541	12.541	12.541	12.541
Stand. Dev.	77.357	77.270	77.240	77.226	77.226	77.226	77.226	77.226	77.226	77.226
Minimum	0.010	0.080	0.318	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250
$ADF_o^f(.) < IADF_o^f(.)$	60	125	103	107	99	85	77	72	62	59
$ADF_o^f(.) = IADF_o^f(.)$	5	29	60	85	93	107	115	120	130	133
$ADF_o^f(.) > IADF_o^f(.)$	86	38	29	0	0	0	0	0	0	0

Analyzing these results in Table 5, we infer the following conclusions. First, the biased industry attainable plant capacity utilization measure $IADF_o^f(x^f, y, \bar{\lambda})$ increases almost monotonically in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Second, $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 3 under C, and the value 2 under

NC. Third, though $IADF_o^f(x^f,y,\bar{\lambda}) \underset{<}{\overset{\geq}{=}} ADF_o^f(x_p^f,y_p,\bar{\lambda})$, for the majority of observations we find $ADF_o^f(x_p^f,y_p,\bar{\lambda}) < IADF_o^f(x^f,y,\bar{\lambda})$ till $\bar{\lambda}$ reaches the value 4 under C and only 2.5 under NC, and $ADF_o^f(x_p^f,y_p,\bar{\lambda}) = IADF_o^f(x^f,y,\bar{\lambda})$ afterward for the majority of observations. Furthermore, $ADF_o^f(x_p^f,y_p,\bar{\lambda}) > IADF_o^f(x^f,y,\bar{\lambda})$ becomes 0 when $IADF_o^f(x^f,y,\bar{\lambda})$ becomes stationary.

Table 6 Descriptive Statistics of Unbiased Industry Plant Capacity Utilization

	$IAPCU_o(x,x^f,y,\bar{\lambda})$									
Convex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	12.698	5.761	0.746	0.591	0.526	0.522	0.522	0.522	0.522	0.522
Stand. Dev.	23.797	19.196	0.624	0.473	0.273	0.269	0.269	0.269	0.269	0.269
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
Maximum	98.250	98.250	3.150	3.150	1.090	1.000	1.000	1.000	1.000	1.000
$APCU_o^f(.) < IAPCU_o^f(.)$	82	78	57	27	24	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	0	2	7	20	38	73	82	93	103	110
$APCU_o^f(.) > IAPCU_o^f(.)$	73	112	128	145	130	119	110	99	89	82
	$IAPCU_o(x,x^f,y,\lambda)$									
Nonconvex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	10.792	0.811	0.673	0.553	0.553	0.553	0.553	0.553	0.553	0.553
Stand. Dev.	20.264	1.006	0.543	0.304	0.304	0.304	0.304	0.304	0.304	0.304
Minimum	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Maximum	98.250	12.543	3.150	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$APCU_o^f(.) < IAPCU_o^f(.)$	86	38	29	0	0	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	5	29	60	85	93	107	115	120	130	133
$APCU_o^f(.) > IAPCU_o^f(.)$	60	125	103	107	99	85	77	72	62	59

Turning to the results in Table 6, the following deductions emerge. First, the unbiased industry attainable plant capacity utilization measure $IAPCU_o^f(x,x^f,y,\bar{\lambda})$ decreases almost monotonically in $\bar{\lambda}$ and on average $IAPCU_o(x,x^f,y,\bar{\lambda})$ is first smaller under NC than under C and then the reverse. Second, $IAPCU_o^f(x,x^f,y,\bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 3 under C, and the value 2 under NC. Third, while $IAPCU_o(x,x^f,y,\bar{\lambda}) \underset{<}{\overset{\geq}{=}} APCU_o(x,x^f,y,\bar{\lambda})$, for the majority of observations we find $APCU_o(x,x^f,y,\bar{\lambda}) > IAPCU_o(x,x^f,y,\bar{\lambda})$ till $\bar{\lambda}$ reaches the value 4 under C and only 2.5 under NC, and $APCU_o(x,x^f,y,\bar{\lambda}) = IAPCU_o(x,x^f,y,\bar{\lambda})$ afterward for the majority of observations. Furthermore, $APCU_o(x,x^f,y,\bar{\lambda}) < IAPCU_o(x,x^f,y,\bar{\lambda})$ becomes 0 when $IAPCU_o(x,x^f,y,\bar{\lambda})$ becomes stationary.

By solving the models in Definition 9 we obtain the two critical points: under C, $L^{I,C} = 0.1199$ and $U^{I,C} = 2.7516$, and under NC, $L^{I,NC} = 0.1199$ and $U^{I,NC} = 1.9947$. We make three comments.

First, while the lower bound is identical under C and NC, the upper bound under NC is substantially lower than under C. Second, based on Proposition 4, for $\bar{\lambda} \geq 2.7516$ in C case and $\bar{\lambda} \geq 1.9947$ in NC case, we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$. Thus, as can be seen in Tables 5 and 6, the five last columns in the C case and seven last columns in the NC case contain identical results. Third, it makes no sense to compare these two critical points L^I and U^I with, for instance, the averages of the corresponding points in the firm models L_p and U_p.

