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Short-Run Johansen Frontier-Based Industry Models: Methodological Refinements and Empirical Illustration on Fisheries

Kristiaan Kerstens
IÉSEG School of Management and LEM-CNRS (UMR 9221), Lille, France

Jafar Sadeghi
IÉSEG School of Management and Ivey Business School, Western University, London, Ontario, Canada

Ignace Van de Woestyne
KU Leuven, Research Unit MEES, Brussels, Belgium

John Walden
NOAA/NMFS, Northeast Fisheries Center, United Sates

IÉSEG School of Management
Lille Catholic University
3, rue de la Digue
F-59000 Lille
Tel: 33(0)3 20 54 58 92
www.ieseg.fr

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Kristiaan Kerstens*, Jafar Sadeghi†, Ignace Van de Woestyne‡, John Walden§

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Abstract

This contribution focuses on extending the current state of the art in the short-run Johansen industry model in two ways. First, instead of only considering output-oriented plant capacity, we allow for alternative plant capacity concepts. In particular, we introduce an input-oriented plant capacity concept, and an alternative attainable output-oriented plant capacity concept that corrects a major empirical issue in the traditional output-oriented plant capacity notion. Second, we correct a long-standing issue of the correct choice of weight variables on the capacity distribution by guaranteeing that these weights determine production combinations that belong to the production technology on which the plant capacity estimates are based in the first place. These double methodological refinements are illustrated with a data set on U.S. fishing vessels by developing a planning model to curb overfishing.

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*Corresponding author: IESEG School of Management, CNRS, Univ. Lille, UMR 9221-LEM, 3 rue de la Digue, F-59000 Lille, France, Tel: +33 320545892 (switchboard), Fax: +33 320574855, k.kerstens@ieseg.fr

†IESEG School of Management, 3 rue de la Digue, F-59000 Lille, France & Ivey Business School, Western University, London, Ontario, Canada j.sadeghi1987@gmail.com

‡KU Leuven, Research Unit MEES, Brussels, Belgium, ignace.vandewoestyne@kuleuven.be

§NOAA/NMFS, Northeast Fisheries Science Center, 166 Water St., Woods Hole, MA 02543, United States, john.walden@noaa.gov

1 Introduction

The short-run Johansen (1972) industry or sectoral model has received attention as a planning tool because it allows analysing industry structure on a disaggregated basis from underlying ex post firm-level inputs and a single output. This model starts from a putty-clay model of production and investment decisions: ex-ante firms are free to choose among several production activities exhibiting smooth substitution possibilities, but ex post these firms face fixed coefficient technologies with capacities that are entirely conditioned by the investment decision made. The short-run industry model nevertheless exhibits substitution possibilities when inputs and outputs can be reallocated across the units composing the industry. Over time, substitution and technical change can be traced via shifts in successive short-run industry models. Surveys of this short-run Johansen (1972) industry model are found in Heathfield and Wibe (1987) and Førsund and Vislie (2016).[1]

The short-run industry or ex post macro (Johansen's terminology) model is derived from the short-run ex post firm functions. It is a simple linear programming model with an objective function maximising the sum of firm outputs subject to capacity constraints related to the aggregate levels of inputs. The weight vectors are subject to an upper bound. Empirical applications of this short-run Johansen (1972) industry model include the following examples in chronological order: Førsund, Gaunitz, Hjalmarsson, and Wibe (1980) analyse the Swedish pulp industry, Hildenbrand (1981) studies the Norwegian tanker fleet and the US electric power-generating industry; Førsund and Hjalmarsson (1983) analyse the Swedish cement industry; Førsund and Jansen (1983) reflect upon the Norwegian aluminum industry; Førsund, Hjalmarsson, and Eitrheim (1985) provide an international comparison of the cement industry in the Nordic countries comparing Denmark, Finland, Norway, and Sweden; the last four empirical chapters in Førsund and Hjalmarsson (1987) focus on a variety of sectors; Wibe (1995) studies the Swedish paper industry; Førsund, Hjalmarsson, and Summa (1996) scrutinise the Finnish brewery industry; Førsund, Hjalmarsson, and Zheng (2011) develop an analysis for Chinese coal-fired electricity generation plants, and Førsund, Heshmati, and Wang (2018) study coal-fired electricity generation plants in South Korean and at the aggregate level of Chinese provinces, among others.

Sengupta (1989) and Färe, Grosskopf, and Li (1992) are the first to establish a link between the short-run Johansen (1972) industry model and the frontier-based production theory that focuses on best practice instead of average practice (see also Dosi, Grazzi, Marengo,

[1] The latter survey positions this short-run industry model within the context of the entire oeuvre of Leif Johansen

and Settepanella (2016) for some further links). Average practice analysis focuses on average behaviour, while best practice analysis concentrates on the best performing units on the boundary of the production possibility set. The book chapter of Dervaux, Kerstens, and Leleu (2000) innovates by developing an entirely non-parametric frontier-based approach to the short-run Johansen (1972) industry model. In particular, this work improves two features. First, it transforms the single output case into a multiple outputs frontier framework.[2] Second, it substitutes the somewhat ad hoc specification of a capacity distribution in the traditional short-run Johansen (1972) industry model by a non-parametric output-oriented plant capacity concept introduced in the literature by Färe, Grosskopf, and Kokkelenberg (1989) in the single output case and by Färe, Grosskopf, and Valdmanis (1989) in the multiple output case using a pair of output-oriented efficiency measures inspired by Johansen (1968).[3] Relaxing the single-output restriction substantially enlarges the scope of empirical applications beyond the historically almost exclusive focus on industry studies. Furthermore, the frontier nature allows for a benchmarking perspective when adopting it for social planning purposes.

Empirical applications of this generalised frontier-based short-run Johansen (1972) industry model include the following examples: Dervaux, Kerstens, and Leleu (2000) analyse French surgery units in 1605 hospitals, Kerstens, Moulaye Hachem, Van de Woestyne, and Vestergaard (2010) provide an analysis of a German bank branch network and how it can be restructured, Färe, Grosskopf, Kerstens, Kirkley, and Squires (2001) provide a first study on how to reduce overfishing in the northwest USA Atlantic sea scallop fishery, Kerstens, Squires, and Vestergaard (2005) and Kerstens, Vestergaard, and Squires (2006) develop a plan to curb overfishing in the Danish fishery fleet under a variety of scenarios with quota and fishing days, while Lindebo (2005), Tingley and Pascoe (2005) and Yagi and Managi (2011) develop a similar plan for the North Sea, Scottish and Japanese fishing fleets, among others.

Note that the short-run frontier-based Johansen (1972) industry model is but one example of a stream of literature on central resource allocation models in the frontier framework. Central resource reallocation models cover a heterogeneous variety of models reallocating

[2]However, in the traditional non-frontier literature Dosi, Grazzi, Marengo, and Settepanella (2016, Appendix B) also develop a multiple output-case. To the best of our knowledge, this multi-outputs approach has never been empirically implemented. Also Sengupta (1989, p. 49-50) outlines some possibilities to develop a multiple outputs approach: also these options have never been implemented empirically.

[3]Johansen (1972) introduces the capacity distribution as a mechanism to derive optimal factor proportions in a dynamic setting. He and followers like Muysken (1985) and Seierstad (1985) explicitly introduce the capacity distribution notion as a continuous or discrete or mixed statistical distribution of the input coefficients when plants are used at full capacity.

some inputs and/or outputs across space and/or time while eventually accounting for multiple objectives (e.g., efficiency, effectiveness, equality). To the best of our knowledge Färe, Grosskopf, and Li (1992) and Golany, Phillips, and Rousseau (1993) are among the first frontier-based central resource reallocation models. Other examples of these models can be found in the work by Athanassopoulos (1998), Golany and Tamir (1995), Korhonen and Syrjänen (2004), Lozano and Villa (2004), and Ylvinger (2000), among others. A preliminary survey of a selection of these frontier-based central resource allocation models is found in Mar-Molinero, Prior, Segovia, and Portillo (2014).

The purpose of this contribution is twofold. First, we want to remedy one remaining problem in the short-run Johansen (1972) industry model: while the output-oriented plant capacity concepts is estimated at the extremes of the empirical data range in the technology, there is currently no guarantee that the scaling of these plant capacity inputs and outputs remains technically feasible by remaining within the frontier technology. By contrast, all frontier-based central resource allocation models in the literature do meet this requirement. This problem is illustrated using a numerical example and a general remedy is proposed. Second, another purpose of this contribution is to widen the methodological choices open to the users of the short-run Johansen (1972) industry model by introducing new plant capacity concepts. On the one hand, we follow Cesaroni, Kerstens, and Van de Woestyne (2017) who define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures. On the other hand, we follow up on Kerstens, Sadeghi, and Van de Woestyne (2019b) who argue and empirically illustrate that the traditional output-oriented plant capacity utilization may be unrealistic in that the amounts of variable inputs needed to reach the maximum capacity outputs may simply be unavailable at either the firm or the industry level. This problem is linked to what Johansen (1968) called the attainability issue and therefore Kerstens, Sadeghi, and Van de Woestyne (2019b) define a new attainable output-oriented plant capacity utilization at the firm level. Throughout this contribution, we contrast the traditional average practice-based short-run Johansen (1972) industry model with the more recent frontier-based short-run industry model to highlight both similarities and differences.

This contribution is structured as follows. The next Section 2 defines the basic technology as well as the efficiency measures needed to define the different frontier-based plant capacity concepts. Section 3 defines the traditional output-oriented plant capacity concept as well as the alternative input-oriented plant capacity measure and the attainable output-oriented plant capacity measure. The basic frontier-based short-run Johansen (1972) industry model is discussed in Section 4. This same section also illustrates the problem that the scaling of the plant capacity inputs and outputs need not remain technically feasible by remaining within the technology. Thereafter, Section 5 develops three new short-run Johansen (1972) industry models. First, we develop a revised version of the short-run industry model based on the output-oriented plant capacity that does respect the technology. Second, we introduce two new plant capacity concepts in the short-run Johansen models: either the attainable output-oriented plant capacity utilization, or the input-oriented plant capacity measure. The differences between old and new short-run Johansen (1972) industry models are empirically illustrated in Section 6 using convex and nonconvex technologies. A final Section 7 concludes.

2 Technology: Basic Definitions

This section introduces some basic notation and defines the technology at the firm level. Given an N-dimensional input vector $x \in \mathbb{R}^N_+$ and an M-dimensional output vector $y \in \mathbb{R}^M_+$, the production possibility set or technology T is defined as follows: $T = \{(x, y) | x \text{ can produce } y\}$. Associated with T, the input set denotes all input vectors x capable of producing a given output vector y: $L(y) = \{x | (x, y) \in T\}$. Analogously, the output set associated with T denotes all output vectors y that can be produced from a given input vector x: $P(x) = \{y | (x, y) \in T\}$.

Throughout this contribution, technology T satisfies some combination of the following standard assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0, 0) \in T$ and if $(0, y) \in T$, then $y = 0$.

(T.2) T is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.

(T.3) Strong input and output disposal, i.e., if $(x, y) \in T$ and $(x', y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(x', -y') \geq (x, -y) \Rightarrow (x', y') \in T$.

(T.4) T is convex.

Briefly discussing these traditional axioms on technology, it is useful to recall the following (see, e.g., Hackman (2008) for details). Inaction is feasible, and there is no free lunch. Technology is closed. We assume free disposal of inputs and outputs in that inputs can be wasted and outputs can be discarded. Finally, technology is convex. In our empirical analysis not all these axioms are simultaneously maintained.[4] In particular, key assumption distinguishing some of the technologies in the empirical analysis is convexity versus nonconvexity.

[4]For instance, note that the convex flexible or variable returns to scale technology does not satisfy inaction.

The radial input efficiency measure characterizes the input set $L(y)$ completely and can be defined as follows:

$$DF_i(x, y) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \quad (1)$$

This radial input efficiency measure has the main properties that it is smaller or equal to unity ($DF_i(x, y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity, and that it has a cost interpretation (see, e.g., Hackman (2008)).

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and can be defined as:

$$DF_o(x, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}. \quad (2)$$

Its main properties are that it is larger than or equal to unity ($DF_o(x, y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that this radial output efficiency measure has a revenue interpretation (e.g., Hackman (2008)).

In the short run, we can partition the input vector into a fixed and variable part. In particular, we denote ($x = (x^f, x^v)$) with $x^f \in \mathbb{R}_+^{N_f}$ and $x^v \in \mathbb{R}_+^{N_v}$ such that $N = N_f + N_v$. Similarly, a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}_+^{N_f} \times \mathbb{R}_+^M \mid$ there exists x^v such that (x^f, x^v) can produce at least $y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}_+^{N_f} \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$ can be defined. Note that technology T^f is in fact obtained by a projection of technology $T \subset \mathbb{R}_+^N \times \mathbb{R}_+^M$ into the subspace $\mathbb{R}_+^{N_f} \times \mathbb{R}_+^M$ (i.e., by setting all variable inputs equal to zero).[5] By analogy, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the output set $P^f(x^f)$ by $DF_o^f(x^f, y)$, this output-oriented efficiency measure can be defined as follows:

$$DF_o^f(x^f, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \quad (3)$$

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$DF_{vi}^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \quad (4)$$

The sub-vector input efficiency measure reducing only the fixed inputs is defined as follows:

$$DF_{fi}^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (\lambda x^f, x^v) \in L(y)\}. \quad (5)$$

[5]See Cesaroni, Kerstens, and Van de Woestyne (2019, p. 388 and following) for more details about this projection.

