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A new approach for quantitative risk analysis

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Abstract

Project risk management aims to provide insight into the risk profile of a project as to facilitate decision makers to mitigate the impact of risks on project objectives such as budget and time. A popular approach to determine where to focus mitigation efforts, is the use of so-called ranking indices (e.g. the criticality index, the significance index etc.). Ranking indices allow the ranking of project activities (or risks) based on the impact they have on project objectives. A distinction needs to be made between activity-based ranking indices (those that rank activities) and risk-driven ranking indices (those that rank risks). Because different ranking indices result in different rankings of activities and risks, one might wonder which ranking index is best? In this article, we provide an answer to this question. Our contribution is threefold: (1) we set up a large computational experiment to assess the efficiency of ranking indices in the mitigation of risks; (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that a risk-driven approach is more efficient than an activity-based approach.

1 Introduction

It is well known that projects worldwide are still struggling to meet their objectives (The Standish Group 2009). During project execution, unforeseen

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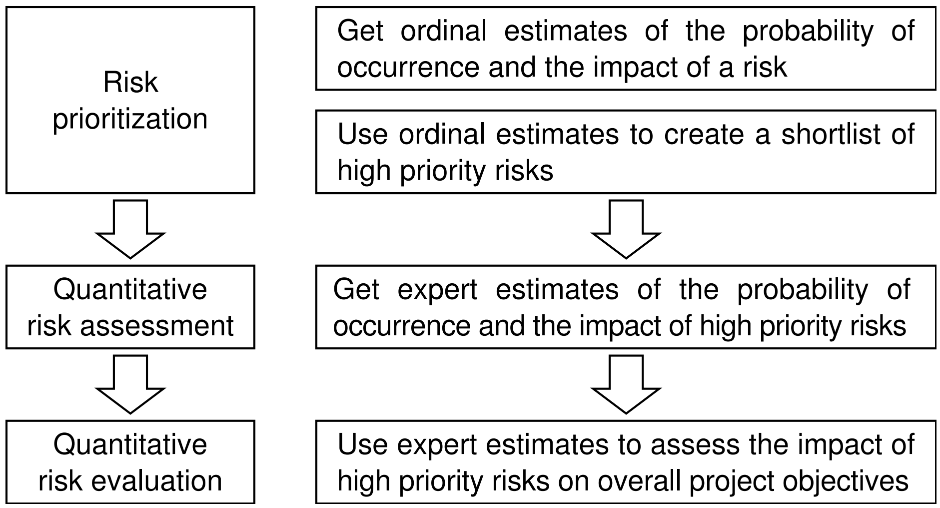

Figure 1: Overview of the risk analysis process

events arise that disrupt plans and budgets and that result in substantial overruns. Risk management is widely recognized as a compulsory discipline to deal with this kind of project uncertainty.

The Project Management Institute (2008) defines risk management as the process that deals with the planning, identification, analyzing, responding, monitoring and controlling of project risks. In this article, we focus on the risk analysis process and its effect on the risk response process. The risk analysis process can be divided into a number of subprocesses: risk prioritization, quantitative risk assessment and quantitative risk evaluation. Risk prioritization is a qualitative procedure that allows to prioritize the risks that were identified in an earlier stage of the risk management process. It requires ordinal estimates of both the probability of occurrence and the impact of a risk. These ordinal estimates are then used to create a shortlist of high priority risks (analogous to the Pareto principle). Further risk analysis efforts should focus on these high priority risks. Quantitative risk assessment is the procedure in which experts provide detailed estimates of the probability of occurrence and the impact of high priority risks. These estimates are used in the quantitative risk evaluation procedure to analyze the impact of the shortlisted risks on overall project objectives. Figure 1 provides a short overview of the dynamics of the risk analysis process.

Good risk management requires a risk analysis process that is scientifically sound and that is supported by quantitative techniques (Hubbard 2008). A wide body of knowledge on quantitative techniques has been accumulated over the last two decades. Monte Carlo Simulation is the predominant quantitative risk evaluation technique in both practice and in literature. Advocates of alternative techniques such as neural networks, fuzzy logic and decision tree analysis have debatable arguments in favor of these techniques, but have so far failed to persuade most project schedulers of their practical usage (refer to Sadeghi et al. (2009) and Georgieva et al. (2009) among others for an evaluation of risk analysis techniques).

The goal of risk analysis is to generate insight into the risk profile of a project and to use these insights to drive the risk response process (The Project Management Institute 2008). The insights generated include: the probability of achieving a specific project outcome, the distribution function of the project completion time, etc. The risk response process will use these insights to define practical risk responses that allow project managers to mitigate risks (i.e. to reduce the impact of risks on project objectives). A popular approach to determine where to focus mitigation efforts is the use of so-called ranking indices (e.g. the criticality index, the significance index, etc.). Ranking indices allow the ranking of project activities (or risks) based on the impact they have on project objectives. A distinction needs to be made between activity-based ranking indices (those that rank activities) and risk-driven ranking indices (those that rank risks). Remark that the impact of an activity (or risk) on a project objective may differ depending on the ranking index used, resulting in the question: which ranking index is best? It is exactly this question that we will address in this article.

The contribution of this article is threefold: (1) we set up a large computational experiment to assess the potential of ranking indices in mitigating risks; (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that a risk-driven approach outperforms an activity-based approach. For our study, we assume risks to impact the duration of activities and hence use the project completion time to gauge the performance of ranking indices (i.e. we assess the potential of ranking indices to mitigate risks that delay the completion time of a project). In order to approximate the distribution of the project completion time, we adopt Monte Carlo simulation.

The remainder of this article is organized as follows: in Section 2 we review the basic principles of stochastic project scheduling. Section 3 introduces the risk-driven approach and compares it to the activity-based approach. Section 4 presents the ranking indices. In Section 5 the computational experiment is outlined and results are discussed. Section 6 draws some conclusions.

2 Stochastic project scheduling

The Critical Path Method (CPM) is developed in the 50's by DuPont Corporation and provides the foundations of modern project scheduling. It represents a project as an activity network which is a graph $G = (N, A)$ that consists of a set of nodes $N = \{1, 2, \ldots, n\}$ and a set of arcs $A = \{(i, j)|i, j \in N\}$. The nodes represent project activities whereas the arcs that connect the nodes represent precedence relationships. Activities 1 and n are referred to as the dummy-start and the dummy-end activity and represent the start and the completion of the project respectively. Each activity j has a deterministic activity duration d_j and can only start when its predecessors have finished. CPM adopts an early start schedule in which activities are scheduled to start as soon as possible. The early start schedule may be represented by a vector of earliest start times $\mathfrak{s} = \{s_1, s_2, \ldots, s_n\}$. The earliest start time of an activity j is defined as follows:

$$s_j = \max \{f_i|(i, j) \in A\}, \tag{1}$$

where f_j is the earliest finish time of an activity j and equals:

$$f_j = s_j + d_j. \tag{2}$$

By convention, the project starts at time instance 0 (i.e. $s_1 = 0$). According to CPM, the project completion time c is computed as follows:

$$c = f_n. \tag{3}$$

The longest path of the scheduled activities is called the critical path and the activities on this path are critical activities.

Since the establishment of CPM, many extensions of the basic model have been introduced: generalized precedence relationships, resource-constrained project scheduling, multi-mode scheduling, critical chain buffer management, etc. We refer to Demeulemeester and Herroelen (2002) for an extensive overview of the field. In this article, we are particularly interested in what is called stochastic project scheduling or stochastic CPM. Stochastic CPM acknowledges that activity durations are not deterministic. We model the duration of an activity j as a positive random variable D_j. Because the duration of an activity is a random variable, the earliest start and finish times of an activity are random variables as well. Let S_j and F_j denote the random variable of the earliest start and finish times of an activity j respectively. The project completion time is a random variable C which is a function of D_j. Calculating the distribution function of C is proven to be $\#P$-complete (Hagstrom 1988) and thus requires approximative methods such as Monte Carlo simulation (Van Slyke 1963). Monte Carlo simulation is used to virtually execute a project a large number of times, providing insight and allowing the project manager to enhance the actual execution of the project.