Instead, Table 7 reports the amount of increase of aggregate variable inputs such that all units obtain the maximum of the standard plant capacity utilization measure $DF_o^f(x_p^f, y_p)$ from both the perspective of firm and industry levels in both the C and NC cases. The second column shows the sum of observed variable inputs. The sum of needed variable inputs with the firm level model (9) under C and NC is reported in the third and fifth columns, respectively. The columns four and six present the sum of needed variable inputs with the industry level model (14) under C and NC, respectively. The second part of the table shows the magnification factors computed by taking the ratios of the sum of needed variable inputs to the sum of observed variable inputs under firm and industry models and under C and NC. The rows denote the two variable inputs: water and workers.

Table 7 Amounts of Variable Inputs Across Models

		Convex		Nonconvex	
Variable inputs	$\sum_{p=1}^K x_p^v$	$\sum_{p=1}^K U_p x_p^v$	$\sum_{p=1}^K U^I x_p^v$	$\sum_{p=1}^K U_p x_p^v$	$\sum_{p=1}^K U^I x_p^v$
Billions of m^3 of water	30718.888	103352.775	84526.092	74867.372	61274.966
# workers	3203.284	94183.888	8814.156	89220.392	6389.590
		Convex		Nonconvex	
Variable inputs		$\frac{\sum_{p=1}^K U_p x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U^I x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U_p x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U^I x_p^v}{\sum_{p=1}^K x_p^v}$
Billions of m^3 of water		3.364	2.752	2.437	1.995
# workers		29.402	2.752	27.853	1.995

Analyzing the results in Table 7, one can deduce the following conclusions. First, firm models need substantially more amounts of variable inputs than industry models. Second, C models need substantially more amounts of variable inputs than NC models. Third, while the industry models with an almost doubling of variable inputs under NC and an almost tripling of variable inputs under C are not necessarily incredible, the firm models with a doubling by a factor of almost 2.5 at minimum and a thirty fold magnification at worst are clearly incredible. For the variable input workers it is simply inconceivable that one could magnify the existing amounts by a factor of 27.85 under NC and a factor of 29.40 under C.

In conclusion, we deduce the following. First, firm models necessitate unlikely amounts of variable inputs, while the results for industry models are not a priori strikingly unrealistic. Second, NC models involve less unrealistic amounts of variable input magnifications than C models.

While some may put their hope in the industry models, it is crucial to remember their limitations. First, these industry models presuppose that there is a central authority coordinating among all firms. If firms are decentralized, this clearly is no option. Second, the industry models are clearly very basic. Any more realistic industry model with additional constraints (e.g., constraints on the amounts of inefficiency that are allowed for (as in Kerstens et al. (2006)), putting lower and upper bounds on changes in variable inputs per firm, etc.) will lead to less spectacular results.

To provide additional empirical evidence, we have also put the empirical results for both the firm and industry level for a data set with multiple outputs in the e-companion in Subsection EC.3. In turns out that the multiple output results are slightly less extreme than the single output results. Clearly, one empirical illustration suffices to make the basic point about the attainability issue of the traditional output-oriented plant capacity measure.

5. Conclusions

The output-oriented plant capacity concept has been around for at least two decades and is quite popular for empirical applications. While it was directly inspired by the informal definition provided by Johansen (1968), the doubts of Johansen (1968) regarding the attainability of the concept have seemingly never been investigated. This paper has tried to dig deeper into this issue of attainability.

In Section 3 we have formally defined both the traditional output-oriented and the rather new input-oriented plant capacity notions. Thereafter, we have argued that the output-oriented plant capacity notion may well fail attainability in general, because the amounts of variable inputs needed to reach the maximum capacity outputs may simply not be available. There does not seem to be such an issue for the input-oriented plant capacity concept. Consequently, we have defined a new attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. It is up to the researcher to determine plausible values limiting the upward scaling of variable inputs.

Using secondary data, we have developed an empirical illustration in Section 4. We can draw several conclusions. First, outputs need to be magnified an unreasonable amounts of times to reach traditional plant capacity outputs. Second, this phenomenon is related to the fact that variable inputs are supposed to be scalable at amounts that are unlikely to be available at either the firm or the industry level. Anyway, the amounts of scaling that need to be applied are ways above the fivefold increase with which we experimented when defining our attainable plant capacity notion. Third, while this scaling of variable inputs is probably ways beyond the reasonable, it is a fact that the computational results on a nonconvex technology are slightly less implausible than the ones obtained on a traditional convex technology. Thus, nonconvexity seems to mitigate partly the extreme results associated with the traditional output-oriented plant capacity notion. Fourth, the industry model (if applicable) leads to less incredible results than the firm model.

In conclusion, it is clear that given the fact that the traditional output-oriented plant capacity concept likely faces serious attainability problems, the new notion of an attainable output-oriented plant capacity concept merits further attention. Furthermore, since the new input-oriented plant capacity notion does not face any attainability issues, it may likely constitute an alternative framework as well.