Next, we need the following particular definition of a technology: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level.[6] The sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is as follows:

$$DF_{vi}^{SR}(x^f, x^v, 0) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \tag{6}$$

Having introduced all necessary efficiency measures needed to define the various plant capacity concepts, we now turn to the algebraic definition of the technologies relative to which plant capacities are estimated. Given data on K observations ($k = 1, \cdots, K$) consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under the flexible or variable returns to scale assumption is possible as follows:

$$T^\Lambda = \left\{(x, y) \mid x \geq \sum_{k=1}^{K} z_k x_k, y \leq \sum_{k=1}^{K} z_k y_k, (z_1, \ldots, z_K) \in \Lambda\right\}, \tag{7}$$

where

(i) $\Lambda \equiv \Lambda^{\mathrm{C}} = \left\{(z_1, \ldots, z_K) \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0\right\};$

(ii) $\Lambda \equiv \Lambda^{\mathrm{NC}} = \left\{(z_1, \ldots, z_K) \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0, 1\}\right\}.$

The activity vector $(z_1, \ldots, z_K)$ of real numbers summing to unity represents the convexity axiom. This same sum constraint with each vector element being a binary integer is representing nonconvexity. The convex technology satisfies axioms (T.1) (except inaction) to (T.4), while the nonconvex technology adheres to axioms (T.1) to (T.3). It is now useful to condition the above notation of the efficiency measures relative to these nonparametric frontier technologies by distinguishing between convexity (convention C) and nonconvexity (convention NC). This firm technology allows to compute a series of different frontier-based concepts of plant capacity to which we now turn.

[6] As already pointed out in Cesaroni, Kerstens, and Van de Woestyne (2019, p. 388), $L(0)$ can also be defined as $L(y_{min}) = \{x \mid (x, y_{min}) \in T\}$, whereby $y_{min} = \min_{k=1,\ldots,K} y_k$ takes the minimum in a component-wise manner for every output y over all observations K.

3 Plant Capacity Notions: Definitions

It is common to distinguish between technical or engineering concepts on the one hand and economic capacity concepts on the other hand. Johansen (1968) develops a technical approach by introducing an informally defined plant capacity notion. This informal definition of plant capacity by Johansen (1968, p. 362) reads:"the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." This clearly output-oriented plant capacity notion has been admirably made operational by Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) using a pair of output-oriented efficiency measures. We now recall the definition of this output-oriented plant capacity utilization.

Definition 3.1. The output-oriented plant capacity utilization PCU_o is defined as follows:

$$PCU_o(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)},$$

where $DF_o(x, y)$ and $DF_o^f(x^f, y)$ are output efficiency measures including, respectively excluding, the variable inputs as defined before in (2) and (3).

Since $1 \leq DF_o(x, y) \leq DF_o^f(x^f, y)$, notice that $0 < PCU_o(x, x^f, y) \leq 1$. Thus, output-oriented plant capacity utilization has an upper limit of unity. Following the terminology introduced by Färe, Grosskopf, and Kokkelenberg (1989), one can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f, y)$ and an unbiased plant capacity measure $PCU_o(x, x^f, y)$ depending on whether the measure ignores inefficiency or adjusts for the eventual existence of inefficiency. Taking the ratio of efficiency measures eliminates any existing inefficiency and yields in this sense a cleaned concept of output-oriented plant capacity utilization. Computational issues are discussed in Section 4.

Recently, Kerstens, Sadeghi, and Van de Woestyne (2019b) have argued and empirically illustrated that the output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ may be unrealistic in that the amounts of variable inputs needed to reach the maximum capacity outputs may simply be unavailable at either the firm or the industry level. This is linked to what Johansen (1968) called the attainability issue. Hence, Kerstens, Sadeghi, and Van de Woestyne (2019b) define a new attainable output-oriented plant capacity utilization at the firm level as follows:

Definition 3.2. An attainable output-oriented plant capacity utilization $APCU_o$ at level

$\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x, y)}{ADF_o^f(x^f, y, \bar{\lambda})},$$

where the attainable output-oriented efficiency measure ADF_o^f at a certain level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$ADF_o^f(x^f, y, \bar{\lambda}) = \max\{\varphi \mid \varphi \geq 0, 0 \leq \lambda \leq \bar{\lambda}, \varphi y \in P(x^f, \lambda x^v)\} \tag{8}$$

Again, for $\bar{\lambda} \geq 1$, since $1 \leq DF_o(x, y) \leq ADF_o^f(x^f, y, \bar{\lambda})$, notice that $0 < APCU_o(x, x^f, y, \bar{\lambda}) \leq 1$. Also, for $\bar{\lambda} < 1$, since $1 \leq ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o(x, y)$, notice that $1 \leq APCU_o(x, x^f, y, \bar{\lambda})$.

One can again distinguish between a so-called biased attainable plant capacity measure $ADF_o^f(x^f, y, \bar{\lambda})$ and an unbiased attainable plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$, whereby the latter is cleaned from any eventual inefficiency. Kerstens, Sadeghi, and Van de Woestyne (2019b) pragmatically experiment with values of $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$.[7] Furthermore, these authors note that if expert opinion cannot determine a plausible value, then it may be better to opt for the next input-oriented plant capacity measure that does not suffer from the attainability issue. Computational issues are treated in Section 5 below.

Cesaroni, Kerstens, and Van de Woestyne (2017) define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures.

Definition 3.3. The input-oriented plant capacity utilization (PCU_i) is defined as follows:

$$PCU_i(x, x^f, y) = \frac{DF_{vi}^{SR}(x^f, x^v, y)}{DF_{vi}^{SR}(x^f, x^v, 0)},$$

where $DF_{vi}^{SR}(x^f, x^v, y)$ and $DF_{vi}^{SR}(x^f, x^v, 0)$ are the sub-vector input efficiency measures defined in (4) and (6), respectively.

Since $0 < DF_{vi}^{SR}(x^f, x^v, 0) \leq DF_{vi}^{SR}(x^f, x^v, y)$, notice that $PCU_i(x, x^f, y) \geq 1$.[8] Thus, input-oriented plant capacity utilization has a lower limit of unity. Similar to the previous cases, one can distinguish between a so-called biased plant capacity measure $DF_{vi}^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i(x, x^f, y)$, the latter being cleaned of any prevailing inefficiency. Computational questions are again dealt with in Section 5.

[7]Notice that $\bar{\lambda} < 1$ is added for completeness sake. Normally there is no need to reduce variable inputs below their currently available levels.

[8]Kerstens, Sadeghi, and Van de Woestyne (2019a, Proposition B.1) prove that $DF_{vi}^{SR}(x^f, x^v, 0) = DF_{vi}^{SR}(x^f, x^v, y_{min})$, where y_{min} is as defined supra.

Note that graphical illustrations of plant capacity Definitions 3.1, 3.2 and 3.3 are provided in Appendix A. Note that Cesaroni, Kerstens, and Van de Woestyne (2019) also define an input-based and output-based long-run plant capacity concept whereby both fixed and variable inputs can adjust. Furthermore, Kerstens, Sadeghi, and Van de Woestyne (2019a) empirically illustrate that both engineering and economic capacity concepts differ systematically when estimated using convex and nonconvex technologies.

As earlier stated, the average practice single output short-run industry models suffer in practice from a rather ad hoc specification of capacity distributions (as also admitted in the recent article of Dosi, Grazzi, Marengo, and Settepanella (2016, footnote 13)). It should be stressed that some substantial efforts are available in the literature to derive a more satisfactory solution for this state of affairs: Muysken (1985) develops continuous capacity distribution, while Seierstad (1985) develops any form of the capacity distribution (discrete, continuous, or a mixture). However, it is clear that the above frontier-based technical or engineering plant capacity concepts are quite appealing.

4 Short-run Johansen Industry Model

4.1 Basic Version

Following Dervaux, Kerstens, and Leleu (2000), the focus is on reallocation of production among units by explicitly allowing improvements in technical efficiency and capacity utilisation rates. One can distinguish between two phases. In the first phase one computes capacity inputs and outputs. In the second phase, one constructs the short-run industry model using the parameters obtained from the first phase. As explained below, this short-run industry model does not inherit the properties of the technology used to compute the plant capacity concept.

In the first phase, the short-run output-oriented radial technical efficiency measure $DF_o^f(x_p^f, y_p)$ (i.e., the denominator in Definition 3.1 and introduced in (3)) of firm p, $(p = 1, \ldots, K)$, with fixed inputs $x_p^f \in \mathbb{R}_+^{N_f}$ and outputs $y_p \in \mathbb{R}_+^M$ requires the optimization of the following program:

$$
\begin{aligned}
DF_o^f(x_p^f, y_p) = \quad & \max_{\varphi, z_k, x^v} \quad \varphi \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq \varphi y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v = x^v, \\
& \qquad (z_1, \ldots, z_K) \in \Lambda, \\
& \qquad \varphi \geq 0, x^v \geq 0,
\end{aligned}
\tag{9}
$$

where Λ determines the convex or nonconvex assumption of the technology as defined in (7). Assume that φ^* is the optimal value of short-run output-oriented model (9). To find a solution that maximizes slacks and surpluses, the following model can be solved:

$$
\begin{aligned}
& \max_{S^+, S^-, z_k, x^v} \quad 1_M.S^+ + 1_{N_f}.S^- \\
& s.t \quad \sum_{k=1}^{K} z_k y_k - S^+ = \varphi^* y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f + S^- = x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v = x^v, \\
& \qquad (z_1, \ldots, z_K) \in \Lambda, \\
& \qquad x^v \geq 0, S^+ \geq 0, S^- \geq 0,
\end{aligned}
\tag{10}
$$

with $1_M = (1, \ldots, 1) \in \mathbb{R}^M$ and $1_{N_f} = (1, \ldots, 1) \in \mathbb{R}^{N_f}$. From model (10), an optimal activity vector $z^{p*} = (z_1^{p*}, \ldots, z_K^{p*})$ is provided for firm p under evaluation. Capacity outputs and its optimal use of fixed and variable inputs can be computed:

$$
y_p^* = \sum_{k=1}^{K} z_k^{p*} y_k; \quad x_p^{f*} = \sum_{k=1}^{K} z_k^{p*} x_k^f; \quad x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v. \tag{11}
$$

This has to be repeated for all firms $p = 1, \ldots, K$.

In a second phase, these 'optimal' frontier results (capacity output, and capacity variable and fixed inputs) at the firm level are used as parameters in the industry model. In particular, the industry model minimises the industry use of fixed inputs in a radial way (using $DF_{fi}^{SR}(x^f, x^v, y)$ from (5)) such that the total production of outputs is at least at the current total level by a reallocation of production between firms. Reallocation is allowed based on the frontier production inputs and outputs usage of each firm. In the short run, it is assumed that current plant capacities cannot be exceeded either at the firm or at the industry level. The formulation of the multi-output and frontier-based short-run Johansen (1972) industry model (hereafter also referred to as the basic version (bv)) can then be specified as follows:

$$\begin{aligned}
\min_{\theta^{bv}, w_k^{bv}, X^v} \quad & \theta^{bv}, \\
s.t. \quad & \sum_{k=1}^{K} w_k^{bv} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{bv} x_k^{f*} \leq \theta^{bv} X^f, \\
& \sum_{k=1}^{K} w_k^{bv} x_k^{v*} \leq X^v, \\
& 0 \leq w_k^{bv} \leq 1, \qquad\qquad k = 1, ..., K, \\
& \theta^{bv} \geq 0, X^v \geq 0,
\end{aligned} \tag{12}$$

where

$$Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right). \tag{13}$$

After solving model (12), the vector $(w_p^{bv^*} x_p^{f*}, w_p^{bv^*} x_p^{v*}, w_p^{bv^*} y_p^*)$ can be a target for firm p where $w_p^{bv^*}$ is an optimal solution of model (12) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (11). Note that X^v in model (12) holding the variable inputs is a vector with decision variables.

In the short-run Johansen (1972) frontier-based industry model (12), focus is on reducing fixed inputs by a scalar θ^{bv}. This is related to the original empirical application in Dervaux, Kerstens, and Leleu (2000) where the purpose is to minimize the number of surgery units. The same motivation applies to the empirical applications curbing overfishing in the fishery industries where output quota are imposed to guarantee biological sustainability constraints. While fixed inputs can normally not be reduced by definition, one can mothball either temporarily or definitively the use of particular fixed inputs. However, it is trivial to define an alternative short-run Johansen (1972) industry model that maximises all industry outputs using an output-oriented efficiency measure similar to (2). This is similar in spirit to the original average practice single output short-run industry model in Johansen (1972), except that in the latter no explicit efficiency measure is available.

Geometrically, this short-run industry model (12) is a set consisting of a finite sum of line segments, or *zonotopes* (see Hildenbrand (1981, p. 1096)).[9] More precisely, under the as-

[9] One may also benefit from consulting the work of Koopmans (1977), Hildenbrand (1983) or Settepanella, Dosi, Grazzi, Marengo, and Ponchio (2015).

sumptions of divisibility and additivity of production processes the industry technology is geometrically represented by the space formed by the finite sum of all the line segments linking the origin and the points representing each production unit (see Dosi, Grazzi, Marengo, and Settepanella (2016, p. 877)). Furthermore, Dosi, Grazzi, Marengo, and Settepanella (2016, footnote 3) remark that convexity comes as a result of the chosen analytical framework: it is not an assumption of some underlying theory of production.

The activity vector $w = (w_1, \ldots, w_K)$ indicates which portions of the line segments representing the firm capacities are effectively used to produce outputs from given inputs. The bounds on the activity vector w ($0 \leq w_k \leq 1$) reflect the assumption of constant returns to scale up to full capacity for individual production units (see Hildenbrand (1981, p. 1096)). The optimal solution to this simple LP gives the combination of firms that can produce the same or more outputs with less or the same use of fixed inputs at the aggregate level.