We will use Monte Carlo simulation to generate random variates of D_j. Let $\mathbf{d}_j = \{d_{j,1}, d_{j,2}, \ldots, d_{j,q}\}$ denote the vector of q random variates of D_j (where q represents the number of simulation iterations). We refer to $\mathbf{d}_j$ as the vector of realized durations of D_j. In addition, define $\mathbf{s}_j$ the vector of realized earliest start times of an activity j:

$$\mathbf{s}_j = \max \left\{ \mathbf{f}_i | (i, j) \in A \right\}, \tag{4}$$

where $\mathbf{f}_j$ is the vector of realized earliest finish times of an activity j and equals:

$$\mathbf{f}_j = \mathbf{s}_j + \mathbf{d}_j. \tag{5}$$

The vector of realized project completion times $\mathbf{c}$ is defined as follows:

$$\mathbf{c} = \mathbf{f}_n. \tag{6}$$

It is clear that $\mathbf{s}_j$, $\mathbf{f}_j$ and $\mathbf{c}$ are vectors of random variates of random variables S_j, F_j and C respectively.

3 Towards a risk-driven approach

One of the main challenges in project risk management is to estimate and to model the uncertainty of activity durations. Often, it is assumed that the duration of an activity follows a distribution that captures all uncertainty that originates from the occurrence of risks (popular distributions include: the triangular distribution, the beta distribution and the normal distribution).

As such, risk assessment boils down to providing estimates of activity duration distribution parameters. We refer to this approach as the activity-based approach.

In this article we argue that the activity-based approach is inherently flawed. As Hulett (2009) points out, there is no clear link between the impact of identified risks on the duration of an activity and the distribution of the activity duration itself (i.e. the activity-based approach is unable to identify the root causes of the uncertainty in the duration of an activity). In addition, our experience learns that practitioners have a hard time assessing uncertainty by estimating the parameters of an activity duration distribution.

To resolve the problems of the activity-based approach, we devise a risk-driven approach in which the impact of each risk is assessed individually and is mapped to the duration of an activity afterwards. Our approach is based on previous work by Schatteman et al. (2008) and Van de Vonder (2006) and is similar to the risk-driver approach of Hulett (2009). Contrary to the activity-based approach, we focus on risks as primary sources of uncertainty. In what follows, we adopt an integrated approach that relies on Monte Carlo simulation to evaluate the impact of risks on activity durations and on the project completion time. Figure 2 presents a visual overview from which it is clear that a risk-driven approach assesses the impact of root risks on the uncertainty of the activities and on the project completion time. An activity-based approach on the other hand, assesses only the uncertainty of the activities without observing the root risks that cause this uncertainty.

To further support the risk-driven approach, we provide the following example. Consider an activity whose duration is impacted by two risks. The first risk has a small impact yet a large probability of occurrence whereas the second risk has a large impact but a small probability of occurrence. The probability distribution of the duration of the activity is presented in Figure 3. From the figure it is clear that fitting a distribution would result in significant errors (the best fit of the triangular distribution is indicated by the dotted line). In addition, it would be very hard for practitioners to estimate the parameters of the fitted distribution. Assessing the probability of occurrence and the impact of both risks on the other hand, would be a manageable task and would result in the correct distribution of the duration of the activity.

In order to formally define risks and their impacts, let $R = \{1, 2, \ldots, r\}$ denote the set of risks and let $\mathbf{M} = \{M_{j,e} | j \in N \wedge e \in R\}$ denote the set of risk impacts, where $M_{j,e}$ is the random variable of the risk impact of a risk

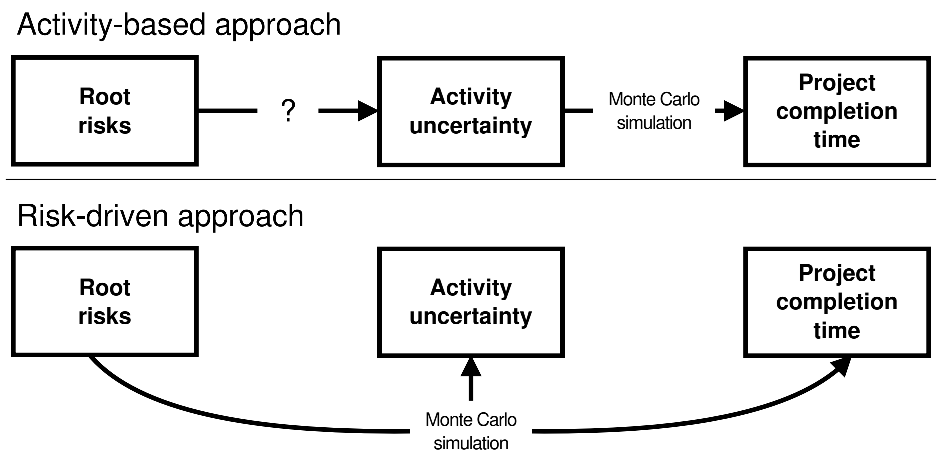

Figure 2: Activity-based versus risk-driven approach

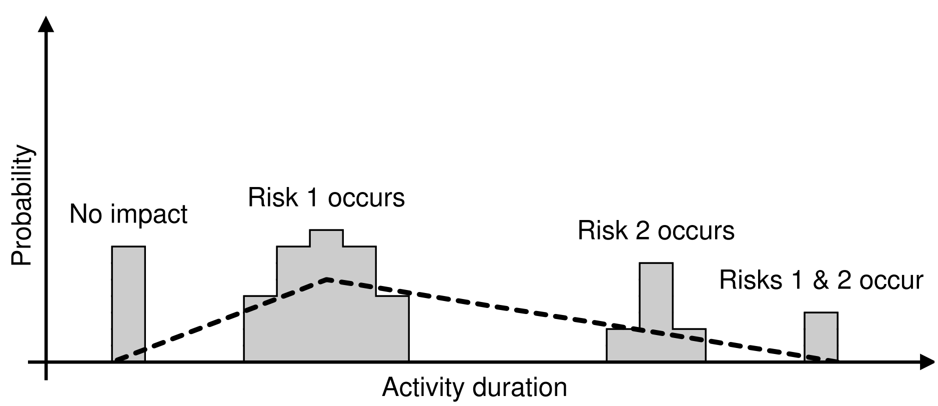

Figure 3: Example distribution of the duration of an activity

e on the duration of an activity j. Let $\mathbf{m}_{j,e}$ represent the vector of random variates of $M_{j,e}$ and define $\mathbf{d}_j^{(E)} = \left\{d_{j,1}^{(E)}, d_{j,2}^{(E)}, \ldots, d_{j,q}^{(E)}\right\}$, the vector of random variates of the duration of an activity j subject to a set of risks $E \subseteq R$:

$$\mathbf{d}_j^{(E)} = d_j + \sum_{e \in E} \mathbf{m}_{j,e}, \tag{7}$$

where d_j is the deterministic (i.e. risk-free) duration of an activity j. From $\mathbf{d}_j^{(E)}$ we obtain $\mathbf{s}_j^{(E)} = \left\{s_{j,1}^{(E)}, s_{j,2}^{(E)}, \ldots, s_{j,q}^{(E)}\right\}$, $\mathbf{f}_j^{(E)} = \left\{f_{j,1}^{(E)}, f_{j,2}^{(E)}, \ldots, f_{j,q}^{(E)}\right\}$ and $\mathbf{c}^{(E)} = \left\{c_1^{(E)}, c_2^{(E)}, \ldots, c_q^{(E)}\right\}$ by generalizing Equations 4, 5 and 6:

$$\mathbf{s}_j^{(E)} = \max\left\{\mathbf{f}_i^{(E)} | (i,j) \in A\right\}, \tag{8}$$

$$\mathbf{f}_j^{(E)} = \mathbf{s}_j^{(E)} + \mathbf{d}_j^{(E)}, \tag{9}$$

$$\mathbf{c}^{(E)} = \mathbf{f}_n^{(E)}. \tag{10}$$

The expected project delay over q simulation iterations is defined as follows:

$$\Delta^{(E)} = \frac{1}{q} \sum_{p=1}^{q} c_p^{(E)} - c, \tag{11}$$

where c is the risk-free project completion time and is computed using Equation 3.