We suggest some avenues for future research. First, our empirical analysis related to the attainability problem of the traditional output-oriented plant capacity concept needs further corroboration. In particular, it would be important to verify whether the attainability problem is equally serious when employing alternative estimators (e.g., stochastic frontier analysis as in Felthoven

(2002)). Furthermore, one major limitation is that we limited our analysis to radial efficiency measures, while it is well-known that the traditional convex and especially the nonconvex technologies suffer from large amounts of unmeasured inefficiency appearing as slacks (see, e.g., De Borger et al. (1998)). There are some indications that slacks may also play a substantial role in the measurement of plant capacity utilization (e.g., Dupont et al. (2002), or Vestergaard et al. (2003)). Therefore, it could be useful to revisit the attainability problem using nonradial rather than radial efficiency measures.

Second, our attainable plant capacity notion could benefit from clarifying the amounts by which variable inputs can reasonably be magnified (i.e., the value of $\bar{\lambda}$). Expert opinion may be one source of inspiration worthwhile exploring. Economic considerations related to, e.g., cost minimization or profit maximization may be another source of inspiration. Otherwise, it remains a conceptual alternative for the traditional output-oriented plant capacity notion, but it has little empirical bite.

Third, in a recent study reported in Kerstens et al. (2017) it turns out that the input-oriented plant capacity notion compares well with cost-based capacity notions, while the output-oriented plant capacity notion performs less well in this respect. One may wonder to which extent the attainability issue of the traditional output-oriented plant capacity plays a role in these results. Perhaps the attainable output-oriented plant capacity would mitigate these differences: this remains an open question.

Endnotes

1. For instance, note that the convex variable returns to scale technology does not satisfy inaction.
2. In case of parametric production technologies with a single output, Färe (1984) formally defines the notions of plant capacity limiting and weakly plant capacity limiting factor combinations and provides necessary and sufficient conditions for a factor combination to be plant capacity limiting, assuming additional restrictions on the class of production functions. However, not all production functions satisfy these additional restrictions. E.g., for the popular production function like the CES with certain parameter values, no factor combination is (weakly) plant capacity limiting.

3. Sahoo and Tone (2009, p. 581) propose another input-oriented capacity notion based on the short-run technology T^f. Its eventual relation with $PCU_i(x, x^f, y)$ remains to be explored.

4. The idea of a kind of limited strong disposability has been pursued in the context of congestion measurement in Briec et al. (2016).

5. While inaction is often phrased mathematically as $(0, 0) \in T$, the occurrence of zero outputs need not imply zero inputs. By the assumption of strong input disposability $(x, 0) \in T$ for $x > 0$. Thus, the use of positive inputs is compatible with zero outputs.

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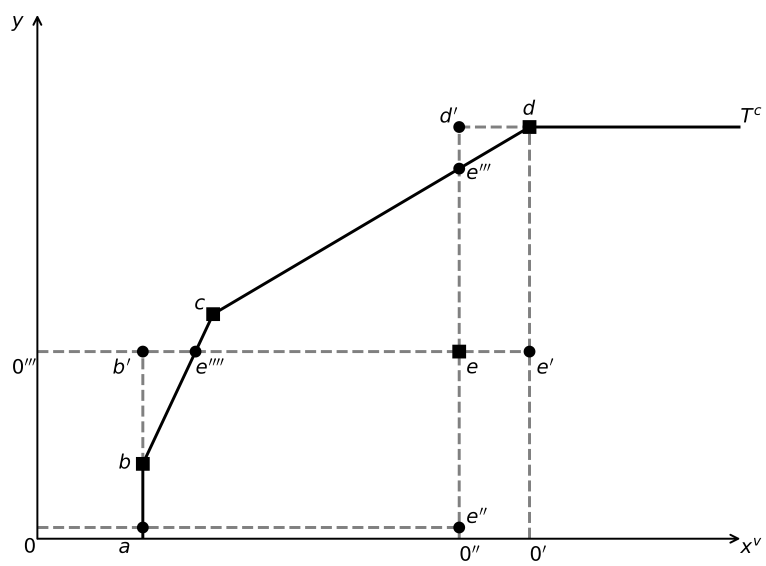
Figure, Proofs, and Multiple Outputs Empirical Example

EC.1. Figure Illustrating Output- and Input-Oriented Plant Capacity

Now we try to clarify both these Definitions 1 and 2 with the help of Figure EC.1 which depicts a single variable input and output space. In particular, Figure EC.1 shows a total product curve for given fixed inputs as the polyline *abcd* and its horizontal extension at *d*. We focus on observation *e*. Note that observations are represented by squares and projection points by circles.

Figure EC.1 Total product curve: Output- and input-oriented plant capacity.



The output-oriented plant capacity measure $PCU_o(x, x^f, y)$ compares point e to its vertical projection point e''' on the frontier on the one hand, and the translated point e' that consumes more variable inputs to its vertical projection point on the horizontal frontier segment emanating from point d with maximal outputs on the other hand. Clearly, the maximal output d can be labeled the plant capacity output. Thus, the unbiased plant capacity measure $PCU_o(x, x^f, y)$ is somehow linked to the distance $e'''d'$, whereby point d' is simply the translation of the maximal output at point d to the output level comparable with point e.