Note that the outcomes of model (12) are not unique in general. Hence, there can be multiple optimal solutions for this model. When facing the problem of multiple optimal solutions, researchers sometimes present secondary goals to partially try to remove this problem. But, in general the problem still remains and there is no guarantee that the formulation of one or more secondary goals leads to a choice of a unique solution among these alternative optimal solutions. As much as it can be undesirable to have multiple optimal solutions in a social planning model, it is up to this date unclear how this problem can be avoided.

In brief, one can state that average practice and best practice models share a very similar formal structure of the short-run Johansen (1972) industry model. The main difference is that only the best practice version is consistent with the idea of an industry frontier, while in the average practice version one is not really sure that one estimates an industry frontier given the uncertainties surrounding the underlying ad hoc capacity estimates. Now, it may be objected that social planning based on an industry frontier notion may be too demanding: perhaps, one should allow for some amount of technical inefficiency persisting among firms. But, as shown in Kerstens, Vestergaard, and Squires (2006) it is straightforward to adjust the frontier-based short-run Johansen (1972) industry model to allow for some amount of technical inefficiency.

In addition, there are some more subtle differences between average practice and best practice models. Average practice models ignore fixed inputs, while best practice models do include these. As a matter of fact, in average practice models the fixed inputs determine the capacities. Furthermore, in addition to technical efficiency some of the average practice authors assume cost minimization (e.g., Hildenbrand (1983, p. 175)). Indeed, average practice models need input prices to determine the cost per output. This is not the case for best practice models in this contribution that depend solely on physical inputs and outputs.

Finally, we mention a series of methodological refinements of the short-run Johansen (1972) industry model. First, it has been rather common to trace the evolution of the short-run average practice Johansen (1972) industry production function over time: examples are found in Førsund and Hjalmarsson (1983), Førsund and Jansen (1983), several chapters in the Førsund and Hjalmarsson (1987) book, Wibe (1995), among others. Second, Dosi, Grazzi, Marengo, and Settepanella (2016) define a normalized volume of the zonotope as a measure of industry heterogeneity. These authors also propose a measure of productivity change based on the zonotope's main diagonal, and assess the role of firm entry and exit on industry level productivity growth (see Settepanella, Dosi, Grazzi, Marengo, and Ponchio (2015) for technical details). These developments so far do not seem to have been implemented in a frontier context.

4.2 Numerical Example

Consider a numerical example containing 13 fictitious observations with two inputs generating a single output: one input is variable, the other one is fixed. The first four columns of Table 1 contain these data. By solving model (12), we obtain $\theta^{bv^*} = 0.638$, where θ^{bv^*} is the optimal value of θ^{bv}. Columns 5 to 7 of Table 1 show the inputs and outputs targets defined in equation (11) which are obtained by solving model (10). The vector $w^{bv^*} = (w_1^{bv^*}, \ldots, w_K^{bv^*})$ is an optimal solution of model (12) and is reported in the 8-th column. The final target points of inputs and outputs obtained by the solving model (12) (i.e., points $(w_p^{bv^*} x_p^{f*}, w_p^{bv^*} x_p^{v*}, w_p^{bv^*} y_p^*)$ corresponding to firm p) are presented in the last three columns.

As can be seen in Table 1, the value of $w_k^{bv^*}$ for all units is unity except for units $4, 5, 6$ and 13. For these four units, we have $w_4^{bv^*} = w_5^{bv^*} = w_6^{bv^*} = 0$ and $w_{13}^{bv^*} = 0.2$. Therefore, for units $4, 5$ and 6, the target points are located at the origin. However, the target point of unit 13 is $(1.2, 0.8, 1)$: this point does not belong to the production possibility set. We show this by reporting the result of the refined output-oriented short-run Johansen industry model in the Appendix B, section B.1.

Also, we can visualize this infeasibility problem in Figures 1 and 2. A three-dimensional representation of the technology resulting from these 16 fictitious observations is provided by Figure 1. This technology consists of two inputs (variable input x^v and fixed input x^f) and one output (y) and is visible by means of its convex boundary. The original observations are

Table 1: Inputs and outputs targets obtained by solving model (12)

DMU_p	x_p^v	x_p^f	y_p	x_p^{v*}	x_p^{f*}	y_p^*	$w_p^{bv^*}$	$w_p^{bv^*}x_p^{v*}$	$w_p^{bv^*}x_p^{f*}$	$w_p^{bv^*}y_p^*$
1	3	3	2	5	3	4	1	5	3	4
2	2	5	2	6	4	5	1	6	4	5
3	2	7	2	6	4	5	1	6	4	5
4	5	2	2	2	2	2	0	0	0	0
5	10	2	2	2	2	2	0	0	0	0
6	2	2	2	2	2	2	0	0	0	0
7	3	7	4	6	4	5	1	6	4	5
8	3	4	4	6	4	5	1	6	4	5
9	5	3	4	5	3	4	1	5	3	4
10	9	3	4	5	3	4	1	5	3	4
11	5	5	5	6	4	5	1	6	4	5
12	6	4	5	6	4	5	1	6	4	5
13	4	6	5	6	4	5	0.2	1.2	0.8	1

visible by means of orange spheres. The projection of the frontier in the vertical plane $x^v = 0$ is visualised by the transparent red region positioned on the x^f axis. The projection of the original observations in the vertical plane $x^v = 0$ is indicated by blue boxes. The optimal 3D points obtained from equation (11) (i.e., $(x_p^{v*}, x_p^{f*}, y_p^*)$) are denoted with green crosses. Finally, the targets points obtained after applying model (12) (i.e., $(w_p^{bv^*}x_p^{f*}, w_p^{bv^*}x_p^{v*}, w_p^{bv^*}y_p^*)$) are illustrated with black boxes.

The gray intersecting plane passes through the origin and the output-oriented target point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$ of observation 13 indicated by label A. Based on the results of Table 1, since $w_{13}^{bv^*} = 0.2$, this target point scales down by 0.2 times to $(1.2, 0.8, 1)$ depicted by the black square indicated by label D in the gray intersecting plane. Obviously, this point does not belong to the technology.

To even better illustrate this technological infeasibility, we present in Figure 2 the intersection of the gray plane and the boundary of technology of Figure 1. The horizontal axis shows the amount of simultaneous change in fixed and variable inputs (α) for the target point 13 in a radial way while the vertical axis shows the amount of changes in outputs (φ). For observation 13, $(\alpha, \varphi) = (1, 1)$ since $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$. Consequently, the target point of observation 13 is depicted as the black solid box with label A. Again based on the results of Table 1, we must scale down point A by a factor 0.2 resulting in the target point $(1.2, 0.8, 1)$ for which $(\alpha, \varphi) = (0.2, 0.2)$. The corresponding point is labelled D in Figure 2). Geometrically, this scaling factor corresponds with the ratio of Euclidean distances $\|0D\|/\|0A\| = 0.2 = w_{13}^{bv^*}$. Obviously, this point D does not belong to the technology and is

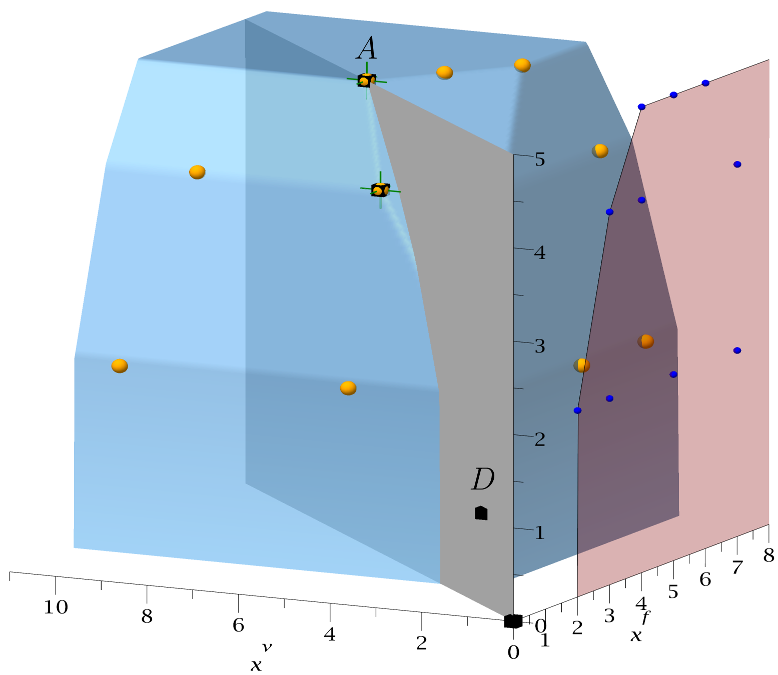

Figure 1: 3-dimensional view of the convex frontier for numerical example

thus not feasible.

Based on the results of Table 1, the output-oriented target points of units $2, 3, 7, 8, 11$ and 13 are identical. Therefore, the intersection of the technology with the plane passing through the origin and the output-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^*)$ of these observations are the same as illustrated in Figure 2. The value of $w_k^{bv^*}$ for these units is unity. Therefore, the target point of these units remains unchanged at point A in Figure 2.

5 Output-, Attainable Output-, and Input-oriented Short-run Johansen Industry Models: New Proposals

This section develops the methodological refinements to the basic short-run Johansen industry model outlined above. We first correct the short-run Johansen industry model based on the output-oriented plant capacity concept such that the scaling of the plant capacity inputs and outputs remains technically feasible by staying within the frontier technology. While this

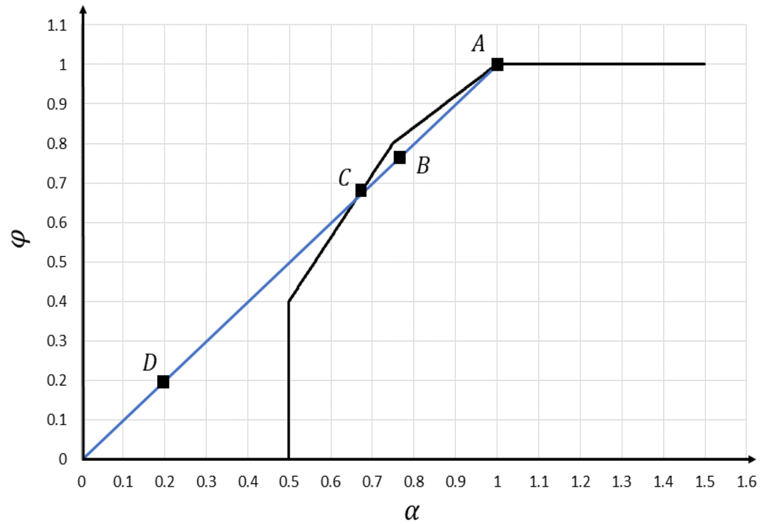

Figure 2: Intersection of the technology with the plane going through the origin and the output-oriented target point of observation 13

refinement is clearly important for the best practice or frontier-based short-run Johansen industry model, its pertinence for the traditional average practice short-run Johansen industry model is harder to assess: we just assume this refinement can never harm. Thereafter, we develop a new short-run Johansen industry model based on the attainable output-oriented plant capacity concept. Finally, we develop a new short-run Johansen industry model based on the input-oriented plant capacity concept.

5.1 Short-run Johansen Industry Model with Output-oriented Capacity Measures: A Revised Version

The method is developed in two steps as follows. In the first step, from models (9) and (10), an optimal activity vector z^{p*} is provided for each firm p. Hence, capacity output and its optimal use of fixed and variable inputs y_p^*, x_p^{f*} and x_p^{v*} can be computed by means of equation (11).

In a second step, these 'optimal' frontier results (capacity output and capacity variable and fixed inputs) at the firm level are used as parameters in the below short-run industry model (hereafter also referred to as the revised version (rv)):

$$
\begin{aligned}
\min_{\theta^{rv}, w_k^{rv}, X^v} \quad & \theta^{rv} \\
s.t. \quad & \sum_{k=1}^{K} w_k^{rv} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{rv} x_k^{f*} \leq \theta^{rv} X^f, \\
& \sum_{k=1}^{K} w_k^{rv} x_k^{v*} \leq X^v, \\
& w^{rv} = (w_1^{rv}, \ldots, w_K^{rv}) \in \mathit{\Gamma}^{rv}, \\
& \theta^{rv} \geq 0, X^v \geq 0.
\end{aligned}
\tag{14}
$$

where

$$
Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right),
$$

and

$$
\mathit{\Gamma}^{rv} = \{(w_1, \ldots, w_K) \mid w_k \leq 1, (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^{\Lambda},\ k = 1, \ldots, K\}. \tag{15}
$$

This set $\mathit{\Gamma}^{rv}$ determines the feasible area of weights $(w_1, \ldots, w_K)$ such that the target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$, $(p = 1, \ldots, K)$, belong to the technology. Note that for feasible weights $(w_1, \ldots, w_K) \in \mathit{\Gamma}^{rv}$, we have $w_p \leq 1$ for all $p = 1, \ldots, K$. Therefore in model (14), the decision variable w_p^{rv} scales down the target point $(x_p^{f*}, x_p^{v*}, y_p^*)$ of firm p such that the technology is respected. Note that in model (14), the variable inputs stored in the vector X^v are decision variables. To obtain a lower bound L_p^{rv} for $w_p^{rv}, (p = 1, ..., K)$, we need to solve model (16):

$$
\begin{aligned}
L_p^{rv} = \min_{\delta, z_k} \quad & \delta \\
s.t. \quad & \sum_{k=1}^{K} z_k y_k \geq \delta y_p^*, \\
& \sum_{k=1}^{K} z_k x_k^f \leq \delta x_p^{f*}, \\
& \sum_{k=1}^{K} z_k x_k^v \leq \delta x_p^{v*}, \\
& (z_1, \ldots, z_K) \in \Lambda, \\
& \delta \geq 0,
\end{aligned}
\tag{16}
$$

where y_p^*, x_p^{f*} and x_p^{v*} are defined in (11). Actually, by solving model (16), we scale down the output and input capacity targets such that they become feasible within the technology. Therefore, model (16) can be interpreted as reducing the capacity targets to obtain the lower bound of weights such that the technology is still respected. This relaxes the assumption of constant returns to scale up to full capacity in the basic version of the model.