4 Effective risk mitigation

Most commercial risk analysis software packages provide the functionality to generate insight into the source of project overruns. The activities (or the risks) that contribute most to the project overrun are identified using ranking indices. Let $(\cdot)_j^{(E)}$ and $(\cdot)_e^{(E)}$ denote the ranking values of a ranking index $(\cdot)$ for an activity j and a risk e when activity durations are subject to a set of risks E. The larger the ranking value, the larger the contribution of the activity (or the risk) to the project overrun. The ranking of activities (or risks) is typically visualized using a tornado graph (see Figure 4 for an example of a tornado graph).

In the remainder of this Section, we will first provide an overview of the existing ranking indices. Next, we will introduce two new ranking indices that will be compared with the existing ones in a computational experiment.

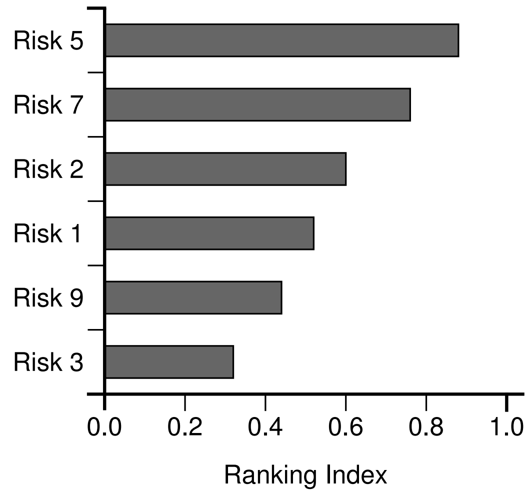

Figure 4: Tornado graph

4.1 Literature review

In this Section, we provide an overview of the existing ranking indices to determine the contribution of activities (or risks) to the project overrun. We refer to Elmaghraby (2000) and Demeulemeester and Herroelen (2002) for a more detailed discussion on the ranking indices discussed below.

4.1.1 Critical Activities (*CA*)

A common practice in project risk management is to focus mitigation efforts on the critical activities of the deterministic early start schedule $\mathfrak{s}$ (Goldratt 1997). The Critical Activities (*CA*) ranking values are computed as follows:

$$\mathrm{CA}_j^{(E)} = \delta_j, \tag{12}$$

where δ_j equals 1 if j is critical in $\mathfrak{s}$ and 0 otherwise.

While easy to implement, *CA* does not recognize the uncertain nature of a project. In addition, all activities on the critical chain have an equal ranking value, thereby severely limiting the discriminative power of the ranking index.

4.1.2 Activity Criticality Index (*ACI*)

In stochastic CPM, the critical path is not fixed. For instance, the occurrence of risks may alter the critical path in a given network. The Activity Criticality Index (*ACI*) recognizes that almost any path and any activity can become critical with a certain probability (Van Slyke 1963). When using Monte Carlo simulation, the *ACI* of an activity is simply the proportion of simulation iterations in which the activity was critical:

$$\mathrm{ACI}_j^{(E)} = \frac{1}{q}\sum_{p=1}^{q}\delta_{j,p}^{(E)}, \tag{13}$$

where $\delta_{j,p}^{(E)}$ equals 1 if j is critical in $\mathfrak{s}_p^{(E)}$ and 0 otherwise ($\mathfrak{s}_p^{(E)}$ is the early start schedule during a simulation iteration p when activity durations are subject to a set of risks E).

4.1.3 Significance Index (*SI*)

The Significance Index (*SI*) was developed by Williams (1992) as an answer to criticism on *ACI*. When using Monte Carlo simulation, *SI* is computed as follows:

$$\begin{aligned}\mathrm{SI}_j^{(E)} &= \frac{1}{q}\left(\frac{1}{\sum\limits_{p=1}^{q} c_p^{(E)}}\right)\left[\sum_{p=1}^{q}\left(\frac{d_{j,p}^{(E)}}{d_{j,p}^{(E)}+\mathrm{TF}_{j,p}^{(E)}}c_p^{(E)}\right)\right], &(14)\\ &= E\left[\frac{\mathbf{d}_j^{(E)}}{\mathbf{c}^{(E)}\mathbf{d}_j^{(E)}\mathbf{TF}_j^{(E)}}\right], &(15)\end{aligned}$$

where $\mathrm{TF}_{j,p}^{(E)}$ is the total float of an activity j during a simulation iteration p when activity durations are subject to a set of risks E and $\mathbf{TF}_j^{(E)} = \left\{\mathrm{TF}_{j,1}^{(E)}, \mathrm{TF}_{j,2}^{(E)}, \ldots, \mathrm{TF}_{j,q}^{(E)}\right\}$ (refer to Demeulemeester and Herroelen (2002) for a definition of total float).

4.1.4 Cruciality Index (*CRI*)

The cruciality index (*CRI*) is defined as the absolute value of the correlation between the duration of an activity and the total project duration. When using Monte Carlo simulation, *CRI* is computed as follows:

$$\mathrm{CRI}_j^{(E)} = \left|\mathrm{corr}\left(\mathbf{d}_j^{(E)}, \mathbf{c}^{(E)}\right)\right|. \tag{16}$$

4.1.5 Spearman Rank Correlation (*SRC*)

Cho and Yum (1997) have criticized *CRI* because it assumes a linear relationship between the duration of an activity and the project completion time. They propose the use of a non-linear correlation measure such as the Spearman rank correlation coefficient. The Spearman Rank Correlation index (*SRC*) is computed as follows:

$$\mathrm{SRC}_j^{(E)} = \left|\mathrm{corr}\left(\mathrm{rank}\left(\mathbf{d}_j^{(E)}\right), \mathrm{rank}\left(\mathbf{c}^{(E)}\right)\right)\right|. \tag{17}$$

4.1.6 Schedule Sensitivity Index (*SSI*)

The PMI Body of Knowledge (2008) and Vanhoucke (2010) define a ranking index that combines *ACI* and the variance of $\mathbf{d}_j^{(E)}$ and $\mathbf{c}^{(E)}$. When using Monte Carlo simulation, *SSI* is computed as follows:

$$\mathrm{SSI}_j^{(E)} = \mathrm{ACI}^{(E)} \sqrt{\frac{\mathrm{Var}\left(\mathbf{d}_j^{(E)}\right)}{\mathrm{Var}\left(\mathbf{c}^{(E)}\right)}}. \tag{18}$$

4.1.7 Risk-Driven Ranking Indices

All prior ranking indices have been criticized in the literature (refer to Williams (1992), Elmaghraby (2000) and Cui et al. (2006)) and are primarily designed to rank activities, not risks. In this (and later) Sections we will introduce risk-driven ranking indices.