The input-oriented plant capacity measure $PCU_i(x, x^f, y)$ focuses on a sub-vector of variable inputs and compares point e to its horizontal projection point e'''' on the frontier on the one hand, and the translated point e'' (consuming equal amounts of variable inputs but at a zero outputs level) to its horizontal projection point on the vertical frontier segment ab with zero outputs on the other hand. Clearly, the minimal variable input a yielding zero output can be labeled the plant capacity input. Thus, the unbiased plant capacity measure $PCU_i(x, x^f, y)$ is somehow linked to the distance $b'e''''$, whereby point b' is the translation of the variable input at point b to the variable input level comparable with point e.

EC.2. Proofs of Propositions

PROPOSITION 1. *The attainable output-oriented plant capacity utilization* $APCU_o(x, x^f, y, \bar{\lambda})$ *converges to the output-oriented plant capacity utilization* $PCU_o(x, x^f, y)$ *as* $\bar{\lambda} \longrightarrow \infty$*, i.e.,* $\lim_{\bar{\lambda}\to\infty} APCU_o(x, x^f, y, \bar{\lambda}) = PCU_o(x, x^f, y)$*.*

Proof The result follows directly by combining (3) with Definitions 1, 4 and 5 together with taking the limit for $\bar{\lambda} \longrightarrow \infty$. $\square$

PROPOSITION 2. *For the biased and unbiased attainable output-oriented plant capacity utilization in both* C *and* NC *technologies, for every observation* (x_p, y_p) *we have:*

(i) If $\bar{\lambda} < L_p$*, then model* (9) *is infeasible.*

(ii) If $L_p \leq \bar{\lambda} < M_p$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > 1$*.*

(iii) If $M_p \leq \bar{\lambda}$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) \leq 1$*.*

Proof (i) Suppose that $\bar{\lambda} < L_p$ and model (9) is feasible with optimal solution (z_k^*, x^{v*}). Hence, $x^{v*} \leq \bar{\lambda} x_p^v < L_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (10) with optimal value $\hat{\theta} = \bar{\lambda}$. But, this is a contradiction since $\bar{\lambda} < L_p$.

(ii) Assume that $L_p \leq \bar{\lambda} < M_p$ and $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (9) such that $\theta^* \geq 1$. Hence, we have $x^{v*} \leq \bar{\lambda} x_p^v < M_p x_p^v$. So $(\hat{z}_k^* = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (11) with optimal value $\hat{\theta} = \bar{\lambda}$. This is contradiction since $\bar{\lambda} < M_p$.

(iii) Assume that $M_p \leq \bar{\lambda}$ and $(z_k^*, \theta^* = M_p)$ is an optimal solution of model (11). Since $M_p x_p^v \leq \bar{\lambda} x_p^v$, hence $(\hat{z}_k = z_k^*, \hat{x}^v = M_p x_p^v, \hat{\theta} = 1)$ is a feasible solution of model (9) with objective value $\hat{\theta} = 1$. Therefore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ because the kind of model is a maximising problem. □

Proposition 3. *For the biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, for every observation (x_p, y_p), we have:*

(i) If $L_p \leq \bar{\lambda} < U_p$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ and $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$.

(ii) If $\bar{\lambda} \geq U_p$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.

Proof (i) Suppose that $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (9). This solution is also a feasible solution of model (8). Since $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$, it is sufficient to show that this solution is not an optimal solution of model (8). By contradiction, suppose that $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (8), since $\bar{\lambda} < U_p$, thus $x^{v*} \leq \bar{\lambda} x_p^v < U_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (13) with objective value $\bar{\lambda} < U_p$, which is a contradiction.
(ii) Assume that $(z_k^*, \theta^* = U_p)$ is an optimal solution of model (13). Since $\bar{\lambda} \geq U_p$, hence $\bar{\lambda} x_p^v \geq U_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{x}^v = U_p x_p^v, \hat{\theta} = DF_o^f(x_p^f, y_p))$ is a feasible solution of model (9). Thus, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq DF_o^f(x_p^f, y_p)$. But, we also know that $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$. Hence, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$. This completes the proof. □

Proposition 4. *For the industry biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, we have:*

(i) If $\bar{\lambda} < L^I$, then model (14) is infeasible.

(ii) If $L^I \leq \bar{\lambda} < U^I$, then at least for one observed observation (x_p, y_p) we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$.

(iii) If $U^I \leq \bar{\lambda}$, then for every observation (x_p, y_p) we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.

Proof (i) Assume that model (14) is feasible with optimal solution $(\theta_p^*, z_k^{p*}, x_p^{v*})$. Since $\bar{\lambda} < L^I$, thus

$$\sum_{p=1}^{K} x_p^{v*} \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v < L^I \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{\theta} = \bar{\lambda}, \hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*})$ is a feasible solution of model (15) with objective value $\hat{\theta} = \bar{\lambda} < L^I$ which is a contradiction.

