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Ilke Aydogan
IESEG School of Managemen,t, UMR 9221 - LEM, F-59000 Lille, France; and Department of Economics, Bocconi University, Italy (i.aydogan@ieseg.fr).

Loïc Berger
CNRS, IESEG School of Management, Univ. Lille, UMR 9221 - LEM, F-59000 Lille, France; RFF-CMCC European Institute of Economics and the Environment (EIEE), and Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (l.berger@ieseg.fr).

Valentina Bosetti
Department of Economics and IGIER, Bocconi University, and RFF-CMCC European Institute on Economics and the Environment (EIEE), Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (valentina.bosetti@unibocconi.it).

Ning Liu
Schoof of Economics and Management, Beihang University, China (nliu2018@buaa.edu.cn).

IÉSEG School of Management Lille Catholic University 3, rue de la Digue F-59000 Lille Tel: 33(0)3 20 54 58 92
www.ieseg.fr

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THREE LAYERS OF UNCERTAINTY AND THE ROLE OF MODEL MISSPECIFICATION

Ilke Aydogan* Loïc Berger† Valentina Bosetti‡ Ning Liu§

Abstract

We explore decision-making under uncertainty using a framework that decomposes uncertainty into three distinct layers: (1) *risk*, which entails inherent randomness within a given probability model; (2) *model uncertainty*, which entails subjective uncertainty about the probability model to be used; and (3) *model misspecification*, which entails uncertainty about the presence of the correct probability model among the set of models considered. Using a new experimental design, we measure individual attitudes towards these different layers of uncertainty and examine the role of each of them in characterizing attitudes towards ambiguity. In addition to providing new insights into the underlying processes behind ambiguity aversion, our study provides the first empirical evidence of the role of model misspecification in decision-making under uncertainty.

Keywords: Ambiguity aversion, model uncertainty, model misspecification, non-expected utility, reduction of compound lotteries

JEL Classification: D81

*IESEG School of Management, UMR 9221 - LEM, F-59000 Lille, France; and Department of Economics, Bocconi University, Italy (i.aydogan@ieseg.fr).

†CNRS, IESEG School of Management, Univ. Lille, UMR 9221 - LEM, F-59000 Lille, France; RFF-CMCC European Institute on Economics and the Environment (EIEE), and Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (l.berger@ieseg.fr).

‡Department of Economics and IGIER, Bocconi University, and RFF-CMCC European Institute on Economics and the Environment (EIEE), Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (valentina.bosetti@unibocconi.it).

§School of Economics and Management, Beihang University, China (nliu2018@buaa.edu.cn).

1 Introduction

Uncertainty is pervasive and plays a major role in economics. Whether economic agents in the market pursuing individual goals, or policymakers pursuing social objectives, decision-makers (DMs) rarely have complete information about (objective) probability distributions over relevant states of the world. A valid understanding of individual behavior in the face of uncertainty is of great importance for the construction of realistic economic models capable of making accurate predictions, as well as for prescriptive applications guiding decision-making processes.

Following the early insights of Arrow (1951) and the recent contributions of Hansen (2014), Marinacci (2015), and Hansen and Marinacci (2016), our investigation focuses on a framework that decomposes uncertainty into three distinct *layers* of analysis. We consider a DM who possesses ex-ante information about a set of possible *probability models* characterizing the inherent randomness within a phenomenon of interest but who is uncertain about the "correct" probability model. In the words of Hansen (2014) and Marinacci (2015), a distinction is thus made among the layers of (i) *risk*, in which the uncertainty is about the possible outcomes within a given probability model; (ii) *model uncertainty*, in which the uncertainty is about which alternative probability model one should use to assign probabilities; and (iii) *model misspecification*, in which the uncertainty pertains to the presence of the correct probability model within the initial set of models under consideration.

More specifically, the first layer of risk characterizes situations in which the consequences of the DM's actions depend on the states of the world over which there is an *objectively* known probability distribution. This uncertainty about consequences represents the inherent variability *within* a particular probability model and, as such, is considered as being of an aleatory (or physical) type, analogous to chance mechanisms. The extra layer of model uncertainty arises when the DM is not able to identify a single probability distribution (among a given set of probability models) corresponding to the phenomenon of interest (Marinacci, 2015). This uncertainty *across* models has an epistemic nature, which may be quantified by means of *subjective* probabilities. Finally, as models are, by design, approximations of more complex phenomena, they are often misspecified (at least along some dimensions). As such, the set of probability models under consideration might not include the correct model, giving rise to the third layer of model misspecification, or epistemic uncertainty *about* models (Hansen 2014; Hansen and Marinacci 2016, see also White 1982).

The three layers of uncertainty are inherent in any decision problem under uncertainty in which a DM adopts probabilistic theories about the outcomes of a phenomenon and forms beliefs about their relevance. Therefore, they provide a useful framework to analyze the vast majority of real-life decision problems under ambiguity.[1] An example is when different experts provide opinions about the probability of the same event (e.g., the chances of developing a disease, of fire risk in buildings, aircraft accidents, etc.). From the DM's perspective, each expert's probability model can be interpreted as a risk. As experts may provide different assessments, the second layer of model uncertainty emerges regarding which expert's model to rely on. Finally, the third layer of model misspecification captures the possibility that none of the experts consulted is correct.

A specific example in which this framework naturally applies in real life is decision-making in the context of global warming. Imagine a policy-maker who has to select the optimal environmental policy to put in place to address climate change. In this problem, the first layer of risk may be represented by a given probability distribution over the long-term temperature response to CO_2 emissions. As multiple instances of this distribution exist (depending on the climate models employed or the type of data used to estimate the probabilistic relationship) a second layer of model uncertainty emerges as regards the uncertainty across these different climate models. Lastly, the potential misspecification of the existing climate models gives rise to the third layer.[2]

This paper presents an empirical account of the three layers of uncertainty and, to our knowledge, the first examination of model misspecification in a laboratory environment. Previous experimental research under the standard Ellsberg (1961) paradigm has so far concerned the layers of risk and model uncertainty exclusively. Specifically, the ambiguous two-color

[1]Since Ellsberg (1961), the term "ambiguity" has emerged in the literature to characterize situations in which probabilities are unknown (Knightian uncertainty). Accordingly, situations encompassing the layers of model uncertainty and model misspecification are *ambiguous*.

[2]For a further discussion in the context of climate change, see Brock and Hansen, 2017, and Berger and Marinacci, 2020.

urn containing N balls provides implicitly $N+1$ potential compositions, each of which constitutes a risk, whereas the distribution over the compositions, unknown to the DM, relates to model uncertainty. However, as the set of possible distributions is fully specified in the Ellsberg urn, such a setting necessarily leaves out the issue of model misspecification. For that reason, our experiment entails a new, *extended* Ellsberg setting, in which the number of possible compositions is first left unspecified. Then, by changing the information about the possible compositions, we implement the instances of (compound) risk, model uncertainty, and model misspecification, in accordance with Hansen (2014), and Hansen and Marinacci (2016). Our design enables us to isolate the effect of the layer of model uncertainty from that of risk by comparing situations characterized by a known probability distribution over possible urn compositions with situations in which the distribution over these possible compositions is unknown. We further isolate the effect due to the layer of model misspecification by considering situations in which alternative compositions, outside the initial set of likely ones, cannot be excluded.

Our investigation complements and extends previous empirical research on ambiguity and its relation to the multiple-stage presentation of uncertainty (e.g., Halevy, 2007; Abdellaoui, Klibanoff, and Placido, 2015; Chew, Miao, and Zhong, 2017). Besides the existence of attitudes towards compound risk, our data document distinct attitudes towards the layers of model uncertainty and model misspecification. These attitudes are shown to explain from 19% to 38% of the overall variation in uncertainty attitudes of a typical sample of university students. Thus, we provide a comprehensive view on the role of stages and layers of uncertainty in explaining individual ambiguity attitudes. Relying on different theoretical approaches, we show that a behavioral distinction can be made between two types of ambiguity preferences: (a) one that specifically relates ambiguity to the epistemic layers of uncertainty (i.e., model uncertainty and model misspecification) and (b) another that relates ambiguity to different stages of uncertainty. The former type of preferences is observed in 22% to 41% of our university student sample, and in 27-28% of a sample of risk professionals. Finally, our experiment reveals the importance of testing the theories of ambiguity on subject pools possessing different backgrounds, as well as of using more realistic decision environments, which are better able to capture the inherent complexity involved in situations of model misspecification.