To the best of our knowledge, Hulett (2009) is the only reference that explicitly refers to a risk-driven ranking index. He proposes a simple adaptation of the *CRI* such that it calculates the absolute correlation between the impact of a risk and the project completion time. When using Monte Carlo Simulation, the Cruciality Index for Risks (*CRIR*) is computed as follows:

$$\mathrm{CRIR}_e^{(E)} = \left|\mathrm{corr}\left(\mathbf{m}_e, \mathbf{c}^{(E)}\right)\right|, \tag{19}$$

where $\left(\mathbf{m}_e = \sum_{j \in N} \mathbf{m}_{e,j}\right)$ and $e \in E$. A similar adaptation may be made with respect to *SRC*:

$$\text{SRCR}_e^{(E)} = \left|\text{corr}\left(\text{rank}\left(\mathbf{m}_e\right), \text{rank}\left(\mathbf{c}^{(E)}\right)\right)\right|. \tag{20}$$

No simple risk-driven adaptation exists for the other activity-based ranking indices (i.e. *CA*, *ACI*, *SI* and *SSI*).

4.2 Two new ranking indices

The aim of the new ranking indices is to redistribute the project delay over the combinations of activities and risks that cause the delay. More formally, the Critical Delay Contribution (CDC) of an activity j and a risk e may be expressed as follows:

$$\text{CDC}_{j,e}^{(E)} = \frac{1}{q}\frac{\sum_{p=1}^{q} m_{j,e,p}\delta_{j,p}^{(E)}\left(c_p^{(E)} - c\right)}{\sum_{j \in N}\sum_{e \in E}\sum_{p=1}^{q} m_{j,e,p}\delta_{j,p}^{(E)}}, \tag{21}$$

$$= E\left[\frac{\mathbf{m}_{j,e}\mathbf{y}_j^{(E)}}{\sum_{j \in N}\sum_{e \in E}\mathbf{m}_{j,e}\mathbf{y}_j^{(E)}}\right]\Delta^{(E)}, \tag{22}$$

where $\mathbf{y}_j^{(E)} = \left\{\delta_{j,1}^{(E)}, \delta_{j,2}^{(E)}, \ldots, \delta_{j,q}^{(E)}\right\}$.

From $\text{CDC}_{j,e}^{(E)}$ it is easy to obtain both an activity-based as well as a risk-driven ranking index:

$$\text{CDCR}_e^{(E)} = \sum_{j \in N} \text{CDC}_{j,e}^{(E)}, \tag{23}$$

$$\text{CDCA}_j^{(E)} = \sum_{e \in E} \text{CDC}_{j,e}^{(E)}. \tag{24}$$

CDCA and *CDCR* are both new ranking indices whose dynamics are best explained using an example. Consider the project presented in Figure 5. The project has three non-dummy activities (i.e. the start and the completion of the project are represented by activity 1 and 5 respectively). The risk-free activity durations are $(d_2 = 2)$; $(d_3 = 3)$ and $(d_4 = 6)$. Precedence relations

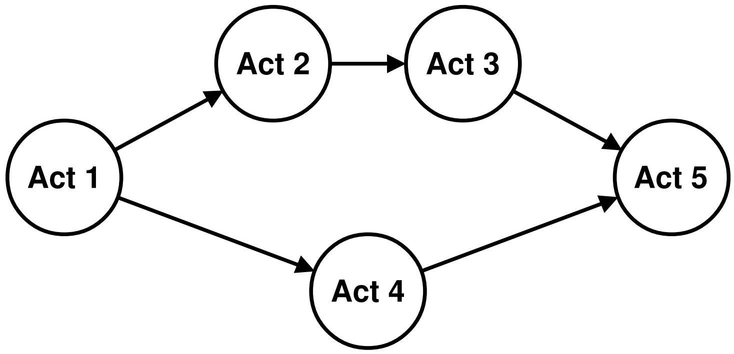

Figure 5: Example project network

Activity	Risk	$m_{j,e,p}\left(\frac{m_{j,e,p}\delta_{j,p}^{(E)}\left(c_p^{(E)}-c\right)}{\sum_{j\in N}\sum_{e\in E} m_{j,e,p}\delta_{j,p}^{(E)}}\right)$ $p=1$	$p=2$	$p=3$	CDC
$j=2$	$e=1$	$+1(0)$	$+1(0.66)$	$+1(0.75)$	$\text{CDC}_{2,1}^{(E)}=0.47$
	$e=2$	$-$	$+2(1.33)$	$-$	$\text{CDC}_{2,2}^{(E)}=0.44$
$j=3$	$e=3$	$-$	$-$	$+3(2.25)$	$\text{CDC}_{3,3}^{(E)}=0.75$
$j=4$	$e=1$	$+1(1)$	$+1(0)$	$+1(0)$	$\text{CDC}_{4,1}^{(E)}=0.33$
$c_p^{(E)}$		7	8	9	$E\left[\mathbf{c}^{(E)}\right]=8$
$c_p^{(E)}-c$		1	2	3	$\Delta^{(E)}=2$

Table 1: Computing the CDC

(finish-start) exist between activities 1 and 2, activities 1 and 4, activities 2 and 3, activities 3 and 5 and activities 4 and 5. The risk-free project completion time is ($c=6$). Three risks have been identified and their respective risk impacts are presented in Table 1. For the sake of simplicity, we consider only three simulation iterations (i.e. q equals 3). For example, we observe that activities 2 and 3 are critical during simulation iteration ($p=2$) and that the critical path has a length of eight time units, resulting in a project delay of two time units (i.e. $c_p^{(E)}-c=8-6=2$). As a consequence, during simulation iteration ($p=2$), the CDC of risk 1 on activity 2 equals $\left(\frac{1(8-6)}{1+2}={}^2\!/_3\right)$.

	Risk 1	Risk 2	Risk 3	$\text{CDCA}_j^{(E)}$
Act 2	0.47	0.45	–	0.92
Act 3	–	–	0.75	0.75
Act 4	0.33	–	–	0.33
$\text{CDCR}_e^{(E)}$	0.8	0.45	0.75	$\Delta^{(E)} = 2$

Table 2: Aggregation of CDC

Table 2 illustrates how the numbers in Table 1 may be aggregated both at the level of each activity as well as at the level of each risk, resulting in ranking values $\text{CDCA}_j^{(E)}$ and $\text{CDCR}_e^{(E)}$ respectively. Similarly to the other activity-based ranking indices, *CDCA* ranks activities. The performance of both *CDCA* and *CDCR* will be evaluated in the upcoming Section.

5 Computational Experiment

Contrary to most of the literature, we will not assess the performance of ranking indices by means of counterexamples. Our goal is to evaluate the resilience of ranking indices in a wide variety of settings, using an extensive experimental design. At the core of our experimental design are the PSPLIB J120 project networks (Kolisch and Sprecher 1996). For each of these networks and for each of the 48 distinct risk profiles defined below, we will evaluate the mitigation potential of the ranking indices discussed in Section 4. A similar approach is followed in Vanhoucke (2010), who considers only activity-based ranking indices.

In what follows, we will first discuss the experimental design itself. Next we deal with the experimental setup and finally, we present the results in Section 5.3.

5.1 Experimental design

For each of the projects in the PSPLIB J120 data set, uncertainty is introduced by modeling a number of risks. Five parameters were selected to characterize the risks: (1) risk uniformity; (2) risk quantity; (3) risk probability; (4) risk impact and (5) risk correlation. The settings of these parameters are based on our experience in the risk management field.

Risk uniformity deals with the number of activities that are impacted by a single risk. Often, clusters of activities have a similar task content and hence are subject to similar risks. We refer to these clusters of activities as activity groups (Schatteman et al. 2008). When risk uniformity is low, the number of activities impacted by any risk $e \in R$ follows a discrete uniform distribution with minimum and maximum equal to 1 and 3 activities respectively. A low risk uniformity setting results in an average of 60 activity groups in a project network. The average number of activities in an activity group equals 2. When risk uniformity is high, the number of activities impacted by any risk $e \in R$ follows a discrete uniform distribution with minimum and maximum equal to 1 and 11 activities respectively. A high risk uniformity setting corresponds to an average of 20 activity groups in a project network whereas the average number of activities in an activity group equals 6.