(ii) Let

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p), \qquad p = 1, ..., K.$$

Also, $(\theta_p^*, z_k^{p*}, x_p^{v*})$ is an optimal solution of model (14) in which $\theta_p^* = IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and

$$\sum_{p=1}^{K} x_p^{v*} \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v < U^I \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{\theta} = \bar{\lambda}, \hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*})$ is a feasible solution of model (16) with objective value $\hat{\theta} = \bar{\lambda} < U^I$ which is a contradiction.

(iii) Assume that $(z_k^{p*}, x_p^{v*}, \theta^* = U^I)$ is an optimal solution of model (16). We have

$$\sum_{p=1}^{K} x_p^{v*} \leq \theta^* \sum_{p=1}^{K} \bar{x}_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*}, \hat{\theta}_p = DF_o^f(x_p^f, y_p))$ is a feasible solution of model (14) in which $DF_o^f(x_p^f, y_p) \leq IADF_o^f(x_p^f, y_p, \bar{\lambda})$. But, we know that $IADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$. Hence, $DF_o^f(x_p^f, y_p) = IADF_o^f(x_p^f, y_p, \bar{\lambda})$ and this completes the proof. $\square$

EC.3. Multiple Outputs Empirical Example

EC.3.1. Description of the Sample with Multiple Outputs

As a second sample we draw upon an unbalanced panel of three years of French fruit producers based on annual accounting data collected in a survey (see Ivaldi et al. (1996) for details). Mainly two criteria were adapted to select these farms: (i) the production of apples must be positive, and (ii) the acreage of the orchard is at least five acres. The short panel covers the three successive years from 1984 to 1986. As a technology, three aggregate inputs produce two aggregate outputs. The three inputs are: (i) capital (including land), (ii) labor, and (iii) materials. The two outputs are (i) the production of apples, and (ii) an aggregate of alternative products. Summary statistics for the 405 observations in total and details on the definitions of all variables are available in Appendix 2 in Ivaldi et al. (1996). Note that the short length of the panel (only three years) justifies the use of an intertemporal approach that ignores technical change. Table EC.1 presents basic descriptive statistics for inputs and outputs. Again one striking feature is the large heterogeneity in terms of size among the different inputs as well as the outputs.

Table EC.1 Descriptive Statistics for French Fruit Producers (1984-1986)

Variable	Trimmed mean[a]	Minimum	Maximum
Capital (fixed input)	85602.58	8891	500452
Labor (variable input)	229569	79569	1682201
Materials (variable input)	157610.9	19566	1523776
Volume of apple production (output)	2.146273	0.00061	37.98153
Volume of other products (output)	1.37793	0.000672	25.895

Note: [a]10% trimming level.

EC.3.2. Empirical Results for Firm Level

Tables EC.2 and EC.3 are structured in a similar way to Tables 2 and 3. While Table EC.2 reports on the biased plant capacity utilization measures $DF_o^f(x^f, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$, Table EC.3 focuses on the unbiased plant capacity utilization measures $PCU_o(x, x^f, y)$ and $APCU_o(x, x^f, y, \bar{\lambda})$. In each table, the second column reports the standard plant capacity utilization measures, while the next ten columns describe the attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5 (thus, $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$). Hence, we somewhat arbitrary assume that variable inputs can be magnified at most fivefold. Obviously, we could have selected a wider range of values to experiment with $\bar{\lambda}$. Based on Proposition 2, note that for 129 observations under C and 134 observations under NC $\bar{\lambda} = 0.5$ is too small for model (9) to be feasible. Hence, these observations are not included in the corresponding descriptive statistics computations.

Analyzing the results in Table EC.2, one can draw the following conclusions. First, on average the biased plant capacity utilization measure $DF_o^f(x^f, y)$ indicates that outputs can be magnified by at

Table EC.2 Descriptive Statistics of Biased Plant Capacity Utilization

		$ADF_o^f(x^f,y,\bar{\lambda})$									
Convex	$DF_o^f(x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	5.415	1.390	3.492	4.673	5.127	5.283	5.339	5.363	5.378	5.389	5.398
Stand. Dev.	4.678	1.005	2.631	3.756	4.369	4.588	4.645	4.652	4.657	4.661	4.664
Minimum	1.000	0.120	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	35.295	4.878	16.287	24.396	29.350	34.735	35.135	35.201	35.266	35.295	35.295
		$ADF_o^f(x^f,y,\bar{\lambda})$									
Nonconvex	$DF_o^f(x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	2.891	0.566	1.648	2.172	2.452	2.573	2.727	2.779	2.828	2.851	2.872
Stand. Dev.	2.935	0.490	1.113	1.810	2.202	2.292	2.701	2.778	2.822	2.938	2.936
Minimum	1.000	0.020	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	32.457	2.316	7.936	18.129	19.538	19.538	32.457	32.457	32.457	32.457	32.457

least 5.41 times under C and 2.89 times under NC. Second, there is a lot of variation in $DF_o^f(x^f,y)$ as indicated by the standard deviation and the range is even huge: the maximum increase in outputs amounts to 35.29 times under C and 32.46 under NC. Third, the biased attainable plant capacity utilization measure $ADF_o^f(x^f,y,\bar{\lambda})$ increases monotonically in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda}=5$), we obtain on average a 5.40 output magnification under C and a 2.87 output magnification under NC. This is very close to the average output magnification computed by the biased plant capacity utilization measure $DF_o^f(x^f,y)$.