Note furthermore that the main difference between the basic version (12) and the revised version (14) of the short-run Johansen industry model is in the range of the weights $(w_1, \ldots, w_K)$: in model (12) we have $0 \leq w_k^{bv} \leq 1$, while in model (14) we have $L_k^{rv} \leq w_k^{rv} \leq 1$. Therefore, after solving model (14), the vector $(w_p^{rv^*} x_p^{f*}, w_p^{rv^*} x_p^{v*}, w_p^{rv^*} y_p^*)$, where $w_p^{rv^*}$ is an optimal solution of model (14), can be a target for firm p which belongs to the technology T^Λ.

Contrasting the basic version (bv) and the revised version (rv) of the short-run Johansen industry model immediately leads to the following result:

Proposition 5.1. *Assume that* $(\theta^{bv^*}, w^{bv^*})$ *and* $(\theta^{rv^*}, w^{rv^*})$ *are an optimal solution of models* (12) *and* (14)*,respectively. In technology* (7)*, we have:*

(i) $\theta^{bv^*} \leq \theta^{rv^*}$ *and* $w_p^{bv^*} \underset{<}{\overset{\geq}{=}} w_p^{rv^*}$.

(ii) If $\theta^{bv^*} < \theta^{rv^*}$*, then for all multiple optimal solutions of model* (12)*, there exists* $k \in \{1, \ldots, K\}$ *such that the corresponding target point* $(w_k^{bv^*} x_k^{f*}, w_k^{bv^*} x_k^{v*}, w_k^{bv^*} y_k^*)$ *does not belong to the technology.*

(iii) If $\theta^{bv^*} = \theta^{rv^*}$*, then there is at least one optimal solution of model* (12) *for which the corresponding target points of all observed units belong to the technology.*

Proof. See Appendix D. □

Interpreting Proposition 5.1, the fact that $\theta^{bv^*} \leq \theta^{rv^*}$ shows the eventual empirical relevance of relaxing the hypothesis of constant returns to scale up to full capacity. Furthermore, it also shows that if we have $\theta^{bv^*} < \theta^{rv^*}$, then for every multiple optimal solution of the basic version of the short-run Johansen industry model (12), there is at least one observation for which its target point does not respect the technology. Also, relation $\theta^{bv^*} = \theta^{rv^*}$ guarantees that there is one optimal solution of the basic version of the short-run Johansen industry model (12) such that all corresponding target points of observations belong to the technology.

It is important to note that the relation $\theta^{bv^*} = \theta^{rv^*}$ does not guarantee that all multiple optimal solutions of model (12) lead to target points belonging to the technology. So, even if $\theta^{bv^*} = \theta^{rv^*}$, the possibility exists of having a target point of some observations not respecting the technology.

By solving model (12) on the data of the numerical example in Table 1, we obtain $\theta^{rv^*} = 0.660$. Hence, we have $0.638 = \theta^{bv^*} < \theta^{rv^*} = 0.660$. Therefore, based on Proposition 5.1, for every multiple optimal solution of the basic version of the short-run Johansen industry model (12), there is at least one observation for which its target point does not respect the technology.

As illustrated in Figure 2, the traditional output-oriented short-run Johansen industry model (12) scales down point A to obtain the target point D which is located outside of the technology. But, by implementing the revised short-run Johansen industry model (14), the target point A translates to the solid black box B: this remains technically feasible by remaining within the technology (for more details, please see Appendix B, section B.1).

5.2 Short-run Johansen Industry Model with Attainable Output-oriented Efficiency Measure: New Proposal

As already mentioned in Section 3, the original output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ has no limitations on the available amounts of variable inputs. However, in most empirical settings this is not realistic and we have to limit the amount of variable inputs available at either the firm or the industry level (see Kerstens, Sadeghi, and Van de Woestyne (2019b) for details). Kerstens, Sadeghi, and Van de Woestyne (2019b) empirically illustrate that variable inputs must somehow be bounded. Thus, $APCU_o(x, x^f, y, \bar{\lambda})$ is a more realistic alternative plant capacity utilization measure provided a reasonable level $\bar{\lambda}$ is chosen.

The attainable output-oriented efficiency measure $ADF_o^f(x_p^f, y_p, \bar{\lambda})$ at level $\bar{\lambda} \in \mathbb{R}_+$ is computed by solving the following linear program:

$$
\begin{aligned}
ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \quad & \max_{x^v, \varphi, z_k} \quad \varphi \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq \varphi y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v = x^v, \\
& \qquad x^v \leq \bar{\lambda} x_p^v, \\
& \qquad (z_1, \ldots, z_K) \in \Lambda, \\
& \qquad x^v \geq 0.
\end{aligned}
\tag{17}
$$

In model (17), the scalar $\bar{\lambda}$ can be varied over some part of the interval $(0, \infty)$. But, when $\bar{\lambda} < 1$, then it is possible that model (17) is infeasible. However, Kerstens, Sadeghi, and Van de Woestyne (2019b) determine the complete feasible interval for $\bar{\lambda}$ by defining three critical points. We only need two critical points for our purpose:

Definition 5.1. For a given observation (x_p, y_p), the following two critical points C_P^1 and C_P^2 can be defined.

$$C_P^1 = DF_{vi}^{SR}(x_p^f, x_p^v, 0), \tag{18}$$

and

$$C_P^2 = DF_{vi}^{SR}(x_p^f, x_p^v, y_p). \tag{19}$$

Note that the critical points C_P^1 and C_P^2 make up the components of the input-oriented plant capacity measure $PCU_i(x, x^f, y)$ in Definition 3.3. Furthermore, Kerstens, Sadeghi, and Van de Woestyne (2019b) have proven that for every observation (x_p, y_p): if $\bar{\lambda} < C_P^1$, then model (17) is infeasible.

Assume that φ^* is the optimal value of model (17), then the following model can be solved to find a solution that maximizes slacks and surpluses:

$$\begin{array}{ll}
\max\limits_{x^v, S^+, S^-, z_k} & 1_M.S^+ + 1_{N_f}.S^- \\
s.t & \sum\limits_{k=1}^{K} z_k y_k - S^+ = \varphi^* y_p, \\
& \sum\limits_{k=1}^{K} z_k x_k^f + S^- = x_p^f, \\
& \sum\limits_{k=1}^{K} z_k x_k^v = x^v, \\
& x^v \leq \bar{\lambda} x_p^v, \\
& (z_1, \ldots, z_K) \in \Lambda, \\
& x^v \geq 0, S^+ \geq 0, S^- \geq 0.
\end{array} \tag{20}$$

The method is developed in two steps as follows. In the first step, from model (20) an optimal activity vector $z^{p^*} = (z_1^{p*}, \ldots, z_K^{p*})$ is provided for firm p under evaluation and hence capacity output and its optimal use of fixed and variable inputs can be computed:

$$y_p^* = \sum_{k=1}^{K} z_k^{p*} y_k; \quad x_p^{f*} = \sum_{k=1}^{K} z_k^{p*} x_k^f; \quad x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v. \tag{21}$$

This has to be repeated for all firms $p = 1, \ldots, K$.

In a second step, these 'optimal' frontier results (capacity output and capacity variable and fixed inputs) at the firm level are used as parameters in the below industry model (hereafter also referred to as the attainable version (att)):

$$\begin{array}{lll} \min\limits_{\theta^{att}, w_k^{att}, X^v} & \theta^{att} & \\ s.t. & \sum\limits_{k=1}^{K} w_k^{att} y_k^* \geq Y, & \\ & \sum\limits_{k=1}^{K} w_k^{att} x_k^{f*} \leq \theta^{att} X^f, & \\ & \sum\limits_{k=1}^{K} w_k^{att} x_k^{v*} \leq X^v, & (22) \\ & (w_1^{att}, \dots, w_K^{att}) \in \Gamma^{att}, & \\ & \theta^{att} \geq 0, X^v \geq 0, & \end{array}$$

where

$$Y = \left(\sum_{k=1}^{K} y_{k1}, \dots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \dots, \sum_{k=1}^{K} x_{kN_f}^f \right),$$

and

$$\Gamma^{att} = \{ (w_1, \dots, w_K) \mid w_k \leq 1, \ (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^{\Lambda}, \ k = 1, \dots, K \}, \qquad (23)$$

where y_p^*, x_p^{f*} and x_p^{v*} are now defined in (21) instead of (11). Note that X^v in model (22) holding the variable inputs is a vector of decision variables. Set Γ^{att} determines the feasible area of weights $(w_1, \dots, w_K)$ such that the target point $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$, where $p = 1, \dots, K$, belongs to the technology.

Notice that the constraints $w_k \leq 1$, $(k = 1, ..., K)$, in set Γ^{att} guarantee that the obtained target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$ can be magnified at most as much as $\bar{\lambda}$ which is an attainable level of variable inputs defined in model (17). Therefore, in model (22) decision variable w_k scales down the target point $(x_k^{f*}, x_k^{v*}, y_k^*)$ of firm p such that the technology is respected. Note that we have no relation between θ^{att^*} and θ^{rv^*} in optimality.

To obtain a lower bound $L_p^{att}, (p = 1, ..., K)$, for w_p^{att} in model (22) we need to solve model (16) where y_p^*, x_p^{f*} and x_p^{v*} are now defined in (21) instead of (11).

Note furthermore that the attainable output-oriented short-run Johansen industry model (22) can lead to infeasibilities in practical applications. Proposition 5.2 proves some necessary and sufficient conditions for which model (22) is feasible.

Proposition 5.2. *In technology* (7)*, for every observation* (x_p, y_p)*:*

(i) Model (22) *is feasible if and only if* $\sum_{k=1}^{K} y_k^* \geq Y$.

(ii) If $C_k^2 \leq \bar{\lambda}$ *for all* $k = 1, \ldots, K$, *then model* (22) *is feasible.*

(iii) If we remove constraint $(w_1^{att}, \ldots, w_K^{att}) \in \Gamma^{att}$ *in model* (22), *then model* (22) *is always feasible.*

Proof. See Appendix D. □

Note that based on Proposition 5.2, if there is an $m \in \{1, ..., M\}$ such that $\sum_{k=1}^{K} y_{km}^* < \sum_{k=1}^{K} y_{km}$, then model (22) is infeasible. Also, if model (22) is infeasible, then there is some $k \in \{1, ..., K\}$ such that we have $C_k^2 > \bar{\lambda}$. However, since $C_k^2 \leq 1$, hence if we assume that $\bar{\lambda} \geq 1$, then the attainable output-oriented short-run Johansen industry model (22) is feasible. Finally, when the attainable output-oriented short-run Johansen industry model need not comply with the technology, this model is always feasible.

After solving model (22), the vector $(w_p^{att^*} x_p^{f*}, w_p^{att^*} x_p^{v*}, w_p^{att^*} y_p^*)$ can be a target for firm p which belongs to the technology (7), and in which $w_p^{att^*}$ is an optimal solution of model (22) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (21).

Note that if in the industry model (22) instead of minimising the fixed inputs, we maximise the outputs in a radial way by a reallocation of production between firms, then Proposition 5.2 becomes redundant.

Note furthermore that by implementing the attainable output-oriented short-run Johansen industry model (22) by using the numerical example in Table 1, we have $\theta^{att^*} = 0.70$ which is higher than θ^{bv^*} and θ^{rv^*}. In this case, the target point A translates to the solid black box C in Figure 2: this remains technically feasible by remaining within the boundary of the frontier technology (for more details, please see Appendix B, section B.2).

5.3 Short-run Johansen Industry Model with Input-oriented Capacity Measures: New Proposal

The input-oriented short-run efficiency measure $DF_{vi}^{SR}(x_p^f, x_p^v, 0)$ is computed by optimizing the following program:

$$\begin{aligned} DF_{vi}^{SR}(x_p^f, x_p^v, 0) = \quad & \min_{\theta, z_k} \quad \theta \\ & s.t \quad \sum_{k=1}^{K} z_k y_k \geq 0, \\ & \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \qquad \sum_{k=1}^{K} z_k x_k^v \leq \theta x_p^v, \\ & \qquad (z_1, \ldots, z_K) \in \Lambda, \\ & \qquad \theta \geq 0. \end{aligned} \tag{24}$$

Note that the observed output levels on the right-hand side of the output constraints are put equal to zero. These zero output levels are compatible with any output levels where production is initiated and differs from zero (i.e., y_{min} as defined supra). The reader is referred to Kerstens, Sadeghi, and Van de Woestyne (2019a, Proposition B.1) for additional interpretations (see also supra).