Risk quantity indicates the number of risks that are identified during the risk identification process. A low risk quantity setting corresponds to a project in which activities are impacted by 25 risks. When risk quantity is high, 50 risks impact the activities of a project. Risks are randomly assigned to a single activity group.

Risk probability indicates the probability of occurrence of a risk whereas *risk impact* defines the impact of a risk on the duration of an activity. We define two types of risks: (1) risks with a large impact but with a small probability of occurrence and (2) risks with a small impact but with a large probability of occurrence. Risks are randomly assigned a risk type, where each risk has a 25 percent chance of being of type 1 (as such, risks have a 75 percent chance of being of type 2). For both risk types, we allow for high and low settings of risk probability and risk impact. Table 3 presents the relevant parameter settings. Remark that: (1) the impact of a risk is modeled as a proportional extension of the duration of an activity and follows a triangular distribution and (2) the risk probability is modeled using a continuous uniform distribution.

Risk correlation indicates whether the occurrences of a risk (on activities in the impacted activity group) are correlated. We investigate three possible scenarios. A first scenario deals with the setting in which there is perfect correlation (i.e. either all activities in the activity group are impacted or none are). The second scenario, assumes that risk occurrences are independent (i.e. there is no correlation between risk occurrences). In a third scenario, we assume that the risk correlation is random, indicating that the occurrences of a risk are correlated with a random correlation factor that is drawn from

Risk Probability	Risk Impact	Risk Type	probability min	probability max	impact min	impact most likely	impact max
High	High	Type 1	0.05	0.05	1.0	2.0	9.0
		Type 2	0.1	0.7	0.0	1.0	2.0
High	Low	Type 1	0.05	0.05	0.5	1.0	4.5
		Type 2	0.1	0.7	0.0	0.5	1.0
Low	High	Type 1	0.025	0.025	1.0	2.0	9.0
		Type 2	0.05	0.35	0.0	1.0	2.0
Low	Low	Type 1	0.025	0.025	0.5	1.0	4.5
		Type 2	0.05	0.35	0.0	0.5	1.0

Table 3: Parameter settings for risk probability and risk impact

a continuous uniform distribution with minimum and maximum equal to 0 and 1 respectively.

The possible settings of the five parameters combine to 48 distinct risk profiles that are to be evaluated. For each risk profile and over all project networks in the PSPLIB J120 data set, we will evaluate the performance of the ranking indices discussed in Section 4.

5.2 Experimental setup

We test the mitigation potential of each ranking index using a stepwise procedure. In each step, the selected ranking index is used to identify the risk that contributes most to the delay of the project. Next, this risk is eliminated (i.e. is fully mitigated). After mitigation, we rerun the simulation and recalculate the expected project delay. Once more, the selected ranking index is used to identify and to mitigate the risk that has the largest impact on the project delay. This process continues until all risks have been mitigated. More formally, let $E\left(\cdot\right)_x$ denote the set of risks after mitigation of x risks using ranking index $(\cdot)$, with $(E\left(\cdot\right)_0 = R)$, $(E\left(\cdot\right)_r = \emptyset)$ and $x \in \{0, 1, \ldots, r\}$. An outline of the procedure is provided in Algorithm 1.

In our experiment, we evaluate a total of 12 ranking indices. The ten ranking indices discussed in Section 4 (*CA*, *ACI*, *SI*, *CRI*, *SRC*, *SSI*, *CRIR*, *SRCR*, *CDCA* and *CDCR*) as well as two additional ranking indices: (1) *RAND* randomly selects a risk from those risks still active and may be considered as a worst-case scenario and (2) *OPT* is a greedy optimal ranking index that evaluates the elimination of all risks after each simulation run and selects the best risk to mitigate. *OPT* may be considered as a best-case scenario but has limited practical value due to its computational requirements.

Algorithm 1 Computational experiment

```
for all Ranking indices (·) do
  for all Project networks in the PSPLIB J120 data set do
    for all Risk uniformity settings do
      Assign activities to activity groups
      for all Risk quantity settings do
        Set r and define R = {1, 2, ..., r}
        for all Risk probability settings do
          for all Risk impact settings do
            for e = 1 to r do
              Set the probability and impact of each risk
            end for
            for all Risk correlation settings do
              Set the correlation of risk occurrences
              Set x = 0 and let E(·)_0 = R
              while x < r do
                for p = 1 to q do
                  Compute 𝔰_p^(E(·)_x) and determine c_p^(E(·)_x)
                end for
                Compute Δ^(E(·)_x) using Equation 11
                From Δ^(E(·)_x) compute performance measure RRD^(E(·)_x) using
                Equation 25 for the current combinations of risk parameter settings
                if (·) is activity-based then
                  For each activity j compute ranking value (·)_j^(E(·)_x) using Equation 24 and Equations 12 − 18
                  Select the highest ranked activity and use the two-step procedure
                  outlined in Section 5.2 to identify e*, the best risk to mitigate
                else
                  For each risk e compute ranking value (·)_e^(E(·)_x) using Equations 19 − 23
                  Select the highest ranked risk e*
                end if
                Mitigate risk e* (i.e. E(·)_{x+1} = E(·)_x \ e*)
                Increment x
              end while
            end for
          end for
        end for
      end for
    end for
  end for
end for
```

Realized impacts					
		Risk			
$m_{j,e,p}$		1	2	3	$\sum_{j \in N} m_{j,e,p}$
Act	1	2	1	1	4
	2	0	0	2	2
	3	0	0	0	0
$\sum_{e \in E} m_{j,e,p}$		2	1	3	6
Expected impacts					
		Risk			
$E\left[\mathbf{m}_{j,e}\right]$		1	2	3	$\sum_{j \in N} E\left[\mathbf{m}_{j,e}\right]$
Act	1	2	1	1	4
	2	0	0	1	1
	3	0	0	1	1
$\sum_{e \in E} E\left[\mathbf{m}_{j,e}\right]$		2	1	3	6

Table 4: Example activity risk impact matrix

With respect to the activity-based ranking indices, selecting the largest risk is a two-step procedure. In a first step, the highest-ranked activity is selected. In a second step, the risk that has the largest expected impact on the selected activity is identified as the highest-ranked risk. For instance, observe the matrix of realized (during a simulation iteration p) and expected risk impacts presented in Table 4. It is clear that activity 1 has the largest realized impact over all simulations. Risk 1 has the largest expected impact on activity 1 and hence is selected as the risk that contributes most to the project overrun (i.e. risk 1 is the highest-ranked risk). It is however clear that risk 3 in fact has the most severe impact on the durations of the different activities.

In order to evaluate the performance of the different ranking indices, we will use Monte Carlo simulation. The simulation experiment was coded in Visual C++ and was executed on a Pentium IV 2.67 GHz personal computer. To obtain statistically significant results, we simulated the execution of a project 1000 times: (1) for each of the 600 projects in the PSPLIB J120 data-set; (2) for each of the 48 risk profiles; (3) for each of the 12 ranking indices and (4) for each step in the mitigation process (i.e. depending on the risk quantity, either 25 or 50 risks are mitigated). In total, almost 13 billion simulation iterations were performed.