Table EC.3 Descriptive Statistics of Unbiased Plant Capacity Utilization

		$APCU_o(x,x^f,y,\bar{\lambda})$									
Convex	$PCU_o(x,x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	0.710	3.164	1.000	0.780	0.735	0.723	0.718	0.715	0.713	0.712	0.712
Stand. Dev.	0.221	2.887	0.000	0.161	0.200	0.212	0.215	0.217	0.218	0.219	0.220
Minimum	0.070	1.011	1.000	0.116	0.080	0.071	0.070	0.070	0.070	0.070	0.070
Maximum	1.000	36.057	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		$APCU_o(x,x^f,y,\bar{\lambda})$									
Nonconvex	$PCU_o(x,x^f,y)$	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	0.691	6.870	1.000	0.836	0.772	0.740	0.715	0.707	0.702	0.701	0.695
Stand. Dev.	0.245	9.114	0.000	0.195	0.221	0.226	0.235	0.238	0.241	0.242	0.243
Minimum	0.097	1.000	1.000	0.172	0.116	0.116	0.116	0.116	0.116	0.097	0.097
Maximum	1.000	53.968	1.000	1.095	1.043	1.028	1.000	1.000	1.000	1.000	1.000

Turning to the analysis of Table EC.3, we can infer the following conclusions. First, on average the unbiased plant capacity utilization measure $PCU_o(x^f,y)$ indicates that current outputs make up 71% from maximal plant capacity outputs under C and 69% under NC. Second, the heterogeneity in $PCU_o(x^f, y)$ is large as indicated by the standard deviation and the range is again huge: the minimum of 7.0% under C and 9.7% under NC is simply very low. Third, the unbiased attainable plant capacity utilization measure $APCU_o(x, x^f, y, \bar{\lambda})$ decreases monotonically in $\bar{\lambda}$ and on average $APCU_o(x, x^f, y, \bar{\lambda})$ is smaller under C than under NC for $\bar{\lambda} \in \{1, 1.5, 2, 2.5\}$ and the reverse for $\bar{\lambda} \in \{3, 3.5, 4, 4.5, 5\}$. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda} = 5$), $APCU_o(x, x^f, y, \bar{\lambda})$ is getting close to $PCU_o(x, x^f, y)$ in the C case (a difference of only about 0.2%), while this gap is slightly larger in the NC case (a difference of about 0.4%).

Table EC.4 reports descriptive statistics on the three critical points L_p, M_p and U_p as defined in Definition 6. The following conclusions can be inferred. First, the average values for L_p and M_p are rather moderate, whereby the values are each time lower under C than under NC. This leads to rather plausible results for the input-oriented plant capacity measure $PCU_i(x, x^f, y)$. Under C one needs on average 1.73 more variable inputs with current outputs than with zero outputs, while under NC one employs 2.54 more variable inputs with current outputs than with zero outputs.

Second, on average the critical point U_p is rather moderate: one only needs 2.60 times more variable inputs than currently in use to reach maximum plant capacity outputs under C, while one can magnify variable inputs by just a factor 2.02 under NC. These amounts are very reasonable compared to our prior value of allowing for only a fivefold increase in variable inputs at most. Third, the variation in this factor U_p is rather substantial. For instance, at the third quartile we obtain a 3.30 magnification factor under C and only a 2.60 magnification factor under NC. The maximal magnification factor of 8.54 and 6.79 under C respectively NC are very similar in magnitude and both are clearly implausible in reality. These very strong requirements on the availability of variable inputs clearly cast doubts on the plausibility of the traditional output-oriented plant capacity measure. Fourth, the last column reporting the difference $U_p^C - U_p^{NC}$ reveals that on average the variable inputs under C should be increasing at least 0.57 times more than under NC. Furthermore, there is quite a bit of heterogeneity in this difference $U_p^C - U_p^{NC}$. Thus, in short, while these magnification factors for the variable inputs are clearly implausible, it seems that the non-convex results are the least implausible.