Assume that θ^* is the optimal value of model (24), the following model can be solved to find a solution that maximizes slacks and surpluses:

$$\begin{aligned} \max_{z_k, S^+, S^{v-}, S^{f-}} \quad & 1_M.S^+ + 1_{N_f}.S^{f-} + 1_{N_v}.S^{v-} \\ s.t \quad & \sum_{k=1}^{K} z_k y_k - S^+ = 0, \\ & \sum_{k=1}^{K} z_k x_k^f + S^{f-} = x_p^f, \\ & \sum_{k=1}^{K} z_k x_k^v + S^{v-} = \theta^* x_p^v, \\ & (z_1, \ldots, z_K) \in \Lambda, \\ & S^+ \geq 0, S^{v-} \geq 0, S^{f-} \geq 0, \end{aligned} \tag{25}$$

with $1_{N_v} = (1, \ldots, 1) \in \mathbb{R}_+^{N_v}$.

Similar to the output-oriented short-run Johansen models treated so far, we have to proceed in two steps as follows. In the first step, from model (25) an optimal activity vector

$z^{p*} = (z_1^{p*}, \ldots, z_K^{p*})$ is provided for firm p under evaluation and hence capacity output and its optimal use of fixed and variable inputs can be computed:

$$y_p^* = \sum_{k=1}^{K} z_k^{p*} y_k; \quad x_p^{f*} = \sum_{k=1}^{K} z_k^{p*} x_k^f; \quad x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v. \tag{26}$$

This has to be repeated for all firms $p = 1, \ldots, K$.

In a second step, these 'optimal' frontier results (capacity output and capacity variable and fixed inputs) at the firm level are used as parameters in the below industry model (hereafter also referred to as the input-oriented version (inp)):

$$\begin{aligned}
\min_{\theta^{inp}, w_k^{inp}, X^v} \quad & \theta^{inp} \\
s.t. \quad & \sum_{k=1}^{K} w_k^{inp} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{inp} x_k^{f*} \leq \theta^{inp} X^f, \\
& \sum_{k=1}^{K} w_k^{inp} x_k^{v*} \leq X^v, \\
& (w_1^{inp}, \ldots, w_K^{inp}) \in \Gamma^{inp}, \\
& \theta^{inp} \geq 0, X^v \geq 0.
\end{aligned} \tag{27}$$

where

$$Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{km} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right), \tag{28}$$

and

$$\Gamma^{inp} = \{(w_1, \ldots, w_K) \mid w_k \geq 1, (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^{\Lambda},\ k = 1, \ldots, K\}. \tag{29}$$

This set Γ^{inp} determines the feasible area for the weights $(w_1, \ldots, w_K)$ such that the target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$, where $p = 1, \ldots, K$, belong to the technology. Note that for feasible weights $(w_1, \ldots, w_K) \in \Gamma^{inp}$, we have $w_p \geq 1$ for all $p = 1, \ldots, K$. Therefore, in model (27) decision variable w_k scales up the target point $(x_k^{f*}, x_k^{v*}, y_k^*)$ of firm p such that the technology is respected. Note that θ^{inp^*} cannot be compared to θ^{bv^*}, θ^{rv^*} and θ^{att^*} in optimality.

To obtain an upper bound U_p^{inp}, where $p = 1, ..., K$, for w_p^{inp} we need to solve the next model (30):

$$
\begin{aligned}
U_p^{inp} = \quad & \max_{\delta, z_k} \quad \delta \\
& s.t. \quad \sum_{k=1}^{K} z_k y_k \geq \delta y_p^*, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq \delta x_p^{f*}, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v \leq \delta x_p^{v*}, \\
& \qquad (z_1, \ldots, z_k) \in \Lambda, \\
& \qquad \delta \geq 0,
\end{aligned}
\tag{30}
$$

where y_p^*, x_p^{f*} and x_p^{v*} are defined in (26). Actually, by solving this model we scale up the output and input capacity targets such that they become feasible within the technology. Indeed, notice that in all previous models based on output-oriented plant capacity we start from output and input capacity targets that are situated in point A at the horizontal section in Figure 2, while here we start from input-oriented plant capacity targets that are situated at the vertical section in Figure 2: in Figure 3 one can note another point A at the vertical section.

Therefore, model (30) can be interpreted as expanding the capacity targets to obtain the upper bound of weights such that the technology is respected. Note that all weights $w_k^{inp} \geq 1$ since the optimal solution starts out from the vertical section in Figure 3 and moves up to the right in input-output space, while all previous models based on output-oriented plant capacity start from output and input capacity targets that are situated at the horizontal section in Figure 2 and move down to the left in input-output space. Hence, in model (30) we need to scale up capacity outputs and capacity variable and fixed inputs to meet all requirements.

Note that the input-oriented short-run Johansen industry model (27) can lead to infeasibilities in practical applications. But, if there are no upper bounds in the input-oriented short-run Johansen industry model (27) (i.e., we do not need to respect the technology by ignoring constraint $(w_1^{inp}, \ldots, w_K^{inp}) \in \Gamma^{inp}$ in model (27)), then model (27) is always feasible. Proposition 5.3 proves some necessary and sufficient conditions for which model (27) is feasible.

Proposition 5.3. *In technology (7), for every observation (x_p, y_p):*

(i) Model (27) is feasible if and only if $\sum_{k=1}^{K} U_k^{inp} y_k^* \geq Y$.

(ii) If we remove constraint $(w_1^{inp}, \ldots, w_K^{inp}) \in \Gamma^{inp}$ in model (27), then model (27) is always

feasible.

Proof. See Appendix D. □

After solving model (27), the vector $(w_p^{inp^*} x_p^{f*}, w_p^{inp^*} x_p^{v*}, w_p^{inp^*} y_p^*)$ can be a target for DMU_p which belongs to the technology (7) where $w_p^{inp^*}$ is an optimal solution of model (27) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (26).

Note again that the outcomes of these three new models in Section 5 (i.e., models (14), (22) and (27)) are not in general unique and there can be multiple optimal solutions for these models.

Figure 3 shows the intersection of the technology with the plane that passes through the origin and the input-oriented target point of observation 13, i.e., point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (2, 2, 2)$ which is obtained from equation (26). The horizontal axis shows the amount of simultaneous changes in fixed and variable inputs (α) for the input-oriented target point 13 in a radial way and the vertical axis shows the amount of changes in outputs (φ). Therefore, for $(\alpha, \varphi) = (1, 1)$ we have $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (2, 2, 2)$ (black solid box A).

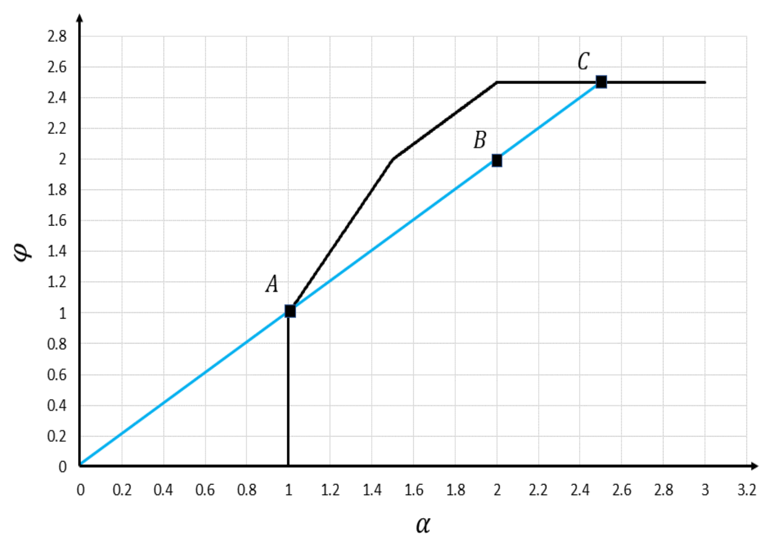

Figure 3: Intersection of the technology with the plane that passes through the origin and the input-oriented target point of observation 13

Note furthermore that by implementing the input-oriented short-run Johansen industry model (27) by using the numerical data in Table 1, we have $\theta^{inp^*} = 0.81$. In this case, the target point A (i.e., the target point of unit 13) remains unchanged at point A in Figure 3 (for more details, please see Appendix B, section B.3).

6 Empirical Illustration

6.1 Data

Our sample is based on 170 steel hulled fishing vessels operating in the northwest Atlantic Ocean during 2014. All vessels have a similar technology and catch their fish by dragging a net behind their vessels just off the ocean floor. Catches were grouped into three distinct categories based on species type: flatfish, roundfish, and "other". There are three fixed inputs: vessel length, engine horsepower, and vessel gross tonnage. The only variable input used is time spent at sea in days.

Table 2 presents basic descriptive statistics for the inputs and the outputs. We report the average, the standard deviation, and the minima and maxima (depending on the context) for both the inputs and outputs. Vessels are between 36 and 88 feet in length, with an average of 63 feet. Their horsepower ranges from 180 to 1,380 (494 average) and their tonnage is between 5 and 199 (average 90). On average, these vessels fish 67 days per year with a range between 2 and 242 days. Their average roundfish catch is 99,113 pounds with a range between zero and 750,976. Flatfish catch is between 9 and 265,617 pounds with an average of 50,602. The "other" category shows an average catch of 154,253 pounds with a range between 299 and 1,462,807 pounds. Basically, one observes a lot of heterogeneity and a rather wide range for all inputs and outputs.

An important remark needs to be made with respect to the sole variable input time spent at sea in days. Based on equation (11) we have $x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v$ and since $\sum_{k=1}^{K} z_k^{p*} = 1$, then $\min_{k=1,\ldots,K} x_{kn}^v \leq x_{pn}^{v*} = \sum_{k=1}^{K} z_k^{p*} x_{kn}^v \leq \max_{k=1,\ldots,K} x_{kn}^v$ for all $n = 1, \ldots, N_v$. Hence, we have $2.222 \leq x_{p1}^{v*} \leq 242.195$ for all $p = 1, \ldots, K$. Thus, the optimal amount of variable inputs is always bounded by the minimum and maximum levels of observed variable inputs in the data, and it can certainly in no way reach the absolute upper bound of 365 days in 2014.

Table 2: Descriptive Statistics for 170 Observed Data

	Fixed input 1	Fixed input 2	Fixed input 3	Variable input	Output 1	Output 2	Output 3
	Horsepower	Length	Tonnage	Days	Roundfish	Flatfish	Other
Average	494.4824	62.67194	90.14706	67.79868	99113.2254	50601.95	154252.701
St. Dev.	210.1697	14.60609	54.59042	66.21814	154640.012	54758.96	233021.661
Min	180	35.8	5	2.222	0	9	299
Max	1380	88.4	199	242.195	750976	265616.9	1462806.89

Table 3 reports the descriptive statistics of input-oriented, output-oriented and attainable output-oriented plant capacity utilisation for 170 DMUs using convex and non-convex technologies, respectively. The main motivation to differentiate between convex and non-convex technologies is that recently Kerstens, Sadeghi, and Van de Woestyne (2019a) revealed significant differences between convex and non-convex plant capacities. Note that for the attainable output-oriented efficiency measure $ADF_o^f(x^f, y, \bar{\lambda})$ as well as the attainable output-oriented plant capacity utilization $APCU_o(x, x^f, y, \bar{\lambda})$, we have chosen $\bar{\lambda} = 2$. Therefore, we simply assume that variable inputs can be magnified at most twofold.

Table 3: Descriptive Statistics of Input and Output Plant Capacity Utilisation for 170 DMUs in both Convex and Non-convex Cases

Convex	$DF_{vi}(x^f, x^v, y)$	$DF_{vi}(x^f, x^v, 0)$	$PCU_i(.)$	$DF_o(.)$	$DF_o^f(.)$	$PCU_o(.)$	$ADF_o^f(.)$	$APCU_o(.)$
Average	0.576	0.201	16.557	2.283	8.056	0.631	3.892	0.712
St. Dev.	0.242	0.279	21.297	1.735	14.286	0.342	3.777	0.246
Min	0.109	0.009	1.000	1.000	1.000	0.022	1.000	0.134
Max	1.000	1.000	108.999	11.546	129.824	1.000	28.865	1.000
Nonconvex								
Average	0.984	0.222	28.120	1.056	3.866	0.679	1.454	0.862
St. Dev.	0.064	0.300	30.095	0.230	10.792	0.344	1.189	0.220
Min	0.543	0.009	1.000	1.000	1.000	0.014	1.000	0.094
Max	1.000	1.000	108.999	2.675	129.558	1.000	11.282	1.000

Analyzing the results in Table 3, one can draw the following conclusions. First, on average the $PCU_i(x, x^f, y)$ indicates that one needs 16.55 times more variable inputs (days) with current outputs than with zero outputs under C, while under NC one employs 28.12 times more variable inputs (days) with current outputs than with zero outputs. Second, on average the biased plant capacity utilization measure $DF_o^f(x^f, y)$ indicates that outputs can be magnified by at least 8.05 times under C and 3.86 times under NC. Also, there is a lot of variation in $DF_o^f(x^f, y)$ as indicated by the standard deviation and the range: the maximum increase in outputs amounts to 129.824 times under C and 129.558 under NC. Third, on average the unbiased plant capacity utilization measure $PCU_o(x, x^f, y)$ indicates that current outputs make up 63% from maximal plant capacity outputs under C and 67% under NC. Also, the heterogeneity in $PCU_o(x, x^f, y)$ is large as indicated by the standard deviation and the range: the minimum of 2.2% under C and 1.4% under NC are simply very low. Fourth, for the biased attainable plant capacity utilization measure $ADF_o^f(x^f, y, \bar{\lambda} = 2)$ the average of the output magnification under C is higher than under NC. Also, for a twofold increase in variable inputs (i.e., $\bar{\lambda} = 2$), we obtain on average a 3.892 output magnification under C and a 1.454 output magnification under NC. Fifth, the average of $APCU_o(x, x^f, y, \bar{\lambda} = 2)$ is smaller under C than under NC.