The paper is organized as follows. Section 2 considers different modern theories of choice under uncertainty and discusses how they deal with the three layers. Readers who are only interested in our experiment can skip this part and directly go to Section 3, which focuses on the design and execution of our experiments. Section 4 defines the measures of absolute and relative premia used to analyze attitudes towards uncertainty and states our predictions. The results of our main experiment, which was conducted with a standard pool of social science students, are presented in Section 5. The results of a follow-up experiment, with a pool of risk professionals, are presented in section 6. We discuss our results in relation to the extant experimental literature in Section 7. Section 8 concludes.

2 Decision theoretic background

In this section, we provide an overview of some prominent ambiguity theories that accommodate a multi-stage representation of uncertainty, and discuss how these theories deal with the three layers of uncertainty.[3] Let S denote the set of states of the world and C the set of consequences. Formally, a prospect (or an act) is a function $\mathcal{P}_i : S \to C$ mapping states into consequences. We consider a DM who has a complete and transitive preference relation $\succsim$ over prospects. We assume that the DM knows that states are generated by a probability model m that is presumed to belong to a collection M, which is taken as a datum of the decision problem.[4] Each model m represents the inherent randomness governing states' realizations. Model misspecification potentially arises by allowing for the possibility of alternative distributions outside the initial set M.[5]

[3]Further theoretical developments are provided in Online Appendix S1.

[4]Following Marinacci (2015), it should be clear that the physical information M is taken as a primitive of the decision-making problem. Formally, we assume the existence of a measurable space (S, Σ) , where Σ is an algebra of events of S. A model $m : \Sigma \to [0, 1]$ is thus a probability measure, and the collection M is a finite subset of $\Delta(S)$, the collection of all probability measures.

[5]Note that the majority of the frameworks presented in this section cannot explicitly accommodate model misspecification as a layer structurally distinct from that of model uncertainty. We thus concentrate on frameworks that take a well-defined set M as a datum of the decision problem. A notable exception is Cerreia-Vioglio et al. (2020), which consists in a generalization of Wald's criterion presented in what follows.

2.1 Subjective expected utility

Traditionally, economists have dealt with uncertainty by following the subjective expected utility (SEU) approach (Savage 1954). In line with the Bayesian tradition, this approach favors quantifying uncertainty in probabilistic terms and treats any source of uncertainty as *risk*, reducing uncertainty *de facto* to its first layer. In the two-layer version of SEU, axiomatized by Cerreia-Vioglio et al. (2013b), it is assumed that the DM has a subjective prior probability measure $\mu : 2^M \to [0,1]$ quantifying the epistemic uncertainty about the models $m \in M$. The subjective probabilities (or priors) reflect the structural information received and some personal information that the DM may have on the models. The SEU of a prospect $\mathcal{P}_i$ is

$$V_{\text{SEU}}(\mathcal{P}_i) = \mathbb{E}_\mu \left(\mathbb{E}_m u \left(\mathcal{P}_i \right) \right), \tag{1}$$

where $u : C \to \mathbb{R}$ is the von Neumann-Morgenstern utility function capturing risk attitude and $\mathbb{E}$ is the expectation operator (taken with respect to the measures μ and m, respectively).[6] Criterion (1) is a two-stage criterion that describes the different layers of uncertainty through standard probability measures. The same attitude is considered towards the layers of risk and model uncertainty, therefore implying ambiguity neutrality.

2.2 Theories making a distinction between layers of uncertainty

The first family of approaches alternative to SEU allows for different attitudes towards different *layers* of uncertainty. The maxmin criteria of Wald (1950) and Gilboa and Schmeidler (1989) make a distinction between the layers of risk and model uncertainty by departing from the Bayesian framework, while the smooth criterion of Klibanoff, Marinacci, and Mukerji (2005) remains in the Bayesian paradigm.

Wald The decision criterion of Wald (1950) considers only the worst model among all the possible models affected by epistemic uncertainty:

$$V_{\text{Wald}}(\mathcal{P}_i) = \min_{m \in M} \mathbb{E}_m u \left(\mathcal{P}_i \right). \tag{2}$$

[6]In other words, expression (1) corresponds to $V_{\text{SEU}}(\mathcal{P}_i) = \sum_m \mu(m) \left(\sum_s p(s|m) u \left(\mathcal{P}_i(s) \right) \right)$ in its discrete version, where $p(s|m)$ is the objective probability of state s conditional on model m, and to $V_{\text{SEU}}(\mathcal{P}_i) = \int_m \left(\int_S u \left(\mathcal{P}_i(s) \right) \mathrm{d}m(s) \right) \mathrm{d}\mu(m)$ in its continuous version.

The layer of risk is not affected by the extreme cautiousness entailed by this criterion. For example, it is perfectly conceivable for a DM to be neutral to compound risk (i.e. when there is an *objectively* determined probability measure μ over the set M) while being extremely averse to model uncertainty.

Multiple priors The multiple priors (MP) criterion axiomatized by Gilboa and Schmeidler (1989) is less extreme than that of Wald. In this framework, the DM's information about epistemic uncertainty is quantified by a set C of priors μ, and the decision is based on the prior giving the least favorable SEU. The two-layer version of this criterion is written

$$V_{\mathrm{MP}}(\mathcal{P}_i) = \min_{\mu \in \mathrm{C}} \mathbb{E}_{\mu} \left[\mathbb{E}_m u \left(\mathcal{P}_i \right) \right]. \tag{3}$$

This formulation encompasses the original version of the MP model proposed by Gilboa and Schmeidler (1989).[7] Under the MP model, which is built within the Anscombe and Aumann (1963) framework, the layer of risk is evaluated under the expected utility criterion of von Neumann and Morgenstern (1944).

Smooth ambiguity model The smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2005, hereafter KMM) adheres to the Bayesian framework, but generalizes the classical SEU approach by allowing different attitudes towards the layers of risk and model uncertainty. The utility of a prospect $\mathcal{P}_i$ under the smooth criterion is

$$V_{\mathrm{KMM}}(\mathcal{P}_i) = \mathbb{E}_{\mu} \phi \left[\mathbb{E}_m u \left(\mathcal{P}_i \right) \right], \tag{4}$$

where $\phi = v \circ u^{-1}$. The strictly increasing continuous function $v : C \to \mathbb{R}$ captures the attitude towards the layer of model uncertainty (Marinacci, 2015). When $v = u$, thus when ϕ is linear, the attitudes towards the layers of risk and model uncertainty are identical. In this case, the smooth ambiguity criterion reduces to the SEU representation (1). Ambiguity aversion results from the concavity of ϕ, which arises when v is more concave than

[7] The original version is recovered when considering the predictive subjective probabilities $\bar{\mu}(s) \equiv \sum_m \mu(m)p(s|m)$ (see Online Appendix S1 for details).

u.[8] Note that when two stages of risk are involved, each stage is evaluated by the same function u, so that criterion (4) collapses to criterion (1).

Robustness Finally, alternative attempts to deal with a general concern about the epistemic uncertainty surrounding the correct probability model also appear in the vast literature on robust control theory (e.g., see Petersen, James, and Dupuis, 2000; Hansen and Sargent, 2001, 2008). A generalized decision criterion under this approach, which is axiomatized by Maccheroni, Marinacci, and Rustichini (2006), is known as the *variational* criterion. In its two-layer version (Cerreia-Vioglio et al., 2013a), this criterion is written as

$$V_{VR}(\mathcal{P}_i) = \min_{\mu \in \Delta(M)} \mathbb{E}_\mu \left[\mathbb{E}_m u\left(\mathcal{P}_i\right) + c\left(\mu\right) \right]. \tag{5}$$

Intuitively, the DM here considers all the possible priors over the models in M and penalizes the uncertainty about each prior by a cost function c. The choice to minimize reflects the aversion to ambiguity over the selection of the prior, or a concern about prior misspecification (Hansen and Marinacci, 2016). When the cost function has a particular form (that assigns 0 to priors inside a convex set C and infinity otherwise), criterion (5) coincides with criterion (3). By contrast, if the cost function has the relative entropy form (with respect to a reference prior), criterion (5) is the *multiplier* decision criterion of Hansen and Sargent (2001, 2008).[9] As the DM, under this approach, typically begins from a unique baseline model without explicitly specifying alternative models, the robust control theory generally does not draw a clear distinction between model uncertainty and model misspecification. In recent contributions, however, Hansen and Sargent (2007, 2018) extend this framework by considering a set of models that are surrounded by a set of alternatives that are nearby in terms of entropy measures.

[8] See also the model of Nau (2006), which, at least in special cases, takes the same representation as (4) and shares the same interpretation as KMM.