5.3 Computational results

In order to compare the performance of the ranking indices, define $\mathrm{RRD}^{(E(\cdot)_x)}$ as the Relative Residual Delay after mitigation of x risks using ranking index $(\cdot)$:

$$\mathrm{RRD}^{(E(\cdot)_x)} = \frac{\Delta^{(E(\cdot)_x)}}{\Delta^{(E(\cdot)_0)}}, \tag{25}$$

where $\Delta^{(E(\cdot)_0)}$ is the expected project delay before any mitigation takes place. It is clear that a smaller value for $\mathrm{RRD}^{(E(\cdot)_x)}$ corresponds to a more effective ranking index.

Another measure to assess the performance of a ranking index $(\cdot)$ is the Mitigation Efficiency Index ($\mathrm{MEI}^{(\cdot)}$). $\mathrm{MEI}^{(\cdot)}$ is defined as follows:

$$\mathrm{MEI}^{(\cdot)} = 1 - 2\frac{\sum_{x=1}^{r} \mathrm{RRD}^{(E(\cdot)_x)}}{r-1} \tag{26}$$

The details of the dynamics of this measure may be found in the Appendix. In short, $\mathrm{MEI}^{(\cdot)}$ is supported on the $[-1, 1]$ real interval, where a value of ($\mathrm{MEI}^{(\cdot)} = 0$) indicates that the performance of the ranking index equals that of the random procedure. A value of ($\mathrm{MEI}^{(\cdot)} = 1$) on the other hand, refers to the optimal case in which mitigating a single risk is sufficient to resolve all project uncertainty. It is clear that a value of ($\mathrm{MEI}^{(\cdot)} = 1$) is unattainable in general.

Figure 6 gives an overview of the average performance of the activity-based ranking indices with respect to measure $\mathrm{RRD}^{(E(\cdot)_x)}$ for the range starting from ($x = 0$) until ($x = 10$) (i.e. ten risks have been mitigated). The data are aggregated over all 600 project networks in the PSPLIB J120 data sets and over all 48 risk profiles. We observe that the mitigation of risks results in a decrease of the expected project delay for each ranking index. Because *RAND* randomly selects risks, its improvement is linear with the number of risks mitigated. All other ranking indices follow a convex function, implying that risks with a larger impact on the project delay are selected first. One might conclude that *CDCA* (the activity-based ranking index proposed in this article) outperforms all other activity-based ranking indices. However, it is clear that there still exists a gap between the performance of the activity-based indices and the *OPT* procedure.

Similarly to Figure 6, Figure 7 presents the performance of risk-driven ranking indices with respect to measure $\mathrm{RRD}^{(E(\cdot)_x)}$. We observe that *CRIR* and *SRCR* have similar performance and are able to outperform the activity-based ranking indices. Of larger importance, however, is the observation that *CDCR* (the risk-driven ranking index proposed in this article) easily outperforms *CRIR* and *SRCR* and even matches the performance of the *OPT* procedure. It is clear that *CDCR* sets a new standard in the field of ranking indices.

Table 5 presents the performance of the different ranking indices with respect to measure $\mathrm{MEI}^{(\cdot)}$. We observe that $\mathrm{MEI}^{(\mathrm{RAND})}$ is close to zero, indicating that the *RAND* procedure has no real mitigation potential. The *OPT* procedure boosts the highest values of $\mathrm{MEI}^{(\cdot)}$ and is rivalled only by *CDCR*. Virtually no difference exists between the performance of the *OPT* procedure and the *CDCR* ranking index. With respect to the activity-based ranking indices, it is clear that *CDCA* takes the pole position.

Furthermore, we observe that risk correlation seems to have a very limited impact on the performance of the ranking indices (certainly for those ranking indices that perform well). Only with respect to *SRCR* (the risk-driven ranking index that adopts a Spearman correlation coefficient) the correlation of the occurrences of risks makes a difference. More specifically, an increased correlation in the occurrence of risks results in a lower $\mathrm{MEI}^{(\mathrm{SRCR})}$. As such, *SRCR* is less robust when compared to the other ranking indices. Similar conclusions hold for risk uniformity and risk probability. Their impact on the performance of ranking indices is subtle to non-existing. Risk quantity on the other hand substantially affects the $\mathrm{MEI}^{(\cdot)}$ of the different ranking indices. We observe that the identification of more risks results in a decreased performance (i.e. if there are more risks, the mitigation of a single risk tends to be less effective). Risk impact has a negative effect on the $\mathrm{MEI}^{(\cdot)}$ of a ranking index. Lower risk impacts correspond to higher values of $\mathrm{MEI}^{(\cdot)}$ (i.e. the relative effect of mitigating a risk increases if there are only few risks that impact project objectives).