In conclusion, the different plant capacity measures behave substantially different under C and NC technologies, which is in line with earlier results reported by Kerstens, Sadeghi, and Van de Woestyne (2019a).

6.2 Key Results

Turning to the results of the four short-run Johansen industry models, Table 4 shows basic descriptive statistics of their efficiency scores (θ), weights (w_p), lower and upper bounds (L_p and U_p), the number of units for which their weights coincide to their lower bound ($\#w_p = L_p$), the number of units for which their weights coincide to their upper bound ($\#w_p = U_p$), and the number of units which are located outside of the technology ($\#\ DMU_p \notin T$), respectively. The rows of this Table 4 include two parts: the first part shows the results under the convex case, and the second part shows the results under the nonconvex case.

Table 4: The results of weights, lower and upper bounds for all methods

		Weights				Lower or upper bound						
Convex	θ	Average	ST. D	Min	Max	Average	ST. D	Min	Max	$\#\ w_p = L_p$	$\#\ w_p = U_p$	$\#\ DMU_p \notin T$
bv	0.3	0.330	0.466	0	1					111	54	117
rv	0.84	0.937	0.108	0.5802	1	0.9366	0.1076	0.580	1	170	170	0
att	0.82	0.946	0.104	0.5802	1	0.9464	0.1040	0.580	1	170	170	0
inp	Inf	Inf	Inf	Inf	Inf	61.0550	25.4301	1	116.19	Inf	Inf	Inf
Nonconvex												
bv	0.35	0.350	0.474	0	1					109	56	114
rv	0.92	0.996	0.025	0.817	1	0.996	0.025	0.817	1	170	170	0
att	0.91	0.995	0.033	0.6858	1	0.995	0.033	0.686	1	170	170	0
inp	Inf	Inf	Inf	Inf	Inf	14.567	35.205	1	116.19	Inf	Inf	Inf

bv: basic version of output-oriented short-run Johansen industry model
rv: revised version of output-oriented short-run Johansen industry model
att: attainable output-oriented short-run Johansen industry model
inp: input-oriented short-run Johansen industry model

We can draw the following conclusions about this Table 4. First, comparing the first two lines, one observes that fixed inputs can be reduced by 70% in the basic version, while these can only be reduced by 16% in the revised version. This dramatic reduction in the basic version is due to the fact that 117 out of 170 vessels are in fact not even part of the frontier technology, an issue that has so far been ignored in the literature on the short-run Johansen model. This is reflected in the low average weights in the basic version compared to the high average weights in the revised version. In the revised version all 170 observations have weights equal to their lower bound. Second, applying a nonconvex technology slightly attenuates these results: fixed inputs can be reduced now by only 65% in the basic version, while they can be reduced by just 8% in the revised version. Average weights are higher under nonconvexity in both the basic and revised versions.

Third, opting for an attainable output-oriented plant capacity measure slightly improves the results compared to the revised version of the output-oriented plant capacity because capacity inputs and outputs are somewhat reduced. Under convexity fixed inputs can be reduced by 16% in the revised version and by 18% in the attainable case, while in the nonconvex case fixed inputs can be reduced by 8% in the revised version and by 9% in the attainable case. While the average weight slightly increases under convexity, it marginally decreases under nonconvexity. Also in the attainable version all 170 observations have weights equal to their lower bound. Fourth, the input-oriented short-run Johansen industry model (27) is infeasible for this empirical application under both convex and nonconvex cases. Thus, it is impossible to scale up the input-oriented capacity targets of units such that these are capable to generate the current aggregate output levels while respecting the technology. The reader must realise that the input-oriented short-run Johansen industry model (27) does yield a solution for the numerical example above, but that the configuration of the empirical data leads to an infeasibility. More detailed results for each of these four short-run Johansen industry models is found in Appendix C.

We think it is safe to make the following conclusions from our empirical illustration. A first conclusion is that the basic version of the short-run Johansen industry model is not only conceptually wrong, but also leads to far too optimistic reductions in fixed inputs. Another conclusion is that the degree of reallocation is somehow conditioned on the type of plant capacity concept one is willing to adhere to. Our results indicate that the traditional output-oriented plant capacity notion may still be a bit too optimistic compared to the attainable output-oriented plant capacity concept that leads to fewer reductions in fixed inputs. It is regrettable that the conceptually appealing input-oriented short-run Johansen industry model does not lead to a feasible solution for our data set.

7 Conclusions

This contribution has provided a cursory review of the historic development of the short-run Johansen industry model. It distinguishes between the traditional average practice version and the best practice or frontier-based version that is of more recent date. The goals of this contribution have been twofold. First, we have remedied a remaining problem in the short-run Johansen (1972) frontier-based industry model: we have relaxed the assumption of constant returns to scale up to full capacity for individual production units. Hence, capacity inputs and outputs remain technically feasible by firmly remaining within the technology. Second, we have opened up the methodological choices open to the users of this short-run Johansen (1972) industry model by introducing new plant capacity concepts: a new input-oriented plant capacity measure, and an attainable output-oriented plant capacity.

On the one hand, we have provided a basic numerical example to illustrate the differences and similarities between these different modeling options. On the other hand, we have developed an empirical illustration using US based fishery data. Both these illustrations have shown the viability of our new modeling options.

To conclude, we mention some avenues for future research. One possibility is to further extend the choice of plant capacity concepts: one option is to include a graph-oriented plant capacity concept (see Kerstens, Sadeghi, and Van de Woestyne (2020)).

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Appendices: Supplementary Material

A Graphical Illustrations

Now we try to clarify Definitions 3.1, 3.2 and 3.3 with the help of a two-dimensional Figure A.1 which depicts a single variable input and an output space. In particular, Figure A.1 shows a total product curve for given variable inputs as the polyline *abcd* and its horizontal extension at d. We focus on observation e. Note that observations are represented by squares and projection points by circles.

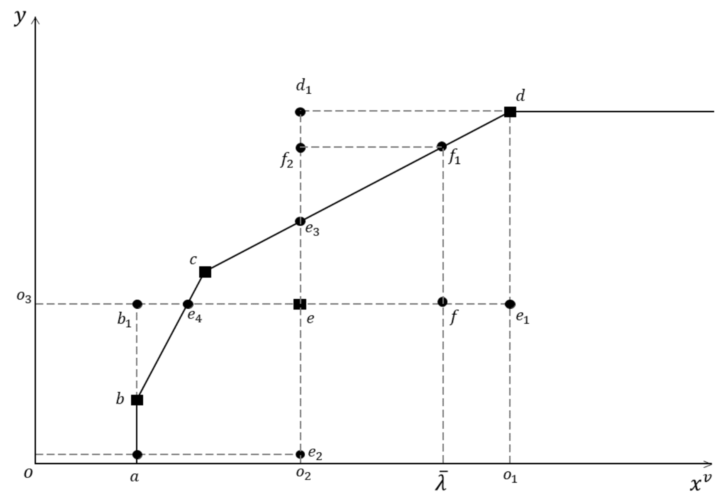

Figure A.1: Total product curve: Output-oriented, attainable output-oriented and input-oriented plant capacities

The output-oriented plant capacity measure $PCU_o(x, x^f, y)$ compares point e to its vertical projection point e_3 on the frontier on the one hand, and the translated point e_1 that consumes more variable inputs to its vertical projection point on the horizontal frontier segment emanating from point d with maximal outputs on the other hand. Clearly, the maximal output d can be labeled the plant capacity output. Thus, the unbiased plant capacity measure $PCU_o(x, x^f, y)$ is somehow linked to the distance e_3d_1, whereby point d_1 is simply the translation of the maximal output at point d to the output level comparable with point e.

The attainable output-oriented plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$ compares point e to its vertical projection point e_3 on the frontier on the one hand, and the translated point f that consumes at most a fraction $\bar{\lambda}$ more variable inputs to its vertical projection point at point f_1 with maximal outputs at level $\bar{\lambda}$ on the other hand. Clearly, the maximal output f_1 at level $\bar{\lambda}$ can be labeled the attainable plant capacity output. Thus, the unbiased attainable plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$ is somehow linked to the distance $e_3 f_2$, whereby point f_2 is simply the translation of the maximal output at point f_1 to the output level comparable with point e.

The input-oriented plant capacity measure $PCU_i(x, x^f, y)$ focuses on a sub-vector of variable inputs and compares point e to its horizontal projection point e_4 on the frontier on the one hand, and the translated point e_2 (consuming equal amounts of variable inputs but at a zero outputs level) to its horizontal projection point on the vertical frontier segment ab with zero outputs on the other hand. Clearly, the minimal variable input a yielding zero output can be labeled the plant capacity input. Thus, the unbiased plant capacity measure $PCU_i(x, x^f, y)$ is somehow linked to the distance $b_1 e_4$, whereby point b_1 is the translation of the variable input at point b to the variable input level comparable with point e.

B Numerical Example: Supplementary Materials

B.1 Section 5.1: Short-run Johansen Industry Model with Output-oriented Capacity Measures: A Revised Version

We illustrate the ease of implementing this revised short-run Johansen industry model with output-oriented capacity measures by using the numerical data in Table 1.

Table B.1 reports input and output targets obtained by solving model (14). The first four columns show the target points of units obtained by relation (11). The lower bound L_p^{rv} and the amounts $w_p^{rv^*}$ are reported in the fifth and sixth columns, respectively. The final targets of inputs and outputs obtained by solving model (14) (i.e., points $(w_p^{rv^*} x_p^{f*}, w_p^{rv^*} x_p^{v*}, w_p^{rv^*} y_p^*)$ corresponding to firm p) are presented in the 7-th, 8-th and 9-th columns. To see the magnification of the variable inputs we report the ratio of variable inputs of the target point over the current variable inputs (i.e., $\frac{w_p^{rv^*} x_p^{v*}}{x_p^v}$) in the very last column.

Analyzing the results in Table B.1, we can draw the following conclusions. First, the minimum amount of lower bound $w_p^{rv^*}$ is 0.667 and its maximum amount remains 1. Comparing with the results of Table 1, this new method puts all target points in the production possibility set by excluding weights below the lower bound $w_p^{rv^*}$ of 0.667. Second, the optimal amount $w_p^{rv^*}$ of units 2, 3, 7 and 8 coincides with their lower bounds, and the amount of $w_p^{rv^*}$ of units 11, 12 and 13 is situated between their lower and upper bounds. Furthermore, the amount of $w_p^{rv^*}$ for the remaining units is unity such that these units reach their upper

Table B.1: Inputs and outputs targets obtained by solving model (14)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	L_p^{rv}	$w_p^{rv^*}$	$w_p^{rv^*}x_p^{v*}$	$w_p^{rv^*}x_p^{f*}$	$w_p^{rv^*}y_p^*$	$\frac{w_p^{rv^*}x_p^{v*}}{x_p^v}$
1	5	3	4	1	1	5	3	4	1.667
2	6	4	5	0.667	0.667	4	3	3.333	2
3	6	4	5	0.667	0.667	4	3	3	2
4	2	2	2	1	1	2	2	2	0
5	2	2	2	1	1	2	2	2	0.2
6	2	2	2	1	1	2	2	2	1
7	6	4	5	0.667	0.667	4	3	3.333	1.333
8	6	4	5	0.667	0.667	4	3	3.333	1.333
9	5	3	4	1	1	5	3	4	1
10	5	3	4	1	1	5	3	4	0.556
11	6	4	5	0.667	0.786	5	3	4	0.943
12	6	4	5	0.667	0.786	5	3	3.929	0.786
13	6	4	5	0.667	0.762	5	3.048	3.810	1.143

bounds. For unit 13, we have $L_{13}^{rv} = 0.667$: this means that if we put $w_{13}^{rv^*} < 0.667$, then the obtained target point $(w_{13}^{rv^*}x_{13}^{v*}, w_{13}^{rv^*}x_{13}^{f*}, w_{13}^{rv^*}y_{13}^*)$ does no longer belong to the production possibility set. Note that in the previous Table 1, since we have $w_{13}^{bv^*} = 0.2 < 0.667 = L_{13}^{rv}$, the obtained target of unit 13 in the basic version is situated outside the production possibility set. As illustrated in Figure 2, the traditional output-oriented short-run Johansen industry model (12) scales down the target of unit 13 (i.e., point A) to obtain the target point D which is located outside of the technology.

To solve this problem of the infeasibility of point D in Figure 2, we have now modified the short-run Johansen industry model (12) such that the scaling of this point A remains technically feasible by remaining within the frontier technology by only moving along the segment AC. We can show this feasibility again by reference to Figure 2. Note that since $w_{13}^{rv^*} = 0.762$, we now scale down the point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$ (solid black box A) by 0.762 times to obtain the target point $(w_{13}^{rv^*}x_{13}^{v*}, w_{13}^{rv^*}x_{13}^{f*}, w_{13}^{rv^*}y_{13}^*) = (5, 3.048, 3.810)$ in Figure 1. As can be seen in Figure 2, the latter target point translates to point $(0.762, 0.762)$ that is represented by the solid black box B: this remains technically feasible by remaining within the technology.

Note that based on the results of Table 1, the output-oriented target point of units $2, 3, 7, 8, 11$ and 12 are identical with unit 13. Therefore, the intersection of the technology with the plane that passes through the origin and the output-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^*)$ of these observations are the same as illustrated in Figure 2. The amount of

$w_p^{rv^*}$ for the units $2, 3, 7$ and 8 coincides with their lower bounds. Hence, their target points are located on point C in Figure 2. The amount of $w_p^{rv^*}$ of units 11 and 12 is situated between its lower and upper bounds and these units have the same behavior as unit 13.