[9] The multiplier decision criterion can also be equivalently written in the smooth ambiguity form when $\phi(x) = -e^{-\lambda x}$ (Hansen and Sargent, 2007; Cerreia-Vioglio et al., 2011).

2.3 Theories making a distinction between stages of uncertainty

Another family of theories models ambiguity as multiple *stages* of uncertainty while not necessarily making a distinction between the layers. For example, Segal's (1987; 1990) and Seo's (2009) approaches take any source of ambiguity as compound risk and relax the reduction principle to capture non-neutral attitudes towards ambiguity.[10]

Recursive rank dependent utility model Segal's (1987; 1990) recursive rank dependent utility (RRDU) approach proposes evaluating the first and second stages of uncertainty by using Quiggin's (1982) rank dependent utility.[11] In the RDU model of Quiggin, the lottery $x = (x_1, p_1; ...; x_n, p_n)$, with $x_1 \geq ... \geq x_n$, is evaluated by

$$V_{\text{RDU}}(x) = u(x_n) + \sum_{s=2}^{n} [u(x_{s-1}) - u(x_s)]\, f\left(\sum_{t=1}^{s-1} p_t\right). \tag{6}$$

In this expression, $f : [0,1] \to [0,1]$, with $f(0) = 0$ and $f(1) = 1$, is an increasing transformation function, which is also convex under (global) uncertainty aversion.[12] Segal's RRDU first computes and ranks the certainty equivalents (CEs) derived for each model $m \in M$ using (6) and then applies formulation (6) recursively to the distribution of these CEs induced by the probability measure μ. Ergin and Gul (2009) generalize Segal's approach by allowing for different transformation functions for the aleatory and epistemic layers of uncertainty present at each stage.[13]

Seo's approach Seo (2009) assumes distinct expected utilities in the different *stages*, using a criterion analogous to that presented in (4). Under

[10]Segal (1987, pp. 178-179) writes "Indeed, most writers in this area, including Ellsberg himself, suggested a distinction between ambiguity (or uncertainty) and risk. One of the aims of this paper is to show that (at least within the anticipated utility framework) there is no real distinction between these two concepts.". Thus, he proposes "risk aversion and ambiguity aversion are two sides of the same coin, and the rejection of the Ellsberg urn does not require a new concept of ambiguity aversion, or a new concept of risk aversion."

[11]An alternative approach, using Gul's (1991) disappointment aversion, is presented in Online Appendix S1.3.

[12]Note that the common empirical finding in the literature is local uncertainty seeking for low likelihood events and uncertainty aversion for moderate and high likelihood events, implying an inverse S-shaped–concave and then convex–f function (see Wakker, 2010).

[13]See also Chew and Sagi (2008), and Abdellaoui et al. (2011), who model ambiguity attitudes by source-dependent preferences in single-stage setups.

this interpretation, u and v each capture the attitude towards one particular stage of uncertainty. As a consequence, ambiguity aversion arises in the same way as nonreduction of compound risk.

3 Experimental design

This section presents our experimental design. We use a within-subject design to examine choices under different sources of uncertainty.[14] These sources potentially encompass different layers of uncertainty. We run two distinct experiments using the same design: the main experiment took place in a laboratory with university students, whereas the follow-up experiment took place in the field, with risk professionals. Both experiments used real monetary incentives.

3.1 The sources of uncertainty

We consider five sources of uncertainty. These sources are constructed in an *extended* Ellsberg two-color setting using decks, which contain an unspecified number of cards.[15] These sources can be characterized by different deck compositions, defined in terms of their proportion of black cards (and the complementary proportion of red cards).

1. *Simple risk*, denoted SR, entails a deck containing an equal proportion of black and red cards;

2. *Compound risk*, denoted CR, entails a deck that contains either $P\%$ black or $(100 - P)\%$ black cards with equal probability;

3. *Model uncertainty*, denoted MU, entails a deck that contains either $P\%$ black or $(100 - P)\%$ black cards with unknown probability;

4. *Model misspecification*, denoted MM, entails a deck that is likely (i.e., with at least 50% probability) to contain either $P\%$ black or

[14] We adopt the definition of "sources" of uncertainty, proposed by Abdellaoui et al. (2011, p. 696), as "groups of events that are generated by the same mechanism of uncertainty, which implies that they have similar characteristics."

[15] For the sake of comprehensiveness and to allow comparisons with previous literature, we also consider the standard two-color Ellsberg ambiguous situation, in which the ambiguous deck contains 100 cards. This source was always presented at last in the experiment to prevent a priming effect about the number of cards in the decks. Online Appendix S2 presents a discussion of this extra source.

$(100 - P)\%$ black cards and may, or may not, contain any other proportion of black cards; and

5. *Extended Ellsberg*, denoted EE, entails a deck that contains an unknown proportion of black and red cards.

The sources CR, MU, and MM differ in terms of the layers of uncertainty they encompass. Specifically, CR entails only the layer of risk, even if it is presented in a compound way (i.e., with two different *stages*). Under CR, the two possible deck compositions, $P\%$ and $(100 - P)\%$ black cards, are each unambiguously assigned an objective probability of 50%. Conversely, the source MU entails two layers of uncertainty: one of model uncertainty and one of risk. Under MU, the two possible deck compositions can only be assigned *subjective* probabilities.[16] The source MM entails all the three layers of uncertainty together, adding the extra layer of model misspecification as follows: although the probability distribution is likely to be characterized by one of the two compositions as in MU, we cannot exclude the possibility of an alternative composition. Therefore, our MM treatment can be interpreted as an extended form of MU, in which the set of potential probability models is enlarged beyond the one explicitly presented. This is a faithful representation of what is generally regarded as model misspecification (Hansen and Sargent, 2001; Hansen and Marinacci, 2016). Finally, EE corresponds, in spirit, to Ellsberg's (1961, p. 643) ambiguous situation in which "numerical probabilities are inapplicable."

3.2 Procedure in the main experiment

The main experiment was run on computers. Subjects were seated in cubicles and could not communicate with one another during the experiment. Each session started with the experimental instructions, examples of the stimuli, and comprehension questions. A typical session lasted approximately one hour, including instructions and payment. Complete instructions are provided in the Online Appendix.

[16]Under the symmetry assumption, which relies on the principle of insufficient reason, these subjective probabilities might be assumed to be 50%. In an additional experiment, we specifically test the symmetry assumption and show that it is respected at both the aggregate and individual levels (see Online Appendix S3).

Subjects Five experimental sessions took place at the Bocconi Experimental Laboratory for Social Sciences in Bocconi University, Italy. The subjects were 125 university students (average age 20.5 years, 52 women). Subjects were recruited on a voluntary basis.

Stimuli Subjects faced monetary prospects under the five sources of uncertainty introduced previously. For CR, MU, and MM, we considered two different cases: one with $P = 0$ and the other with $P = 25$. For example, CR with $P = 0$ entails a deck that contains either 0% black or 100% black cards with equal probability, whereas CR with $P = 25$ entails a deck that contains either 25% black or 75% black cards with equal probability. MU and MM are built analogously. We denote the respective cases as $CR0$, $CR25$, and so forth. Thus, there were eight monetary prospects in total: SR, $CR0$, $CR25$, $MU0$, $MU25$, $MM0$, $MM25$, and EE. Each prospect gave the subjects either €20 or €0, depending on the color of a card randomly drawn from a deck. In every prospect, the color giving €20 (black or red) was selected by the subjects themselves.

Table 1 provides a summary of the eight prospects. They were im-

Table 1: Summary of the prospects used in the experiment

Source	Prospect	# of layers of uncertainty	Set of possible proportions of black cards (M)	Information available about the distribution over the set of possible proportions (μ) [b]
SR	SR	1	$\{50\%\}$	$\mu(50\%) = 1$
CR	$CR0$	1	$\{0\%, 100\%\}$	$\mu(0\%) = \mu(100\%) = 0.5$
	$CR25$	1	$\{25\%, 75\%\}$	$\mu(25\%) = \mu(75\%) = 0.5$
MU	$MU0$	2	$\{0\%, 100\%\}$	$\mu(0\%) + \mu(100\%) = 1$
	$MU25$	2	$\{25\%, 75\%\}$	$\mu(25\%) + \mu(75\%) = 1$
MM	$MM0$	3	$X \cup \{0\%, 100\%\}$ [a]	$\mu(0\%) + \mu(100\%) \geq 0.5$
	$MM25$	3	$X \cup \{25\%, 75\%\}$ [a]	$\mu(25\%) + \mu(75\%) \geq 0.5$
EE	EE	3	X [a]	$\sum_{m \in M} \mu(m) = 1$

[a] For $MM0$, $MM25$, and EE, X is an unspecified subset of $[0, 1]$.