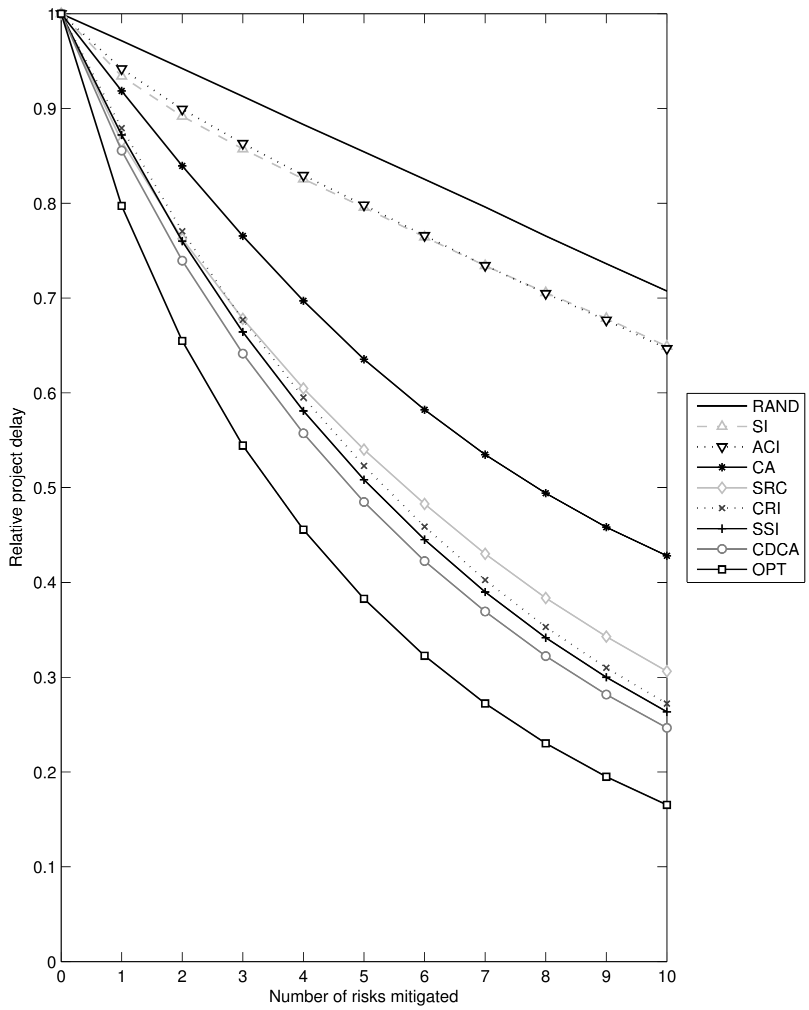

Figure 6: Mitigation potential of activity-based ranking indices

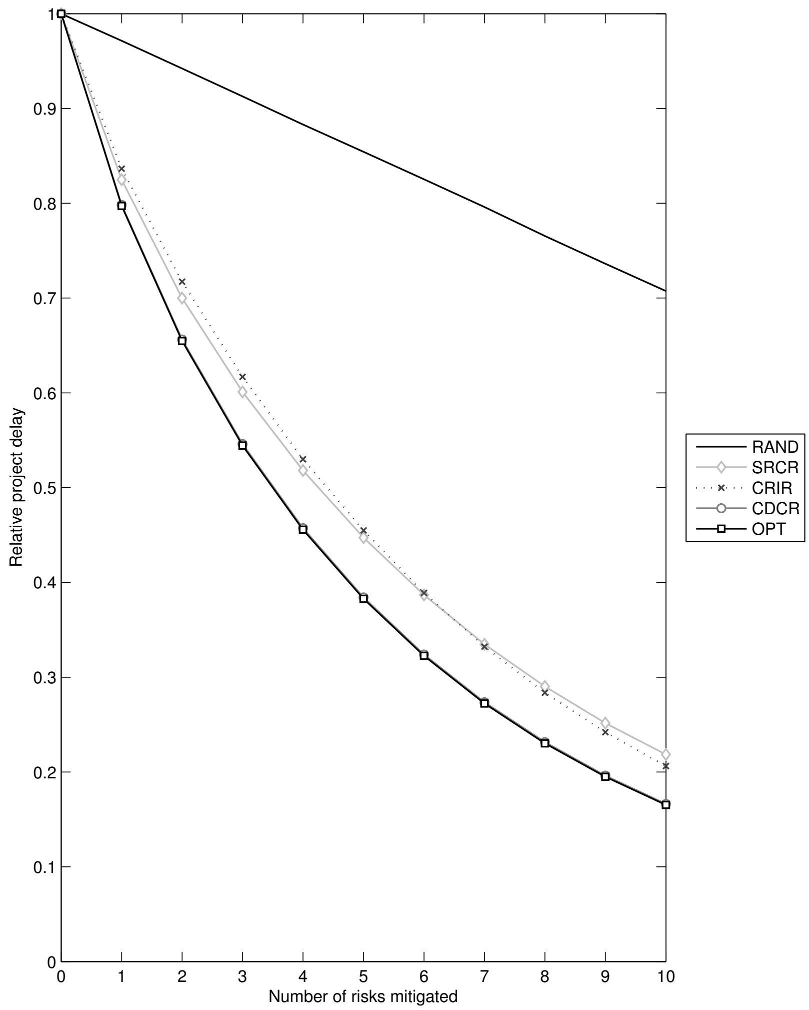

Figure 7: Mitigation potential of risk-driven ranking indices

Index	Avg	Corr	MEI$^{(\cdot)}$															
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	-.02	0 %	-.04	-.02	-.05	-.03	-.03	-.02	-.01	-.01	-.04	-.02	-.02	-.01	-.01	-.00	-.01	-.01
RAND	-.02	RND	-.04	-.02	-.02	-.01	-.03	-.02	-.01	-.00	-.02	-.01	-.02	-.02	-.02	-.01	-.01	-.00
	-.01	100 %	-.03	-.01	-.01	-.00	-.01	-.00	-.01	-.00	-.00	-.00	-.01	-.01	-.01	.00	.01	-.01
	.74	0 %	.64	.67	.66	.69	.76	.81	.78	.82	.66	.69	.68	.70	.79	.83	.80	.84
OPT	.73	RND	.63	.66	.65	.68	.76	.81	.78	.82	.65	.69	.67	.70	.78	.83	.79	.83
	.73	100 %	.63	.66	.64	.68	.76	.81	.77	.82	.64	.68	.66	.69	.78	.83	.79	.83
	.13	0 %	.09	.12	.10	.13	.10	.15	.11	.16	.10	.13	.12	.15	.13	.18	.13	.18
CA	.13	RND	.08	.11	.10	.13	.10	.15	.11	.16	.10	.13	.12	.14	.13	.17	.13	.18
	.13	100 %	.08	.11	.11	.13	.09	.14	.11	.16	.10	.13	.12	.14	.12	.17	.13	.18
	.07	0 %	.01	.07	.03	.08	.03	.09	.05	.10	.03	.08	.04	.09	.06	.12	.08	.14
ACI	.08	RND	.02	.07	.04	.08	.03	.09	.05	.10	.06	.10	.06	.10	.08	.14	.08	.13
	.09	100 %	.05	.09	.06	.10	.05	.10	.04	.10	.07	.11	.07	.11	.07	.13	.08	.13
	.06	0 %	.03	.07	.04	.08	.04	.08	.04	.08	.03	.08	.04	.08	.05	.09	.07	.12
SI	.08	RND	.04	.08	.05	.08	.04	.08	.04	.08	.07	.09	.06	.09	.08	.13	.07	.12
	.09	100 %	.08	.10	.08	.10	.05	.09	.05	.09	.09	.11	.08	.10	.07	.10	.08	.10
	.62	0 %	.44	.49	.47	.52	.66	.73	.69	.75	.51	.55	.53	.57	.72	.77	.73	.78
CRI	.62	RND	.44	.49	.47	.52	.67	.73	.69	.75	.51	.55	.53	.57	.71	.77	.72	.78
	.62	100 %	.45	.50	.47	.52	.66	.73	.68	.74	.50	.55	.53	.58	.71	.77	.73	.78
	.61	0 %	.47	.53	.47	.52	.68	.74	.66	.72	.53	.57	.48	.52	.70	.75	.66	.74
SRC	.60	RND	.46	.52	.46	.52	.67	.73	.65	.71	.52	.56	.47	.51	.68	.73	.65	.72
	.59	100 %	.45	.51	.45	.51	.65	.72	.64	.70	.51	.55	.46	.50	.67	.72	.64	.71
	.63	0 %	.46	.52	.48	.54	.68	.74	.68	.75	.53	.57	.53	.58	.72	.78	.72	.78
SSI	.63	RND	.46	.51	.48	.54	.68	.75	.69	.76	.52	.57	.53	.58	.71	.77	.71	.77
	.62	100 %	.46	.51	.48	.54	.68	.74	.68	.75	.50	.55	.52	.57	.72	.77	.71	.77
	.65	0 %	.46	.53	.48	.55	.70	.77	.72	.78	.54	.59	.56	.60	.75	.80	.76	.81
CDCA	.65	RND	.45	.52	.48	.54	.70	.77	.72	.78	.53	.59	.55	.60	.75	.80	.76	.81
	.65	100 %	.46	.53	.48	.55	.70	.76	.72	.78	.53	.58	.55	.60	.75	.80	.76	.81
	.69	0 %	.57	.61	.60	.63	.72	.77	.74	.78	.60	.62	.63	.65	.74	.79	.76	.80
CRIR	.69	RND	.57	.60	.60	.63	.72	.77	.74	.79	.59	.63	.62	.65	.74	.79	.75	.80
	.69	100 %	.57	.61	.59	.63	.72	.77	.74	.79	.59	.62	.61	.65	.75	.80	.76	.80
	.70	0 %	.59	.63	.63	.66	.74	.78	.75	.78	.63	.66	.66	.67	.76	.79	.74	.78
SRCR	.66	RND	.56	.60	.59	.62	.70	.74	.71	.74	.59	.62	.62	.64	.71	.75	.70	.74
	.63	100 %	.53	.56	.56	.59	.66	.70	.67	.69	.56	.59	.59	.60	.67	.71	.66	.70
	.74	0 %	.63	.67	.66	.69	.76	.81	.78	.82	.66	.69	.68	.70	.79	.83	.79	.83
CDCR	.73	RND	.63	.66	.65	.68	.76	.81	.77	.82	.65	.68	.67	.69	.78	.83	.79	.83
	.73	100 %	.62	.66	.64	.68	.76	.81	.77	.82	.64	.68	.66	.69	.78	.83	.79	.83
Risk uniformity			High								Low							
Risk quantity			High				Low				High				Low			
Risk probability			High		Low		High		Low		High		Low		High		Low	
Risk impact			H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L

Table 5: Mitigation efficiency of the different ranking indices

6 Conclusions

In this article we introduced a quantitative, new approach to project risk analysis that allows to address the risk response process in a scientifically-sound manner. We have shown that a risk-driven approach is more efficient than an activity-based approach when it comes to analyzing risks. Therefore, project risk management should focus on assessing the uncertainty caused by risks themselves (i.e. the root cause) rather than evaluating the uncertainty at the level of activities.