Finally, the last column of Table B.1 indicates the amounts by which the variable inputs can be magnified. There is rather a large amount of variation in these variable inputs. Indeed, the range is broad: the minimum change in variable inputs amounts to 0.2 times and the maximum increase in variable inputs amounts to 2 times.

B.2 Section 5.2: Short-run Johansen Industry Model with Attainable Output-oriented Efficiency Measure: New Proposal

Table B.2 reports the results of the short-run industry model with attainable output-oriented efficiency measure on the numerical example. It is structured in a way similar to the previous Table B.1. For this numerical example, we have chosen $\bar{\lambda} = 2$. Thus, we believe that an increase of the variable inputs with a factor more than 2 is implausible. We make three observations. First, as can be seen in the last column of Table B.2, the variable input can be magnified by maximum 1.667 times. Only for the first unit this magnification is 1.667 and for the other units, it is smaller than 1.667. Second, the optimal amount $w_p^{att^*}$ of units $2, 3, 7$ and 13 coincides with their lower bounds. The amount of $w_p^{att^*}$ of units $8, 11$ and 12 is situated between their lower and upper bounds. Furthermore, the amount of $w_p^{att^*}$ for the remaining units is unity such that these units reach their upper bounds. Third, for none of the observations in this numerical example we reach the upper bound $\bar{\lambda} = 2$.

Note that based on the results of Table B.2, the attainable output-oriented target point of units $7, 8, 11, 12$ and 13 are $(x_p^{v*}, x_p^{f*}, y_p^*) = (6, 4, 5)$. Therefore, the intersection of the technology with the plane that passes through the origin and the attainable output-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^*)$ of these observations are the same as illustrated in Figure 2. Note that since $w_{13}^{att^*} = 0.667$, we need to scale down the point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$ (solid black box A) by 0.667 times to obtain the target point $(w_{13}^{att^*} x_{13}^{v*}, w_{13}^{att^*} x_{13}^{f*}, w_{13}^{att^*} y_{13}^*) = (4, 2.667, 3.333)$ in Figure 1 and its projection $(0.677, 0.677)$ in Figure 2 (solid black box C). Also, unit 7 has the same behavior as unit 13. The amount of $w_p^{att^*}$ of units $8, 11$ and 12 is situated between their lower and upper bounds.

Table B.2: Inputs and outputs targets obtained by solving model (22)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	L_p^{att}	$w_p^{att^*}$	$w_p^{att^*} x_p^{v*}$	$w_p^{att^*} x_p^{f*}$	$w_p^{att^*} y_p^*$	$\frac{w_p^{att^*} x_p^{v*}}{x_p^v}$
1	5	3	4	1	1	5	3	4	1.667
2	4	5	4.667	0.6	0.6	2	3	2.8	1.2
3	4	6	5	0.667	0.667	3	4	3.333	1
4	5	2	2	1	1	5	2	2	1
5	2	2	2	1	1	2	2	2	0.200
6	2	2	2	1	1	2	2	2	1
7	6	4	5	0.667	0.667	4	3	3	1
8	6	4	5	0.667	0.829	5	3	4	1.658
9	5	3	4	1	1	5	3	4	1
10	5	3	4	1	1	5	3	4	0.556
11	6	4	5	0.667	0.778	5	3	4	0.933
12	6	4	5	0.667	0.833	5	3	4.167	0.833
13	6	4	5	0.667	0.667	4	2.667	3.333	1

B.3 Section 5.3: Short-run Johansen Industry Model with Input-oriented Capacity Measures: New Proposal

Table B.3 reports the results for the short-run Johansen industry model with input-oriented plant capacity. It is structured in a similar way as Tables B.1 and B.2. The only difference between these tables is in the 5-th column: while in Tables B.1 and B.2 this column reports the lower bound of w_p, in Table B.3 it shows the upper bound for w_p^{inp} (i.e., U_p^{inp}).

Table B.3: Inputs and outputs target obtained by solving of model (27)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	U_p^{inp}	$w_p^{inp^*}$	$w_p^{inp^*} x_p^{v*}$	$w_p^{inp^*} x_p^{f*}$	$w_p^{inp^*} y_p^*$
1	2	2	2	2.5	1	2	2	2
2	2	2	2	2.5	1	2	2	2
3	2	2	2	2.5	2.5	5	5	5
4	2	2	2	2.5	2.5	5	5	5
5	2	2	2	2.5	2.5	5	5	5
6	2	2	2	2.5	2.417	4.833	4.833	4.833
7	2	2	2	2.5	1.833	3.667	3.667	3.667
8	2	2	2	2.5	2	4	4	4
9	2	2	2	2.5	1	2	2	2
10	2	2	2	2.5	1.75	3.5	3.5	3.5
11	2	2	2	2.5	1	2	2	2
12	2	2	2	2.5	1	2	2	2
13	2	2	2	2.5	1	2	2	2

Analyzing the results in Table B.3, we can draw the following conclusions. First, the upper bound w_p^{inp} of all units is 2.5. This new method keeps all target points within the production possibility set by excluding weights above the upper bound $w_p^{inp^*}$ of 2.5. Second, the optimal amount $w_p^{inp^*}$ of units $3, 4$ and 5 are bigger that unity and coincides with their upper bounds. The amount of $w_p^{inp^*}$ of unit $6, 7, 8$ and 10 is situated between its lower and upper bounds. Furthermore, the amount of $w_p^{inp^*}$ for the remaining units $1, 2, 9, 11, 12$ and 13 is unity such that these units reach their lower bound that is smaller than their upper bound.

Based on the results of Table B.3, since $U_{13}^{inp} = 2.5$, hence it can be scaled up 2.5 times such that its target point remains technologically feasible. But, since we have $w_{13}^{inp^*} = 1$, therefore unit 13 remains unchanged at point A in Figure 3. Note that based on the results of Table B.3, the input-oriented target points of all units are identical with the input-oriented target point of unit 13. Therefore, the intersection of the technology with the plane that passes through the origin and the input-oriented target points $(x_p^{v*}, x_p^{f*}, y_p^*)$ of all observations are the same as illustrated in Figure 3. The amount of $w_p^{inp^*}$ for the units $1, 2, 9, 11$ and 12 is unity, hence these units have the same behavior as unit 13 and their targets remain unchanged at point A in Figure 2.

The optimal amount $w_p^{inp^*}$ of unit $3, 4$ and 5 is bigger than unity and coincides with their upper bounds. Therefore, the target point of these three units translates to point $(2.5, 2.5)$ that is represented by the solid black box B in Figure 3. The amount of $w_p^{inp^*}$ of units $6, 7, 8$ and 10 is situated between their lower and upper bounds. For example, if we focus on unit 8, since $w_8^{inp^*} = 2$, hence we must scale up point A by 2 times to obtain the target point B of unit 8 (i.e., $(4, 4, 4)$ in Figure 1 and its projection $(2, 2)$ in Figure 3).

C Empirical Illustration: Supplementary Material

C.1 Output-oriented Short-run Johansen Industry Model: Basic Version

Table C.1 shows basic descriptive statistics for all normalised inputs and outputs defined in equation (11) which are obtained by solving model (10). The rows of this table include two parts: first part shows the results under convex case, and the second shows the results under non-convex case. In both parts, we report the arithmetic averages, the standard deviation, the minima and maxima depending on the context.

Table C.1: Descriptive Statistics of Normalised Inputs and Outputs Defined in (11)

Convex	$\frac{x^{f*}_{p1}}{x^{f}_{p1}}$	$\frac{x^{f*}_{p2}}{x^{f}_{p2}}$	$\frac{x^{f*}_{p3}}{x^{f}_{p3}}$	$\frac{x^{v*}_{p1}}{x^{v}_{p1}}$	$\frac{y^{*}_{p1}}{y_{p1}}$	$\frac{y^{*}_{p2}}{y_{p2}}$	$\frac{y^{*}_{p3}}{y_{p3}}$
Average	0.922	0.926	0.907	3.465	273.639	10.528	16.029
St. Dev.	0.126	0.074	0.138	4.691	1136.602	33.802	55.466
Min	0.477	0.636	0.396	0.425	1.000	1.000	1.000
Max	1.000	1.000	1.000	46.697	11869.221	390.978	670.947
Nonconvex							
Average	0.885	0.928	0.815	3.056	211.902	21.029	8.685
St. Dev.	0.141	0.087	0.207	4.680	892.748	204.597	48.571
Min	0.450	0.660	0.323	0.844	1.000	1.000	1.000
Max	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

Turning to the analysis of Table C.1, we can draw several conclusions. First, the average magnification of three fixed inputs are smaller and close to unity under both C and NC. Second, the result indicates that the variable input can be magnified by at least 3.46 times under C and 3.05 times under NC, on average. Also, the range is broad: the maximum increase in variable input amounts to 46.70 times under C and 49.07 times under NC.[10] Third, the results show that three outputs can be magnified by at least 273.64, 10.53 and 16.03 times under C and 211.90, 21.03 and 8.68 times under NC, on average. There is also a great amount of variation, as indicated by the standard deviation, and the range is broad: for example the maximum increase in the first output amounts to 11869.22 times under C and 6015.03 times under NC.

Table C.2 shows the basic descriptive statistics for all normalised inputs and outputs obtained by solving model (12), i.e., points $(\frac{w^{bv^*}_p x^{f*}_p}{x^f_p}, \frac{w^{bv^*}_p x^{v*}_p}{x^v_p}, \frac{w^{bv^*}_p y^*_p}{y_p})$ corresponding to DMU_p where $w^{bv^*}_p$ is an optimal solution of model (12) and x^{f*}_{pn}, x^{v*}_{pn} and y^*_{pm} are obtained from the relations (11). The rows of this table include again two parts. The first part reports the results under the convex case, and the second part reports the results under the non-convex case. In both parts, we report the arithmetic averages, the standard deviation, the minima and maxima depending on the context.[11]

Analyzing the results in Table C.2, we can draw the following conclusions. First, the average of $w^{bv^*}_k$ indicates that for scaling down capacity outputs and capacity variable and fixed inputs to meet all requirements, we need on average a 0.335 scaling under C and a

[10]Based on equation (11) we have $x^{v*}_p = \sum_{k=1}^K z^{p*}_k x^v_k$ and since $\sum_{k=1}^K z^{p*}_k = 1$, then $\min_{k=1,\ldots,K} x^v_{kn} \leq x^{v*}_{pn} = \sum_{k=1}^K z^{p*}_k x^v_{kn} \leq \max_{k=1,\ldots,K} x^v_{kn}$ for all $n = 1, \ldots, N_v$. Hence, based on the information in Table 2, we find that $2.222 \leq x^{v*}_{p1} \leq 242.195$ for all $p = 1, \ldots, K$. Therefore, the optimal amount of variable inputs is always bounded by the minimum and maximum levels of observed variable inputs in the data.

[11]Note that the first output is zero for 6 DMUs: hence, we do not consider these DMUs in the descriptive statistics.

Table C.2: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (12)

Convex	$w_p^{bv^*}$	$\frac{w_p^{bv^*} x_{p1}^{f*}}{x_{p1}^{f}}$	$\frac{w_p^{bv^*} x_{p2}^{f*}}{x_{p2}^{f}}$	$\frac{w_p^{bv^*} x_{p3}^{f*}}{x_{p3}^{f}}$	$\frac{w_p^{bv^*} x_{p1}^{v*}}{x_{p1}^{v}}$	$\frac{w_p^{bv^*} y_{p1}^{*}}{y_{p1}}$	$\frac{w_p^{bv^*} y_{p2}^{*}}{y_{p2}}$	$\frac{w_p^{bv^*} y_{p3}^{*}}{y_{p3}}$
Average	0.335	0.299	0.303	0.301	1.319	151.982	2.838	4.771
St. Dev.	0.468	0.425	0.425	0.427	2.918	991.182	6.627	16.113
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	1.000	1.000	1.000	1.000	19.322	11869.221	45.386	146.072
Nonconvex								
Average	0.357	0.311	0.325	0.282	1.465	102.164	18.705	5.748
St. Dev.	0.476	0.422	0.436	0.394	4.574	586.743	204.704	48.382
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	1.000	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

0.357 scaling under NC. Second, the minimum amount of $w_p^{bv^*}$ is zero, Therefore, for some units the target points are located on the origin.

C.2 Output-oriented Short-run Johansen Industry Model: Revised version

Table C.3 is structured in a similar way as Table C.2. In this table, the basic descriptive statistics for all normalised inputs and outputs obtained by solving model (14) are reported. The amounts of $w_p^{rv^*}$ and lower bound L_p^{rv} are reported in the second and third columns, respectively. To see the magnification of the fixed, variable inputs and outputs we report the ratio of their target point over their current amount, i.e., points $(\frac{w_p^{rv^*} x_p^{f*}}{x_p^{f}}, \frac{w_p^{rv^*} x_p^{v*}}{x_p^{v}}, \frac{w_p^{rv^*} y_p^{*}}{y_p})$ corresponding to DMU_p where $w_p^{rv^*}$ is an optimal solution of model (14) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (11), in the fourth to tenth columns.