[b] $\mu(m)$ represents the subjective prior probability attached to model $m \in M$.

plemented as follows: In SR, the subjects were instructed that the deck contained an equal proportion of red and black cards. In the cases of CR, MU, MM, and EE, subjects were instructed that the deck was going to be picked randomly from a pile of decks. In $CR0$ ($CR25$), the pile consisted of decks containing 0% (25%) black cards and decks containing 100%

(75%) black cards, with an equal proportion of each. In $MU0$ ($MU25$), the pile also consisted of decks containing 0% (25%) black and decks containing 100% (75%) black cards, but with an unknown proportion of each. In $MM0$ ($MM25$), the subjects were instructed that at least half of the pile consisted of decks containing 0% (25%) black and decks containing 100% (75%) black cards with an unknown proportion of each. Notably, the subjects were also informed that the pile considered may (or may not) contain decks with compositions other than the two described. In EE, the pile consisted of decks containing red and black cards, each with an unknown composition.

One of the collaborators, who was not present in the room during the experimental sessions, constructed all the decks and piles. Thus, no one in the room, including the experimenters, had any additional information about the content of the decks and piles, other than what was described in the experimental instructions.[17] The subjects were also reminded that they could check the piles and the decks at the end of the experiment to verify the truthfulness of the information provided. During the instructions, subjects were given examples of the different sources of uncertainty that they would face throughout the experiment. We tested their understanding of the differences between the sources through comprehension questions, for which they were given automatic feedback.

We elicited the certainty equivalents (CEs) of the eight prospects using a choice list design. Specifically, in each prospect, the subjects were asked to make 12 binary choices between the prospect of receiving €20 and receiving a sure monetary amount ranging from €0 to €20. The sure amounts were incremented by €2 between €1 and €19, and the order of the prospects was randomized. After completing the choice lists, the subjects answered a short sociodemographic survey.

Incentives Subjects received a €5 show-up fee. In addition, they received a variable amount depending on one of the choices they made during the experiment. The choice situation on which the payment was based was the same for all the subjects in the same experimental session. In practice, the 12 binary choice questions of the choice lists and the descriptions of

[17]The subjects were informed accordingly to prevent the effects of comparative ignorance, according to which a comparison with a more knowledgeable individual (in this case, the experimenter) may induce ambiguity aversion (Fox and Tversky, 1995).

the uncertain situations were printed on paper and physically enclosed in sealed envelopes before every session. In each session, a volunteer from the subject pool randomly picked two envelopes before the experiment started: one containing an uncertain situation and the other containing a question from the choice lists. After the two envelopes had been picked, they were attached, still sealed, to a white board visible to all participants. The subjects were informed that the choice situation that would matter for their payment was contained in the envelopes, which would remain visible and closed until the end of the experiment. When all subjects had completed the questionnaire, the envelopes were opened and their contents were revealed. The draws from the piles and/or from the decks were made according to the uncertain situation contained in the first envelope and the subjects were paid according to their response to the choice question contained in the second envelope.[18]

3.3 Procedure in the follow-up experiment with actuaries

The subject pool in the follow-up experiment consists of 84 risk professionals (average age 40 years, 37 women), who attended the 31st International Congress of Actuaries in Berlin, Germany. The majority of the subjects were highly educated in the fields of mathematics, statistics, and actuarial science. The subjects had an average of 13 years of work experience in the insurance and finance industries. The subjects faced the same stimuli as in the main experiment, except that the stakes were multiplied by a factor of 10 (a correct bet yielded €200, instead of €20). To reduce

[18] The random incentive system (RIS) is one of the most commonly used mechanisms for individual choice experiments in economics. However, its empirical validity has often been a topic of debate (Starmer and Sugden, 1991; Cox, Sadiraj, and Schmidt, 2015). Assuming statewise monotonicity and nothing else, Azrieli, Chambers, and Healy (2018) recently proved that the RIS is essentially the only incentive-compatible mechanism. Note, however, that monotonicity is violated when expected utility is violated, causing potential violation of incentive compatibility under ambiguity aversion (e.g., if subjects integrate their decisions into one larger lottery, see Holt, 1986). Yet the *isolation* assumption of Kahneman and Tversky (1979) is equivalent to monotonicity and therefore sufficient to guarantee incentive compatibility: as long as subjects approach each task as if it were an isolated decision, the RIS is incentive compatible regardless of the subjects' preferences. The isolation hypothesis has been supported empirically by Hey and Lee (2005). The *prior* incentive system of Johnson et al. (2015) that we use (and which performs the randomization *before*, rather than after, the choices and the resolutions of uncertainty) precisely aims to enhance isolation to minimize potential biases, thereby preventing subjects from hedging over the randomization between problems (see Baillon et al., 2015; for a demonstration of its incentive compatibility, and Epstein and Halevy, 2019; for a recent application).

the expected experimental costs and monetary transactions during the conference schedule, only a fraction of the subjects (one out of 10) was paid based on one of their choices. More details on the subject pool and the procedures are presented in Appendix B.[19]

4 Uncertainty premia and predictions

In what follows, we take the midpoint of an indifference interval implied by the switching point in the choice list as a proxy for the CE of the prospect. Switching in the middle of the list implies a CE equal to the expected value (EV) of the prospect, which is €10 in our main experiment (and €100 in our follow-up experiment).

4.1 Absolute and relative uncertainty premia

We use the following notions of uncertainty premium to analyze our results.

Definition 1. The *absolute uncertainty premium* Π_i is defined as

$$\Pi_i \equiv EV_i - CE_i$$

for all $i \in \{SR, CR0, CR25, MU0, MU25, MM0, MM25, EE\}$.

In words, the absolute premium represents the amount of money that an individual is willing to pay to receive the expected value of the prospect with certainty, rather than facing the uncertainty. The premium is positive (resp. zero, or negative) when a subject is averse (resp. neutral, or loving) to the uncertainty in prospect i. The most well-known absolute uncertainty premium is the standard risk premium, noted Π_{SR}.

Because we are interested in comparing the way people react to different sources and layers of uncertainty, a more directly relevant measure for our purposes is the notion of relative premium.

Definition 2. The *relative uncertainty premium* $\Pi_{i,j}$ is defined as

$$\Pi_{i,j} \equiv \Pi_j - \Pi_i$$

[19]Note that a portion of the same data set was used, in comparison with our student sample, in a distinct follow-up project investigating the reduction principle (Aydogan et al., 2019).

for all $i, j \in \{SR, CR0, CR25, MU0, MU25, MM0, MM25, EE\}$.

Given that all the prospects in our experiment have the same EV, the relative premium may equivalently be expressed as the difference between the CEs of the prospects i and j:

$$\Pi_{i,j} = CE_i - CE_j.$$

The relative premium is positive (resp. zero, or negative) when a subject is more (resp. as much, or less) averse to the uncertainty in prospect j than that in prospect i. The version of this premium that has been typically considered in the literature is the one relative to simple risk, so that $\Pi_{SR,j}$ refers to the so-called *compound risk premium* when $j \in \{CR0, CR25\}$ (see, e.g., Abdellaoui, Klibanoff, and Placido, 2015), and to the *ambiguity premium* when $j \in \{MU0, MU25, MM0, MM25, EE\}$ (see, e.g., Maccheroni, Marinacci, and Ruffino, 2013). In our experiment, two other relative premia will be of special importance for analyzing the results. The first, which is called the *model uncertainty premium*, refers to the marginal effect between the first and the second layers of uncertainty. It measures what an individual is ready to pay to avoid being confronted with epistemic uncertainty in the second stage. To isolate the effect of model uncertainty alone (filtering out the effect due to multiple stages), it is measured in relation to compound risk, by $\Pi_{CR0,MU0}$ and $\Pi_{CR25,MU25}$. The second important premium for our analysis, called the *model misspecification premium*, measures the marginal effect between the second and the third layers of uncertainty. It is written $\Pi_{MU0,MM0}$ or $\Pi_{MU25,MM25}$. Overall, the total effect due to the epistemic layers of uncertainty can thus be measured by $\Pi_{CR,MM} (= \Pi_{CR,MU} + \Pi_{MU,MM})$.