Table EC.4 Descriptive Statistics for Three Critical Points

	Convex				Nonconvex				
	L_p^C	M_p^C	U_p^C	$PCU_i(.)$	L_p^{NC}	M_p^{NC}	U_p^{NC}	$PCU_i(.)$	$U_p^C - U_p^{NC}$
Average	0.423	0.588	2.597	1.734	0.431	0.830	2.022	2.540	0.575
Stand.Dev.	0.195	0.192	1.621	1.636	0.202	0.207	1.134	2.157	1.313
Minimum	0.047	0.187	0.513	1.000	0.047	0.270	0.386	1.000	-3.214
1st Quartile	0.281	0.441	1.458	1.093	0.285	0.680	1.058	1.380	-0.149
Median	0.406	0.573	2.093	1.260	0.415	0.955	1.704	1.799	0.332
3rd Quartile	0.542	0.717	3.305	1.659	0.556	1.000	2.598	2.628	1.158
Maximum	1.000	1.000	8.544	21.141	1.000	1.000	6.786	21.141	5.645

EC.3.3. Empirical Results for Industry Level

Tables EC.5 and EC.6 are structured in a way similar to the corresponding firm level tables. While Table EC.5 reports on the industry biased plant capacity utilization measure $IADF_o^f(x^f, y, \bar{\lambda})$, Table EC.6 focuses on the industry unbiased plant capacity utilization measures $IAPCU_o(x, x^f, y, \bar{\lambda})$. Again, we have ten columns describing the industry attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5. New is that the three last rows of Tables EC.5 and EC.6 show the number of observed units that have the amounts $ADF_o^f(.) < IADF_o^f(.)$, $ADF_o^f(.) = IADF_o^f(.)$ and $ADF_o^f(.) > IADF_o^f(.)$, respectively. Thus, these lines focus on comparing firm level and industry level results.

Table EC.5 Descriptive Statistics of Biased Industry Plant Capacity Utilization

	$IADF_o^f(x^f, y, \lambda)$									
Convex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	3.641	5.189	5.373	5.412	5.415	5.415	5.415	5.415	5.415	5.415
Stand.Dev.	5.130	4.815	4.697	4.680	4.678	4.678	4.678	4.678	4.678	4.678
Minimum	0.002	0.002	0.565	0.921	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	35.073	35.295	35.295	35.295	35.295	35.295	35.295	35.295	35.295	35.295
$APCU_o^f(.) < IAPCU_o^f(.)$	115	300	220	204	161	113	91	68	55	39
$APCU_o^f(.) = IAPCU_o^f(.)$	0	15	48	157	244	292	314	337	350	366
$APCU_o^f(.) > IAPCU_o^f(.)$	161	90	137	44	0	0	0	0	0	0
	$IADF_o^f(x^f, y, \lambda)$									
Nonconvex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	2.191	2.833	2.891	2.891	2.891	2.891	2.891	2.891	2.891	2.891
Stand.Dev.	3.173	2.967	2.935	2.935	2.935	2.935	2.935	2.935	2.935	2.935
Minimum	0.002	0.156	1	1	1	1	1	1	1	1
Maximum	32.457	32.457	32.457	32.457	32.457	32.457	32.457	32.457	32.457	32.457
$APCU_o^f(.) < IAPCU_o^f(.)$	142	275	238	170	114	61	43	25	21	9
$APCU_o^f(.) = IAPCU_o^f(.)$	13	101	167	235	291	344	362	380	384	396
$APCU_o^f(.) > IAPCU_o^f(.)$	116	29	0	0	0	0	0	0	0	0

Analyzing these results in Table EC.5, we infer the following conclusions. First, the biased industry attainable plant capacity utilization measure $IADF_o^f(x^f, y, \bar{\lambda})$ increases almost monotonically in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Second, $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 2.5 under C, and the value 1.5 under NC. Third, though $IADF_o^f(x^f, y, \bar{\lambda}) \substack{\geq \\ = \\ <} ADF_o^f(x_p^f, y_p, \bar{\lambda})$, for the majority of observations we find $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < IADF_o^f(x^f, y, \bar{\lambda})$ till $\bar{\lambda}$ reaches the value 2 under C and just 1.5 under NC, and $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = IADF_o^f(x^f, y, \bar{\lambda})$ afterward for the majority of observations. Furthermore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) > IADF_o^f(x^f, y, \bar{\lambda})$ becomes 0 when $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary.

Table EC.6 Descriptive Statistics of Unbiased Industry Plant Capacity Utilization

	$IAPCU_o(x, x^f, y, \bar{\lambda})$									
Convex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	44.056	4.683	0.728	0.712	0.710	0.710	0.710	0.710	0.710	0.710
Stand.Dev.	99.170	40.687	0.239	0.222	0.221	0.221	0.221	0.221	0.221	0.221
Minimum	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070
Maximum	569.071	569.071	1.771	1.085	1	1	1	1	1	1
$APCU_o^f(.) < IAPCU_o^f(.)$	161	90	137	44	0	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	0	15	48	157	243	287	308	330	343	359
$APCU_o^f(.) > IAPCU_o^f(.)$	115	300	220	204	161	113	91	68	55	39
	$IAPCU_o(x, x^f, y, \bar{\lambda})$									
Nonconvex	$\bar{\lambda}=0.5$	$\bar{\lambda}=1$	$\bar{\lambda}=1.5$	$\bar{\lambda}=2$	$\bar{\lambda}=2.5$	$\bar{\lambda}=3$	$\bar{\lambda}=3.5$	$\bar{\lambda}=4$	$\bar{\lambda}=4.5$	$\bar{\lambda}=5$
Average	34.216	0.792	0.691	0.691	0.691	0.691	0.691	0.691	0.691	0.691
Stand.Dev.	78.895	0.605	0.245	0.245	0.245	0.245	0.245	0.245	0.245	0.245
Minimum	0.097	0.097	0.097	0.097	0.097	0.097	0.097	0.097	0.097	0.097
Maximum	569.071	6.418	1	1	1	1	1	1	1	1
$APCU_o^f(.) < IAPCU_o^f(.)$	116	29	0	0	0	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	12	101	167	235	291	344	362	380	384	396
$APCU_o^f(.) > IAPCU_o^f(.)$	143	275	238	170	114	61	43	25	21	9