Table C.3: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (14)

Convex	L_p^{rv}	$w_p^{rv^*}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^{f}}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^{f}}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^{f}}$	$\frac{w_p^{rv^*} x_{p1}^{v*}}{x_{p1}^{v}}$	$\frac{w_p^{rv^*} y_{p1}^{*}}{y_{p1}}$	$\frac{w_p^{rv^*} y_{p2}^{*}}{y_{p2}}$	$\frac{w_p^{rv^*} y_{p3}^{*}}{y_{p3}}$
Average	0.934	0.934	0.863	0.864	0.846	3.309	261.448	10.323	14.839
St. Dev.	0.109	0.109	0.164	0.119	0.157	4.658	1061.042	33.828	44.615
Min	0.580	0.580	0.383	0.576	0.396	0.425	0.580	0.580	0.580
Max	1.000	1.000	1.000	1.000	1.000	46.697	10919.902	390.978	517.788
Nonconvex									
Average	0.996	0.996	0.880	0.924	0.811	3.047	211.887	21.022	8.679
St. Dev.	0.026	0.026	0.141	0.088	0.206	4.679	892.751	204.598	48.571
Min	0.817	0.817	0.450	0.660	0.323	0.817	0.817	0.817	0.817
Max	1.000	1.000	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

Analyzing the results in Table C.3, we can draw the following conclusions. First, the minimum amount of lower bound L_p^{rv} as well as $w_p^{rv^*}$ is 0.580 times under C and 0.817 times under NC and their maximum amount remains 1 for both C and NC. Second, the amount of lower bound L_p^{rv} and $w_p^{rv^*}$ are identical for all units, hence the optimal amount $w_p^{rv^*}$ of all units coincides with their lower bounds. Third, since we have $w_p^{rv^*} \leq 1$, comparing with the results of Table C.1, all final target points obtained by solving model (14) are smaller than the target points obtained from equation (11). Fourth, comparing with the results of Table C.2, this new method puts all target points within the production possibility set by excluding weights below the lower bound $w_p^{rv^*}$ of 0.580 under C and 0.817 under NC. Fifth, comparing with the results of Table C.2, in the basic version of the output-oriented short run Johansen Industry model the average of $w_p^{bv^*}$ is 0.335 under both C and NC, while in the revised version of output-oriented short run Johansen Industry model the average of $w_p^{rv^*}$ is 0.934 under C and 0.996 under NC. It means that for most units, the target points obtained from model (12) do no longer belong to the production possibility set. Finally, the seventh column of Table C.3 indicates that the variable inputs can be magnified by at least 3.31 times under C and 3.05 times under NC, on average. There is a large amount of variation in the variable inputs. Indeed, the range is broad: the minimum changes in variable inputs amounts to 0.42 times under C and 0.82 times under NC and the maximum increase in variable inputs amounts to 46.70 times under C and 49.08 times under NC.

C.3 Attainable Output-oriented Short-run Johansen Industry Model

Table C.4 reports the results of the short-run industry model with attainable output-oriented efficiency measure (22). It is structured in a similar way as the previous Table C.3. For this empirical example, we have chosen $\bar{\lambda} = 2$. Thus, we believe that an increase of the variable inputs with a factor more than 2 is implausible.

We make three observations. First, the minimum amount of the lower bound L_p^{att} as well as $w_p^{att^*}$ is 0.580 times under C and 0.686 times under NC and their maximum amount remains 1 for both C and NC. Second, the amount of the lower bound L_p^{att} and $w_p^{att^*}$ are identical for all units, hence the optimal amount $w_p^{att^*}$ of all units coincides with their lower bounds. Third, as can be seen in the seventh column of Table C.4, the variable input can be magnified at least 1.476 times under C and 1.207 times under NC, on average, and it can be magnified by about maximum 2 times.

Table C.4: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (22)

Convex	L_p^{att}	$w_p^{att^*}$	$\frac{w_p^{att^*} x_{p1}^{f*}}{x_{p1}^f}$	$\frac{w_p^{att^*} x_{p2}^{f*}}{x_{p2}^f}$	$\frac{w_p^{att^*} x_{p3}^{f*}}{x_{p3}^f}$	$\frac{w_p^{att^*} x_{p1}^{v*}}{x_{p1}^v}$	$\frac{w_p^{att^*} y_{p1}^*}{y_{p1}}$	$\frac{w_p^{att^*} y_{p2}^*}{y_{p2}}$	$\frac{w_p^{att^*} y_{p3}^*}{y_{p3}}$
Average	0.944	0.944	0.857	0.847	0.762	1.476	34.246	5.304	7.029
St. Dev.	0.105	0.105	0.166	0.124	0.206	0.518	151.191	15.470	18.207
Min	0.580	0.580	0.383	0.576	0.226	0.425	0.580	0.580	0.580
Max	1.000	1.000	1.000	1.000	1.000	2.000	1781.082	182.033	221.958
Nonconvex									
Average	0.995	0.995	0.878	0.923	0.830	1.207	9.052	2.358	2.052
St. Dev.	0.033	0.033	0.168	0.110	0.238	0.324	49.919	6.221	2.494
Min	0.686	0.686	0.333	0.505	0.113	0.686	0.686	0.686	0.686
Max	1.000	1.000	1.000	1.000	1.000	1.994	610.176	62.861	19.585

C.4 Input-oriented Short-run Johansen Industry Model

Table C.5 shows basic descriptive statistics for all normalised inputs and outputs defined in equation (26) which is obtained by solving model (25). Turning to the analysis of Table C.5, we can draw two conclusions. First, the average of magnification of all fixed, variable inputs and outputs except for the first output are smaller than unity under both C and NC. For the first output, there is a great amount of variation, as indicated by the standard deviation, and the range is broad: for example the maximum increase in the first outputs amounts to 1516.43 times under both C and NC. Second, the fixed input and three variable inputs can be magnified by maximum 1 times under both C and NC.

Table C.5: Descriptive Statistics of Normalised Inputs and Outputs Defined in Relation (26)

Convex	$\frac{x_{p1}^{f*}}{x_{p1}^f}$	$\frac{x_{p2}^{f*}}{x_{p2}^f}$	$\frac{x_{p3}^{f*}}{x_{p3}^f}$	$\frac{x_{p1}^{v*}}{x_{p1}^v}$	$\frac{y_{p1}^*}{y_{p1}}$	$\frac{y_{p2}^*}{y_{p2}}$	$\frac{y_{p3}^*}{y_{p3}}$
Average	0.8161	0.7346	0.4852	0.1907	13.6486	0.8339	0.3732
St. Dev.	0.2062	0.1614	0.2855	0.2730	120.1295	4.8842	1.3003
Min	0.2855	0.5079	0.1479	0.0092	0.0004	0.0112	0.0032
Max	1.0000	1.0000	1.0000	1.0000	1516.4267	61.1712	15.5652
Nonconvex							
Average	0.7953	0.7232	0.4738	0.2100	11.4405	0.9093	0.4244
St. Dev.	0.1967	0.1550	0.2678	0.2927	118.4416	5.9376	1.5092
Min	0.2855	0.5079	0.1479	0.0092	0.0000	0.0112	0.0031
Max	1.0000	1.0000	1.0000	1.0000	1516.4267	74.9765	15.5652

Note that the input-oriented short-run Johansen industry model (27) is infeasible for this empirical application under both convex and nonconvex cases. Thus, it is simply impossible to scale up the input-oriented capacity targets of units such that these can generate the current aggregate output levels while respecting the technology.

D Proofs of Propositions

Proof of Proposition 5.1:

(i) Suppose that the vector $(\theta^{rv^*}, w^{rv^*}, X^{v^*})$ is an optimal solution of model (14). Since the target points $(y_p^*, x_p^{f*}, x_p^{v*})$ for models (12) and (14) are the same, hence $(\theta^{rv^*}, w^{rv^*}, X^{v^*})$ is a feasible solution for model (12). Therefore, $\theta^{bv^*} \leq \theta^{rv^*}$ because this kind of model (12) is a minimising problem. To complete the proof, note that we have $w_p^{bv^*} \overset{\geq}{\underset{<}{=}} w_p^{rv^*}$ because the results of the numerical as well as empirical examples show that $w_p^{bv^*}$ can be equal, bigger or smaller than $w_p^{rv^*}$.

(ii) Assume that $\theta^{bv^*} < \theta^{rv^*}$ and $(w_1^{bv^*}, \ldots, w_K^{bv^*})$ is an optimal solution of model (12). This optimal solution is not a feasible solution of model (14), because if we assume that $(w_1^{bv^*}, \ldots, w_K^{bv^*})$ is a feasible solution of model (14), then we have $\theta^{rv^*} \leq \theta^{bv^*}$ and based on the part (i), we have $\theta^{bv^*} \leq \theta^{rv^*}$. Hence, we have $\theta^{rv^*} = \theta^{bv^*}$ which it is a contradiction because we assume that $\theta^{bv^*} < \theta^{rv^*}$. Therefore, we have $(w_1^{bv^*}, \ldots, w_K^{bv^*}) \notin \Gamma^{rv}$. Based on equation (15), there is $k \in \{1, \ldots, K\}$ such that $(w_k^{bv^*} x_k^{f*}, w_k^{bv^*} x_k^{v*}, w_k^{bv^*} y_k^*) \notin T^{\Lambda}$.

(iii) Assume that $(w_1^{rv^*}, \ldots, w_K^{rv^*})$ is an optimal solution of model (14) with the optimal value θ^{rv^*}. Therefore, it is a feasible solution of model (12) with the objective value θ^{rv^*}. Assume that θ^{bv^*} is an optimal value of model (12). Since we assume that $\theta^{bv^*} = \theta^{rv^*}$, hence $(w_1^{rv^*}, \ldots, w_K^{rv^*})$ is an optimal solution of model (12) and for this optimal solution we have $(w_1^{rv^*}, \ldots, w_K^{rv^*}) \in \Gamma^{rv}$. Thus based on equation (15), we have $(w_k^{rv^*} x_k^{f*}, w_k^{rv^*} x_k^{v*}, w_k^{rv^*} y_k^*) \in T^{\Lambda}$ for all $k \in \{k = 1, \ldots, K\}$.

Proof of Proposition 5.2:

(i) Suppose that model (22) is feasible and $(w_1^{att^*}, ..., w_K^{att^*})$ is an optimal solution for decision variables $(w_1^{att}, ..., w_K^{att})$. Hence, we have $\sum_{k=1}^{K} w_k^{att^*} y_k^* \geq Y$. Since $w_k^{att^*} \leq 1$, we have $\sum_{k=1}^{K} y_k^* \geq Y$.
Now, assume that $\sum_{k=1}^{K} y_k^* \geq Y$. Letting,

$$w_k^{att^*} = 1, \theta^{att^*} = \max_{n=1,\ldots,N_f} \frac{\sum_{k=1}^{K} x_{kn}^{f*}}{\sum_{k=1}^{K} x_{kn}^{f}}, X^{v*} = \sum_{k=1}^{K} x_k^{v*}.$$

Hence, $(w_k^{att^*}, \theta^{att^*}, X^{v*})$ is a feasible solution of model (22).

(ii) Suppose that $C_k^2 \leq \bar{\lambda}$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \varphi^* \geq 1$. For this reason, assume that $(z_k^*, \theta^* = C_k^2)$ is an optimal solution of model (19). Since $C_k^2 x_p^v \leq \bar{\lambda} x_p^v$, hence $(\hat{z}_k = z_k^*, \hat{x}^v = C_k^2 x_p^v, \hat{\theta} = 1)$ is a feasible solution of model (17) with objective value $\hat{\theta} = 1$. Therefore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ because this kind of model is a maximising problem. Thus, based on model (20), we have $\sum_{k=1}^{K} y_k^* \geq Y$. Hence, based on part (i), model (22) is feasible.

(iii) If we define $(w_k^{att^*}, \theta^{att^*}, X^{v^*})$ as follows:
$w_1^{att^*} = \max_{m=1,\ldots,M} \frac{\sum_{k=1}^{K} y_{km}}{y_{1m}^*}$ and $w_k^{att^*} = 0$ for all $k = 2, \ldots, K$,
$\theta^{att^*} = \max_{n=1,\ldots,N_f} \frac{w_1^{att^*} x_{1n}^{f*}}{\sum_{k=1}^{K} x_{kn}^f}$,
$X^{v^*} = w_1^{att^*} x_1^{v*}$.
Then, $(w_k^{att^*}, \theta^{att^*}, X^{v^*})$ is a feasible solution of model (22).

Proof of Proposition 5.3

(i) Suppose that model (27) is feasible and $(w_1^{inp^*}, ..., w_K^{inp^*})$ is an optimal solution for decision variables $(w_1^{inp}, ..., w_K^{inp})$. Hence, we have $\sum_{k=1}^{K} w_k^{inp^*} y_k^* \geq Y$. Since $w_k^{inp^*} \leq U_k^{inp}$, we have $\sum_{k=1}^{K} U_k^{inp} y_k^* \geq Y$.

Now, assume that $\sum_{k=1}^{K} U_k^{inp} y_k^* \geq Y$. Letting,

$$w_k^{inp^*} = U_k^{inp}, \theta^{inp^*} = \max_{n=1,\ldots,N_f} \frac{\sum_{k=1}^{K} U_k^{inp} x_{kn}^{f*}}{\sum_{k=1}^{K} x_{kn}^f}, X^{v*} = \sum_{k=1}^{K} U_k^{inp} x_k^{v*}.$$

Hence, $(w_k^{inp^*}, \theta^{inp^*}, X^{v*})$ is a feasible solution of model (27).

(ii) If we define $(w_k^{inp^*}, \theta^{inp^*}, X^{v^*})$ as follows:
$w_1^{inp^*} = \max_{m=1,\ldots,M} \frac{\sum_{k=1}^{K} y_k m}{y_{1m}^*}$ and $w_k^{inp^*} = 0$ for all $k = 2, \ldots, K$,
$\theta^{inp^*} = \max_{n=1,\ldots,N_f} \frac{w_1^{inp^*} x_{1n}^{f*}}{\sum_{k=1}^{K} x_k n^f}$,
$X^{v^*} = w_1^{inp^*} x_1^{v*}$.
Then, $(w_k^{inp^*}, \theta^{inp^*}, X^{v^*})$ is a feasible solution of model (27).