4.2 Predictions

We can now use these premia to make predictions following the different theories of choice under uncertainty presented in Section 2. We first use the sources CR, MU, and MM to draw general predictions under three hypotheses, labelled *Expected Utility (EU) hypothesis*, *layer hypothesis* and *stage hypothesis*. We then use the source EE to examine what these hypotheses imply for attitudes towards Ellsberg ambiguity.

Under the EU hypothesis, the ambiguity and the compound risk premia are all equal to zero.

EU hypothesis:

$$\Pi_{SR,CR} = \Pi_{SR,MU} = \Pi_{SR,MM} = 0. \tag{7}$$

This hypothesis thus predicts reduction of compound risk as well as ambiguity neutrality. In other words, distinct *layers* of uncertainty are treated in the same way, and different *stages* are reduced to a single one. In consequence, the EU hypothesis also predicts zero model uncertainty and model misspecification premia, i.e., $\Pi_{CR,MU} = 0$ and $\Pi_{MU,MM} = 0$.

Instead, if a distinction is made between different *layers* of uncertainty in accordance with the theories presented in Section 2.2, we expect the model uncertainty premium and/or the model misspecification premium to be non-zero (i.e., $\Pi_{CR,MU} \neq 0$ or $\Pi_{MU,MM} \neq 0$, or both). Because both sources SR and CR only entail the layer of risk, these theories often assume compound risk reduction (see, e.g., the smooth model of Klibanoff et al. (2005), or the multiple priors model of Gilboa and Schmeidler (1989), which adopts an Anscombe and Aumann (1963) framework). Hence, the layer hypothesis is summarized as follows.

Layer hypothesis:

$$\begin{cases} \Pi_{SR,CR} = 0 \\ \Pi_{CR,MU} \neq 0 \text{ or } \Pi_{MU,MM} \neq 0 \text{ (or both).} \end{cases} \tag{8}$$

Finally, if we follow the theories that relate ambiguity attitudes to attitudes towards different *stages* of uncertainty, we predict violation of the reduction of compound risk axiom (i.e., non-indifference between a compound risk and its reduced–simple risk–form). We formulate the stage hypothesis as follows.

Stage hypothesis:

$$\Pi_{SR,CR} \neq 0.$$

Note that Segal's (1987; 1990) theory assumes a weaker condition than the reduction of compound risk axiom, which is known as *time neutrality* (i.e., indifference between the resolution of uncertainty in the first or in the second stage) and implies $\Pi_{SR,CR0} = 0$ in our experiment. Other models in general also predict violation of time neutrality by assuming distinct attitudes within different stages (e.g., Seo, 2009). Some two-stage models can further capture the distinction between aleatory and epistemic uncertainty by incorporating source preferences within stages (e.g., Ergin and Gul, 2009). In principle, our stage hypothesis encompasses two different types of behavior pertaining to attitudes towards the layers of uncertainty: one that further distinguishes the different layers (for which $\Pi_{CR,MU} \neq 0$ *or* $\Pi_{MU,MM} \neq 0$, or both) and another that does not make any distinction between different layers (for which $\Pi_{CR,MU} = 0$ and $\Pi_{MU,MM} = 0$).

We can then investigate what these hypotheses imply for attitudes towards Ellsberg ambiguity. Specifically, consistent with what precedes, it should be clear that the EU hypothesis predicts Ellsberg ambiguity neutrality (i.e., $\Pi_{SR,EE} = 0$). Next, because EE encompasses both model uncertainty and model misspecification,[20] the layer hypothesis predicts a positive correlation between the total effect of these two epistemic layers and the attitude towards Ellsberg ambiguity: $\operatorname{corr}(\Pi_{SR,EE}, \Pi_{CR,MM}) > 0$. Finally, as ambiguity non-neutrality corresponds to nonreduction of compound risk under the theories making a distinction between the stages of uncertainty, we expect to observe a positive correlation between attitudes towards compound risk and towards Ellsberg ambiguity under the stage hypothesis: $\operatorname{corr}(\Pi_{SR,EE}, \Pi_{SR,CR}) > 0$.

5 Results of the main experiment

5.1 Data

The data we collected in the main experiment consist of 124 observations for $MU25$ and 125 observations for the rest of the prospects.[21] We did not include 36 (3.6% of all) choice lists from 13 different subjects in the analysis, because they involve multiple-switching, no-switching, or reverse-switching patterns.[22] The proportion of these patterns is significantly less

[20]The source EE entails a form of model misspecification because even if a set of possible compositions can be postulated, this set may still not contain the correct composition.

[21]One subject omitted answering the choice situation $MU25$ by mistake.

[22]This way to proceed is standard in the experimental literature using choice list designs (see, e.g., Dean and Ortoleva, 2019). It is justified on the grounds that these data are not compatible with standard assumptions on preferences (e.g., monotonicity in money) and that

than the typical 10% observed in the literature (Yu, Zhang, and Zuo, 2020). However, our results are also robust to the inclusion of data with multiple switching patterns. We do not observe any order treatment effect on the CEs. (These results are reported in Online Appendices S4 and S5, respectively.)

5.2 Attitudes towards uncertainty

We first examine attitudes towards uncertainty based on the absolute and relative uncertainty premia. Unless mentioned otherwise, we report the results with two-sided t-tests. The use of non-parametric tests does not alter our main conclusions, thus we mention them only when they affect the results.

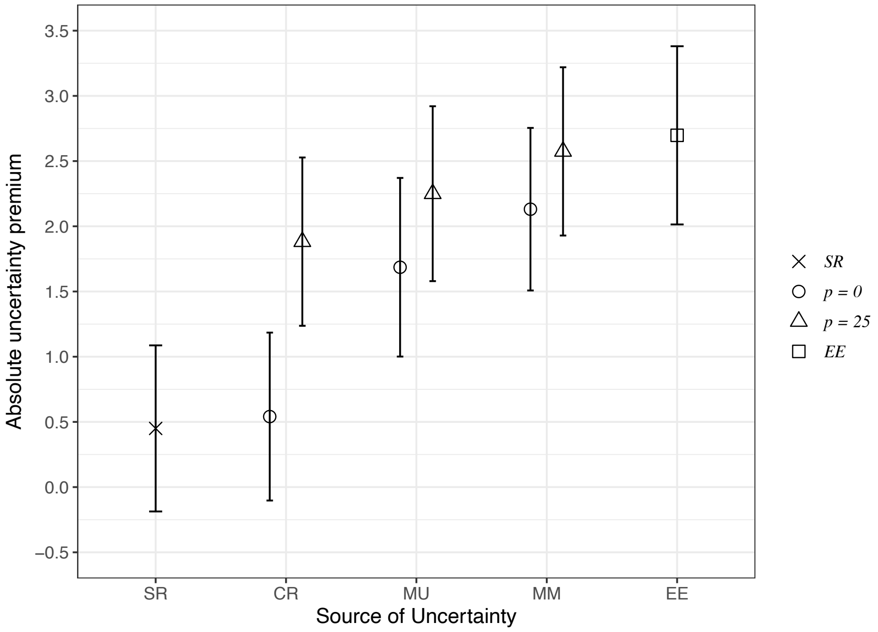

Figure 1: Mean absolute uncertainty premia and corresponding 95% confidence intervals for the eight prospects.

Figure 1 summarizes the mean *absolute* uncertainty premia (Π_i) for the different prospects. The data indicate an increasing trend in absolute premia from the sources of risk (SR and CR) to the sources of ambiguity (MU, MM, and EE). The average risk premium Π_{SR} is not significantly different from zero, indicating risk neutrality (p=0.16), which is reasonable for

they might be due to a lack of understanding of the choice tasks.

small and moderate monetary gains as the ones in our experiment. On the contrary, we observe positive absolute premia for the rest of our sources of uncertainty (p<0.001, for all), except for compound risk with $P = 0$ (p=0.099). An analysis of variance (ANOVA) with repeated measures indicates different absolute premia for CR, MU and MM in both treatments with $P = 0$ and $P = 25$ ($p < 0.001$ and $p = 0.001$, respectively).

Table 2 summarizes the mean *relative* uncertainty premia. We first focus on those relative to simple risk (i.e., compound risk and ambiguity premia). Our data indicate ambiguity aversion: we find positive ambiguity premia under the sources MU, MM, and EE (p<0.001, for all). The compound risk premium is also positive when $P = 25$ (p<0.001) but not when $P = 0$ (p=0.58). The proportions of zero, positive, and negative relative premia are reported in Appendix A.