In addition, we developed two new ranking indices to assist project managers in determining where to focus their risk mitigation efforts. Ranking indices allow to identify the activities (or risks) that contribute most to the delay of a project (popular ranking indices include the criticality index and the significance index). We developed both an activity-based ranking index (that ranks activities) and a risk-driven ranking index (that ranks risks). We refer to these ranking indices as *CDCA* and *CDCR* respectively. Both ranking indices outperform existing ranking indices, with *CDCR* nearly matching the performance of a greedy-optimal procedure. *CDCR* sets a new standard in the field of ranking indices.

Our conclusions are supported by an extensive simulation experiment and were proven to be robust for a broad range of parameter settings. The contributions of this article may be summarized as follows: (1) we assess the performance of a wide variety of ranking indices using a large simulation experiment; (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that risk analysis should be risk-driven rather than activity-based.

Appendix

The efficiency of a ranking index may be seen as its ability to correctly identify those risks that have the largest impact on project objectives. As such, for any good ranking index the following holds: $\mathrm{RRD}^{(E(\cdot)_{x-1})} - \mathrm{RRD}^{(E(\cdot)_x)} \geq \mathrm{RRD}^{(E(\cdot)_x)} - \mathrm{RRD}^{(E(\cdot)_{x+1})}$ (i.e. $\mathrm{RRD}^{(E(\cdot)_x)}$ has to be convex in the interval $x \in [1, r]$). We illustrate this logic in Figure 8. $\mathrm{RRD}^{\left(E_x^{(\cdot)}\right)}$ is convex if:

$$\forall x \in [1, r] : \mathrm{RRD}^{(E(\cdot)_x)} \leq \frac{x\mathrm{RRD}^{(E(\cdot)_r)} + \left[(r - x)\,\mathrm{RRD}^{(E(\cdot)_0)}\right]}{r}. \tag{27}$$

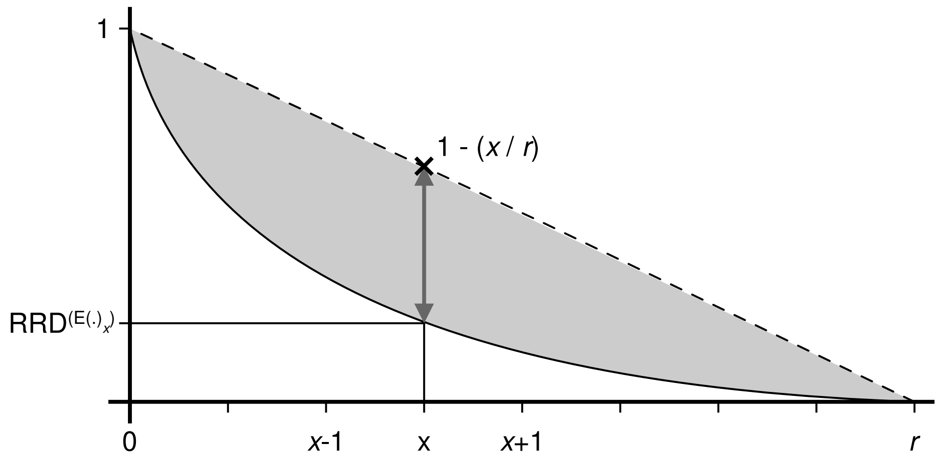

Figure 8: Illustration of mitigation efficiency

Because $\left(\mathrm{RRD}^{(E(\cdot)_r)} = 0\right)$ and $\left(\mathrm{RRD}^{(E(\cdot)_0)} = 1\right)$, the condition translates into:

$$\forall x \in [1, r] : 1 - \frac{x}{r} - \mathrm{RRD}^{(E(\cdot)_x)} \geq 0. \tag{28}$$

To assess the mitigation potential of a ranking index, we want to evaluate the level of convexity of $\mathrm{RRD}^{(E(\cdot)_x)}$. For this purpose, we develop the Mitigation Efficiency Index:

$$\widehat{\mathrm{MEI}}^{(\cdot)} = \sum_{x=1}^{r} 1 - \frac{x}{r} - \mathrm{RRD}^{(E(\cdot)_x)}, \tag{29}$$

$$= \frac{r-1}{2} - \sum_{x=1}^{r} \mathrm{RRD}^{(E(\cdot)_x)}, \tag{30}$$

which corresponds to the surface of the gray area in the graph presented in Figure 8. In order to obtain a relative measure, we divide $\widehat{\mathrm{MEI}}^{(\cdot)}$ by $\left(\frac{r-1}{2}\right)$:

$$\mathrm{MEI}^{(\cdot)} = 1 - 2\frac{\sum_{x=1}^{r} \mathrm{RRD}^{(E(\cdot)_x)}}{r-1}. \tag{31}$$

References

Cho, J.G., & Yum, B.J. (1997). An uncertainty importance measure of activities in PERT networks. *International Journal of Production Research*, *35(10)*, 2737–2758.

Cui, W., Jiajun, Q., & Chaoyuan, Y. (2006). Criticality measurement in PERT networks. *IEEE International Conference on Systems, Man, and Cybernetics*. Taipei, Taiwan.

Demeulemeester, E., & Herroelen, W. (2002). *Project scheduling: A research handbook*. Boston: Kluwer Academic Publishers.

Elmaghraby, S. (2000). On criticality and sensitivity in activity networks. *European Journal of Operational Research*, *127(2)*, 220–238.

Georgieva, K., Farooq, A., & Dumke, R.R. (2009). Analysis of the risk assessment methods A survey. *Proceedings of the International Conferences on Software Process and Product Measurement*. Amsterdam, The Netherlands.

Goldratt, E.M. (1997). *Critical chain*. Great Barrington: The North River Press Publishing Corporation.

Hagstrom, J.N. (1988). Computational complexity of PERT problems. *Networks*, *18(2)*, 139–147.

Hubbard, D.W. (2008). *The failure of risk management: Why it's broken and how to fix it*. New York: Wiley.

Hulett, D.T. (2009). *Practical Schedule Risk Analysis*. England: Gower Publishing Limited.

Kolisch, R., & Sprecher, A. (1996). PSPLIB - A project scheduling problem library. *European Journal of Operational Research*, *96*, 205–216.

Schatteman, D., Herroelen, W., Boone, B., & Van de Vonder, S. (2008). A methodology for integrated risk management and proactive scheduling of construction projects. *Journal of Construction Engineering & Management*, *134(11)*, 885–895.

The Project Management Institute. (2008). *Guide to the project management body of knowledge (PMBOK guide)*. Newton Square: The Project Management Institute.

The Standish Group. (2009). *Chaos report*. Boston: The Standish Group.

Van de Vonder, S. (2006). *Proactive-reactive procedures for robust project scheduling*. PhD Disseration. Leuven: K.U.Leuven.

Sadeghi, N., Fayek, A.R., & Pedrycz, W. (2009). Fuzzy Monte Carlo simulation and risk assessment in construction. *Computer-Aided Civil and Infrastructure Engineering*, *00*, 1–15.

Van Slyke, R.M. (1963). Monte Carlo methods and the PERT problem. *Operations Research*, *11(5)*, 839–860.

Vanhoucke, M. (2010). Using activity sensitivity and network topology information to monitor project time performance. *Omega*, *38(5)*, 359–370.

Williams, T.M. (1992). Criticality in stochastic networks. *The Journal of the Operational Research Society*, *43(4)*, 353–357.