Turning to the results in Table EC.6, the following deductions emerge. First, the unbiased industry attainable plant capacity utilization measure $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ decreases almost monotonically in $\bar{\lambda}$ and on average $IAPCU_o(x, x^f, y, \bar{\lambda})$ is smaller under NC than under C. Second, $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 2.5 under C, and the value 1.5 under NC. Third, while $IAPCU_o(x, x^f, y, \bar{\lambda}) \substack{\geq \\ = \\ <} APCU_o(x, x^f, y, \bar{\lambda})$, for the majority of observations we find $APCU_o(x, x^f, y, \bar{\lambda}) > IAPCU_o(x, x^f, y, \bar{\lambda})$ till $\bar{\lambda}$ reaches the value 2 under C and only 1.5

under NC, and $APCU_o(x, x^f, y, \bar{\lambda}) = IAPCU_o(x, x^f, y, \bar{\lambda})$ afterward for the majority of observations. Furthermore, $APCU_o(x, x^f, y, \bar{\lambda}) < IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes 0 when $IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes stationary.

By solving the models in Definition 9 we obtain the two critical points: under C, $L^{I,C} = 0.3190$ and $U^{I,C} = 2.1179$, and under NC, $L^{I,NC} = 0.320$ and $U^{I,NC} = 1.4103$. We make three comments. First, while the lower bound is close to identical under C and NC, the upper bound under NC is substantially lower than under C. Second, based on Proposition 4, for $\bar{\lambda} \geq 2.1179$ in C case and $\bar{\lambda} \geq 1.4103$ in NC case, we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$. Thus, as can be seen in Tables EC.5 and EC.6, the six last columns in the C case and eight last columns in the NC case contain identical results. Third, it makes no sense to compare these two critical points L^I and U^I with, for instance, the averages of the corresponding points in the firm models L_p and U_p.

Instead, Table EC.7 reports the amount of increase of aggregate variable inputs such that all units obtain the maximum of the standard plant capacity utilization measure $DF_o^f(x_p^f, y_p)$ from both the perspective of firm and industry levels in both the C and NC cases. The second column shows the sum of observed variable inputs. The sum of needed variable inputs with the firm level model (9) under C and NC is reported in the third and fifth columns, respectively. The columns four and six present the sum of needed variable inputs with the industry level model (14) under C and NC, respectively. The second part of the table shows the magnification factors computed by taking the ratios of the sum of needed variable inputs to the sum of observed variable inputs under firm and industry models and under C and NC. The rows denote the two variable inputs: Labor and Materials.

Analyzing the results in Table EC.7, one can deduce the following conclusions. First, firm models need substantially more amounts of variable inputs than industry models. Second, C models need substantially more amounts of variable inputs than NC models. Third, while the industry models with a less than doubling of variable inputs under NC and a doubling of variable inputs under

Table EC.7 Amounts of Variable Inputs Across Models

		Convex		Nonconvex	
Variable inputs	$\sum_{p=1}^{K} x_p^v$	$\sum_{p=1}^{K} U_p x_p^v$	$\sum_{p=1}^{K} U^I x_p^v$	$\sum_{p=1}^{K} U_p x_p^v$	$\sum_{p=1}^{K} U^I x_p^v$
Labor	101923208	249781392	215863162.2	182837273.8	143742300.2
Materials	74548523	169235569	157886316.9	123199828.2	105135782
		Convex		Nonconvex	
Variable inputs		$\frac{\sum_{p=1}^{K} U_p x_p^v}{\sum_{p=1}^{K} x_p^v}$	$\frac{\sum_{p=1}^{K} U^I x_p^v}{\sum_{p=1}^{K} x_p^v}$	$\frac{\sum_{p=1}^{K} U_p x_p^v}{\sum_{p=1}^{K} x_p^v}$	$\frac{\sum_{p=1}^{K} U^I x_p^v}{\sum_{p=1}^{K} x_p^v}$
Labor		2.4506822	2.1179	1.793872832	1.4103
Materials		2.2701398	2.1179	1.652612596	1.4103

C are not necessarily incredible, the firm models with a doubling by a factor of almost 1.65 at minimum and a 2.45 fold magnification at worst are clearly less credible.

In conclusion, we deduce the following conclusions. First, firm models necessitate unlikely amounts of variable inputs, while the results for industry models are not a priori completely unrealistic. Second, NC models involve less unrealistic amounts of variable input magnifications than C models. But, while some may put their hope in the industry models, it is crucial to remember the limitations already spelled out at the end of Subsection 4.3.