Table 2: Relative uncertainty premia

	$P = 0$		$P = 25$	
Source	Attitude towards compound risk and ambiguity	Attitude towards model uncertainty and model misspecification	Attitude towards compound risk and ambiguity	Attitude towards model uncertainty and model misspecification
CR	$\Pi_{SR,CR0} = 0.10$	$\Pi_{CR0,MU0} = 1.11^{***}$	$\Pi_{SR,CR25} = 1.40^{***}$	$\Pi_{CR25,MU25} = 0.50^{**}$
MU	$\Pi_{SR,MU0} = 1.18^{***}$	$\Pi_{MU0,MM0} = 0.50^{*}$	$\Pi_{SR,MU25} = 1.81^{***}$	$\Pi_{MU25,MM25} = 0.37$
MM	$\Pi_{SR,MM0} = 1.71^{***}$		$\Pi_{SR,MM25} = 2.10^{***}$	
EE	$\Pi_{SR,EE} = 2.30^{***}$		$\Pi_{SR,EE} = 2.30^{***}$	

Notes: *** p-value<0.001 ** p-value<0.01 * p-value<0.05, based on two-sided t-tests.

Next, we focus on the relative premia for the layers of model uncertainty and model misspecification. The data indicate positive model uncertainty premia (p<0.001 for $\Pi_{CR0,MU0}$, and p=0.009 for $\Pi_{CR25,MU25}$), corroborating the hypothesis of a clear distinction in attitudes towards the layers of risk and model uncertainty. Similarly, we observe a positive model misspecification premium, which is significant in the treatment with $P = 0$ (p=0.031)[23] although not significant at 5% level in the treatment with $P = 25$ (p=0.099). We summarize our first main result as follows.

Result 1: The layers of risk, model uncertainty, and model misspecification are perceived and treated differently by the subjects.

[23]The non-parametric sign-rank test rejects $\Pi_{MU0,MM0} = 0$ at 10% level (p=0.07).

5.3 Decomposing attitudes towards uncertainty

We now explore the role of stages and layers in the overall attitude towards uncertainty. For this, we conduct a principle component analysis of the *relative* premia for compound risk ($\Pi_{SR,CR}$) and ambiguity ($\Pi_{SR,MU}$ and $\Pi_{SR,MM}$).[24] The results are reported in Table 3.

Table 3: Principal Component Analysis of the relative Compound Risk and Ambiguity Premia

Variable	$P = 0$			$P = 25$		
	1st comp.	2nd comp.	3rd comp.	1st comp.	2nd comp.	3rd comp.
$\Pi_{SR,CR}$	0.54	0.82	0.21	0.57	-0.68	0.45
$\Pi_{SR,MU}$	0.58	-0.54	0.61	0.57	0.73	0.37
$\Pi_{SR,MM}$	0.61	-0.20	-0.77	0.58	-0.04	-0.81
Eigenvalue of the component	1.86	0.68	0.48	2.43	0.31	0.26
Proportion of variance explained	0.62	0.22	0.16	0.81	0.10	0.09

In both treatments with $P = 0$ and $P = 25$, the first component has positive and roughly equal loadings on the three premia. Hence, the first component can be interpreted as capturing the general attitude towards uncertainty presented in multiple stages, which is shared by all the three sources. This first component explains 62% of the variance in the treatment with $P = 0$ and 81% of the variance in the treatment with $P = 25$. The relatively low importance of the attitudes towards multiple stages in the treatment with $P = 0$ may possibly be explained by the fact that, despite being technically presented in two stages, this treatment essentially has only one *stage of uncertainty*. The second component has loadings of opposite signs on compound risk and model uncertainty, whereas its loading on model misspecification is relatively small. Hence, the second component corresponds to the attitude towards the layer of model uncertainty. Confirming this interpretation, the second component is indeed highly correlated with the model uncertainty premium in both treatments: its cor-

[24] Principal components are orthogonal linear combinations of the observed variables that are determined sequentially by maximizing the variance of the projected data. The variance explained by each component is its eigenvalue, which decreases monotonically from the first component to the last. Correlations between the original variables and the unit-scaled principal components are known as loadings.

relation is 0.93 with $\Pi_{CR0,MU0}$ and 0.998 with $\Pi_{CR25,MU25}$. The second component explains respectively 22% and 10% of the variance in the two treatments. Finally, the third component has high and negative loadings on model misspecification compared to the positive loadings on the other sources. The third component may thus be interpreted as capturing the attitude towards the layer of model misspecification. Indeed, the correlation between the third component and the model misspecification premium $\Pi_{MU0,MM0}$ ($\Pi_{MU25,MM25}$) is 0.96 (0.82).[25] The third component explains respectively 16% and 9% of the variance in each of the two treatments. We summarize our second main result as follows.

Result 2: The attitudes towards the sources CR, MU, and MM share a common component, which can be viewed as a general attitude towards uncertainty presented in multiple *stages*. This component explains the majority (in this case, 62% to 81%) of the variance in attitudes towards uncertainty. The rest of the variance is explained by components specific to the layers of model uncertainty (10% to 22%) and model misspecification (9% to 16%).

5.4 Individual-level analysis

We now present an individual-level analysis that classifies subjects' preferences according to the predictions made under the hypotheses presented in Section 4.2. Table 4 summarizes the results. As can be observed, 18% and 21% of the subjects (for the treatments with $P = 0$ and $P = 25$, respectively) made choices consistent with the EU hypothesis, thus exhibiting neutrality towards compound risk and ambiguity. Turning to the layer hypothesis, we observe that 41% and 22% of the subjects (for $P = 0$ and $P = 25$, respectively) reduce compound risk while at the same time exhibiting non-neutrality towards ambiguity under the sources MU and/or MM. Finally, for what concerns the stage hypothesis, we find that 34.5% and 46.5% of the subjects exhibit nonreduction of compound risk as well as distinct attitudes towards different layers (last column of Table 4); and only a few subjects exhibit both $\Pi_{CR,MU} = 0$ and $\Pi_{MU,MM} = 0$ under

[25]This third component also correlates strongly with the total premium for the epistemic layers of uncertainty for $P = 25$: its correlation with $\Pi_{CR25,MM25}$ is 0.88.

this hypothesis (respectively, 6% and 10.5% in the treatments $P = 0$ and $P = 25$).

Table 4: Classification of subjects

	EU hypothesis	Layer hypothesis	Stage hypothesis	
			Non distinction between layers	Distinction between layers
$P = 0$	18.1% (21/116)	41.4% (48/116)	6.0% (7/116)	34.5% (40/116)
$P = 25$	21.1% (24/114)	21.9% (25/114)	10.5% (12/114)	46.5% (53/114)

The validity of our individual classification is assessed by examining the subjects' attitudes towards Ellsberg ambiguity under the three hypotheses. In line with our predictions, we find that the majority of the subjects classified under the EU hypothesis also exhibit neutrality towards Ellsberg ambiguity ($\Pi_{SR,EE} = 0$): 17 out of 21 for $P = 0$; and 21 out of 24 for $P = 25$. Likewise, we find strong positive correlations between Ellsberg ambiguity and the total effect of the epistemic layers of uncertainty for the subjects classified under the layer hypothesis: Spearman's rank correlation coefficient corr($\Pi_{SR,EE}, \Pi_{CR0,MM0}$)=0.60 ($p$<0.001); and corr($\Pi_{SR,EE}, \Pi_{CR25,MM25}$)=0.76 ($p$<0.001).[26] Finally, the correlation between attitudes towards Ellsberg ambiguity and compound risk is positive for the subjects classified under the stage hypothesis: corr($\Pi_{SR,EE}, \Pi_{SR,CR0}$) =0.39 ($p$=0.008) and corr($\Pi_{SR,EE}, \Pi_{SR,CR25}$)=0.53 ($p$<0.001). We however find a weaker correlation between Ellsberg ambiguity and the epistemic layers of uncertainty for subjects under the stage hypothesis, which is only significant for $P = 0$ (corr($\Pi_{SR,EE}, \Pi_{CR0,MM0}$)=0.33, p=0.025), but not for $P = 25$ (corr($\Pi_{SR,EE}$,$\Pi_{CR0,MM0}$)=0.21, p=0.094). Overall, these results are summarized as follows.

Result 3.1: The large part of heterogeneity among individuals can be explained by three main types of behavior towards the layers and stages of uncertainty: *(a)* 35% to 47% of the subjects do not reduce compound risk

[26]Remark here that the correlation of Ellsberg ambiguity with the layer of model uncertainty alone is also significant: corr($\Pi_{SR,EE}, \Pi_{CR0,MU0}$)=0.40 ($p$=0.005); and corr($\Pi_{SR,EE}, \Pi_{CR25,MU25}$)=0.48 ($p$=0.018).

and, at the same time, exhibit different attitudes towards distinct layers; *(b)* 22% to 41% of the subjects have different attitudes towards distinct layers but reduce compound risk; and *(c)* 18% to 21% of the subjects present neutrality towards both compound risk and ambiguity. The proportion of a fourth type of behavior, associated with nonreduction of compound risk and no further distinction between different layers, is marginal (6% to 11% of the subjects).

Result 3.2: The attitude towards Ellsberg ambiguity encompasses attitudes towards the epistemic layers of uncertainty, as well as an attitude towards compound risk. In particular, Ellsberg ambiguity strongly correlates with the epistemic layers of uncertainty for subjects classified under the layer hypothesis (22-41%), and with compound risk for a significant proportion of subjects classified under the stage hypothesis (41-57%).

5.5 Discussion

Two important points are worth noting about this first set of results. First, consistent with the previous theoretical and empirical literature, our results confirm the importance of the presentation of uncertainty in multiple stages in explaining ambiguity attitudes. Here, we in addition show that the attitudes towards the epistemic layers of uncertainty alone (model uncertainty and misspecification) are relevant for explaining the behavior of 22% to 41% of the subjects. Second, our data suggest the existence of a specific attitude towards the layer of model misspecification. The marginal impact of the attitude towards this third layer of uncertainty, although smaller than that towards the layer of model uncertainty, accounts for 9% to 16% of the variation in overall attitudes towards uncertainty.

Different considerations can be made regarding these findings. First, the results we obtain may possibly be specific to our subject pool, and its potential limitations. Indeed, the relative importance of stages and layers of uncertainty might differ for groups of subjects with different characteristics. In particular, we expect that the role played by the presentation of uncertainty in multiple stages of risk could be less important for subjects who are more able to deal with computationally complex problems. A second consideration relates to the methodological limitations of the Ellsberg paradigm, and in particular, to its inability to significantly capture misspecification concerns. Whereas we are convinced that the three layers of uncertainty naturally arise in many real-life decision situations, it is plausible that the classical Ellsberg setting is too simplistic or too structured to capture the full impact of the third layer of model misspecification.

In what follows, we report the results of a second experiment aiming to investigate the first consideration. This follow-up experiment was conducted as an artefactual field experiment with a pool of risk professionals, who possess a high level of proficiency in probabilistic reasoning. The investigation of the three layers of uncertainty in more realistic environments is left to future research.

6 Results of the field experiment with actuaries

Table 5 reports the mean relative premia for the follow-up experiment run on risk professionals. We replicate the findings of our main experiment and observe positive ambiguity premia (p<0.01, for all $\Pi_{SR,MU}$, $\Pi_{SR,MM}$, and $\Pi_{SR,EE}$, in both treatments with $P = 0$ and $P = 25$), and a positive compound risk premium in the treatment with $P = 25$ (p=0.03), but not in the treatment with $P = 0$ (p=0.50). For what concerns the attitudes towards the layers of model uncertainty and model misspecification alone, we also replicate the positive model uncertainty premium (p<0.001 for $P = 0$ and p=0.009 for $P = 25$), whereas the model misspecification premium, although positive, is not significant (p>0.1 for both $P = 0$ and $P = 25$). The proportions of zero, positive, and negative relative premia are reported in Appendix B.

Our results start to differ starkly from those of the main experiment when we turn to the principle component analysis. As Table 6 suggests, the first component, which has higher loadings on the ambiguity premia than on the compound risk premium, now corresponds to a general attitude towards ambiguity. The first component explains 59% and 57% of the variation in the two treatments, respectively. The second component, on the other hand, has a high loading on CR and distinguishes it from MU and MM. Hence, the second component can be interpreted as an index of attitude towards compound risk. Indeed, the correlation of this component

Table 5: RELATIVE UNCERTAINTY PREMIA: ACTUARIES

Source	$P=0$		$P=25$	
	Attitude towards compound risk and ambiguity	Attitude towards model uncertainty and model misspecification	Attitude towards compound risk and ambiguity	Attitude towards model uncertainty and model misspecification
CR	$\Pi_{SR,CR0} = -1.08$	$\Pi_{CR0,MU0} = 13.36^{***}$	$\Pi_{SR,CR25} = 2.93^{*}$	$\Pi_{CR25,MU25} = 7.01^{**}$
MU	$\Pi_{SR,MU0} = 12.16^{***}$	$\Pi_{MU0,MM0} = 3.03$	$\Pi_{SR,MU25} = 10.53^{***}$	$\Pi_{MU25,MM25} = 3.91$
MM	$\Pi_{SR,MM0} = 15.27^{***}$		$\Pi_{SR,MM25} = 14.01^{***}$	
EE	$\Pi_{SR,EE} = 21.27^{***}$		$\Pi_{SR,EE} = 21.27^{***}$	

Notes: *** p-value<0.001 ** p-value<0.01 * p-value<0.05, based on two-sided t-tests.

with $\Pi_{SR,CR0}$ is 0.94 and with $\Pi_{SR,CR25}$ is 0.98. The second component captures 33% and 32% of the variation in the data. Lastly, the third component corresponds specifically to the attitude towards the layer of model misspecification: its correlation with $\Pi_{MU0,MM0}$ and $\Pi_{MU25,MM25}$ is 0.96 in both cases.

Table 6: PRINCIPAL COMPONENT ANALYSIS OF THE RELATIVE COMPOUND RISK AND AMBIGUITY PREMIA: ACTUARIES

Variable	$P=0$			$P=25$		
	1st comp.	2nd comp.	3rd comp.	1st comp.	2nd comp.	3rd comp.
$\Pi_{SR,CR}$	0.14	0.99	0.06	0.26	0.96	0.08
$\Pi_{SR,MU}$	0.70	-0.06	-0.71	0.68	-0.24	0.70
$\Pi_{SR,MM}$	0.70	-0.14	0.70	0.69	-0.12	-0.71
Eigenvalue of the component	1.77	0.99	0.24	1.70	0.96	0.35
Proportion of variance explained	0.59	0.33	0.08	0.57	0.32	0.12

In Table 7, we report the results of the individual-level analysis based on the predictions of Section 4.2. We observe that the majority of the subjects in this sample are classified as following the EU hypothesis.[27] This should not be surprising given the familiarity of this pool of subjects with the Bayesian approach, coming from their occupational practice and training.[28] The second most common behavioral pattern is consistent with the layer hypothesis: this is

[27]Among the 41 actuaries classified under the EU hypothesis for $P = 0$ ($P = 25$), 19 (18) actuaries are also risk neutral (i.e., they have an absolute risk premium $\Pi_{SR} = 0$).

[28]Informal post-experiment interviews with a subset of actuaries confirmed the use of Bayesian arguments to justify the equal treatment of subjective and objective probabilities in the choices they made.

the case for 27% and 28% (for $P = 0$ and $P = 25$, respectively) of the actuaries. The rest of the sample consists of subjects exhibiting nonreduction of compound risk (17.6% and 17.4%), which is significantly less prevalent than in our student sample ($p<0.001$ for both $P = 0$ and $P = 25$). Finally, we remark that the majority of the actuaries classified under the stage hypothesis further make a distinction between different layers of uncertainty. Overall, in contrast with the student sample, we observe a remarkable stability of the actuaries' behavior between the treatments $P = 0$ and $P = 25$.

Table 7: Classification of subjects: Actuaries

	EU hypothesis	**layer hypothesis**	**Stage hypothesis**	
			Non distinction between layers	Distinction between layers
$P = 0$	55.4% (41/74)	27.0% (20/74)	1.4% (1/74)	16.2% (12/74)
$P = 25$	54.7% (41/75)	28.0% (21/75)	2.7% (2/75)	14.7% (11/75)

Studying the subjects' classification together with their attitude towards Ellsberg ambiguity, we find that (1) almost all the subjects following the EU hypothesis (39 of 41 for both $P = 0$ and $P = 25$) are Ellsberg ambiguity neutral ($\Pi_{SR,EE} = 0$); (2) the correlation between the Ellsberg ambiguity premium and the premium for the epistemic layers of uncertainty is strong and positive for the subjects following the layer hypothesis (Spearman's rank correlation coefficients: corr($\Pi_{SR,EE}, \Pi_{CR0,MM0}$)=0.87, $p<0.001$, and corr($\Pi_{SR,EE}, \Pi_{CR25,MU25}$)=0.69, $p<0.001$); and (3) the correlation between Ellsberg ambiguity and compound risk is not significant for subjects classified under the stage hypothesis: corr($\Pi_{SR,EE}$, $\Pi_{SR,CR0}$) $= -0.15$ (p=0.634) and corr($\Pi_{SR,EE}, \Pi_{SR,CR25}$)= 0.30 ($p$=0.329). Under this hypothesis, the correlation of Ellsberg ambiguity with the epistemic layers is significant for $P = 0$ (corr($\Pi_{SR,EE}, \Pi_{CR0,MM0}$)=0.72, p=0.003) but not for $P = 25$ (corr($\Pi_{SR,EE}, \Pi_{CR25,MM25}$)=0.33, p=0.272).

Overall, we can summarize the results of our follow-up experiment as follows.

Result 4: In contrast with our student sample, the majority of the variance in risk professional's attitudes towards uncertainty is explained by the nature of uncertainty featured (aleatory vs. epistemic) rather than by the multiple-stage presentation of uncertainty.

Result 5: In contrast with our student sample, we observe that the most common behavior among risk professionals (55% of the sample) is to follow the EU hypothesis. The next most common behavior is consistent with the layer hypothesis (27-28%). The proportion of risk professionals making a distinction between stages (17–18%) is significantly lower than that of students.

7 Relationship to previous experimental literature

Since Ellsberg (1961), ambiguity aversion has been one of the most intensively investigated phenomena in experimental economics (see e.g., Trautmann and van de Kuilen, 2015). Following the theoretical claims of Segal (1987; 1990), various experimental studies have focused on testing the hypothesis of characterizing ambiguity by means of compound risks. In an early attempt, Bernasconi and Loomes (1992) reported less aversion to compound risk than what is typically found under ambiguity. More recently, Halevy (2007) reported a tight association between ambiguity neutrality and reduction of compound risk, in line with Segal's predictions. Later studies by Chew, Miao, and Zhong (2017)[29] and Dean and Ortoleva (2019) reported qualitatively similar findings. By contrast, using an experimental setup close to Halevy's (2007), Abdellaoui, Klibanoff, and Placido (2015) found a lower association between compound risk reduction and ambiguity neutrality, thus questioning the robustness of this association. Finally, Berger and Bosetti (2020) also reported similar findings for policymakers.

The analysis with the three layers of uncertainty that we present in this paper reconciles the previous findings. In particular, our results confirm the role played by the presentation of uncertainty in multiple stages (e.g., as in the compound risk situations) in explaining attitudes towards ambiguity. As typically reported by previous studies, this effect is the most important in our student sample. Yet our results also show that ambiguity cannot simply be reduced to the layer of risk, but that the recognition of the

[29]Note that the two-point compound risk and partial ambiguity treatments of Chew, Miao, and Zhong (2017) coincide with our CR and MU treatments. A comparison of this part of our data with that of Chew, Miao, and Zhong (2017) is provided in Online Appendix S7.

layers of model uncertainty and model misspecification is crucial in dealing with ambiguity. The role of these epistemic layers is found particularly important when more sophisticated subjects are considered. In Appendix C, we further explore the relative role of stages and layers of uncertainty in explaining *Ellsberg* ambiguity attitudes, which is more directly in line with what has been traditionally done in the literature.

8 Conclusion

The three-layer representation of uncertainty, proposed by Hansen (2014), Marinacci (2015), and Hansen and Marinacci (2016), provides a useful framework for many real-life decision problems under uncertainty. In this paper, we present the first experimental investigation of this framework. Using a design that captures the distinctions among the layers of risk, model uncertainty, and model misspecification, while remaining in a simple environment of urns and balls, we show that the three layers of uncertainty are perceived and treated differently by the majority of subjects. Comparing the three layers with the situation à la Ellsberg (1961), we show that the attitudes towards Ellsberg ambiguity are better explained by reckoning with model uncertainty and model misspecification.

Addressing the issue of model misspecification in a laboratory environment involving urns and balls is necessarily simplistic, as it, by construction, does not provide the relevant complexity for model misspecification to be a central concern. Our treatment of model misspecification should thus be viewed as a proxy to gain insights into the importance of the third layer in a controlled environment. Extrapolating our experimental findings to real-life decision situations, in which model misspecification arises naturally, we conjecture that the role of model misspecification is potentially greater than what we capture in our experiment.

Appendix

A Attitudes towards the sources and layers of uncertainty

Table A.1 presents the proportions of aversion, neutrality, and seeking attitudes towards compound risk and ambiguity, and towards the layers of model uncertainty and model misspecification. The two-sided sign-tests indicate significant aversion towards ambiguity under MU and MM, and towards the layer of model uncertainty. We also observe aversion to compound risk in the treatment with $P = 25$, but not with $P = 0$.

Table A.1: Proportions of zero, positive, and negative premia

	$P = 0$			$P = 25$		
	$\Pi_{i,j} > 0$	$\Pi_{i,j} = 0$	$\Pi_{i,j} < 0$	$\Pi_{i,j} > 0$	$\Pi_{i,j} = 0$	$\Pi_{i,j} < 0$
$\Pi_{SR,CR}$	20.34%	58.47%	21.19%	47.01%	43.59%***	9.4%
$\Pi_{SR,MU}$	47.90%	38.66%***	13.45%	52.99%	39.32%**	7.69%
$\Pi_{SR,MM}$	57.14%	33.61%***	9.24%	58.33%	30.00%***	11.67%
$\Pi_{CR,MU}$	48.74%	38.66%***	12.60%	34.48%	50.86%**	14.66%
$\Pi_{MU,MM}$	27.50%	55.00%	17.50%	26.27%	50.85%	22.88%

Notes: Star signs indicate rejection of the null hypothesis H_0: Proportion($\Pi_{i,j} > 0$)=Proportion($\Pi_{i,j} < 0$) based on two-sided sign-tests.

B Field experiment with actuaries

The follow-up experiment took place as an artefactual field experiment run on a unique pool of risk professionals during the 31st International Congress of Actuaries (ICA). ICA is a conference organized by the International Actuarial Association every four years. It gathers more than 2,500 actuaries, among which academics and high-ranking representatives from the international insurance and financial industry. The 31st congress was held from June 4 to 8, 2018, in Berlin, Germany. At this occasion, a specific conference room with 20 computers was made available for the purpose of the experiment. The experiment was organized in 12 sessions of 45 minutes (including instructions and payment). Subjects were recruited on a voluntary basis.

Sample We collected data from 84 subjects from 33 different countries. The majority (77%) of the subjects were from the European Union, among which most were from Germany (43 out of 84). Most of the subjects were highly educated: 58 subjects (69%) held a master's degree and 18 subjects (21%) held a PhD. 46 subjects reported that their highest degree was obtained in a field related to mathematics and statistics, while 17 subjects reported it related to actuarial sciences. The remaining subjects reported diplomas in physics (2), engineering (1), finance (1), economics and management (3), or did not report anything (14).

Incentives In this follow-up experiment, the prospects yielded either €200 or €0. To equate the expected experimental costs in the two experiments and limit monetary transactions at ICA, only a fraction of actuaries (one out of 10) was paid based on one of their choices (i.e. between-subject random incentives). At the end of the experiments, subjects were offered goodies and drinks for their participation. The choice question implemented for determining the payment was selected randomly prior to the choices and resolutions of uncertainty (Johnson et al., 2015).

Data The data from 4 subjects were dropped as they show inconsistencies in all the eight choice lists. For the rest of the subjects, we proceeded as in the main experiment and did not include those choice lists that indicate multiple, reverse- or no-switching patterns (3.3% of all).

Further results Table B.1 presents the proportions of aversion, neutrality, and seeking attitudes towards the sources of compound risk and ambiguity, and towards the layers of model uncertainty and model misspecification alone in the case of the follow-up experiment. Similar to Table A.1, we find significant aversion towards ambiguous sources and towards the layer of model uncertainty. We observe higher proportions of neutrality in this sample of risk professionals compared to our student sample (two-sided tests of proportion, $p < 0.05$ for all).

Table B.1: PROPORTIONS OF ZERO, POSITIVE, AND NEGATIVE PREMIA: ACTUARIES

	$P = 0$			$P = 25$		
	$\Pi_{i,j} > 0$	$\Pi_{i,j} = 0$	$\Pi_{i,j} < 0$	$\Pi_{i,j} > 0$	$\Pi_{i,j} = 0$	$\Pi_{i,j} < 0$
$\Pi_{SR,CR}$	8.11%	82.43%	9.46%	13.33%	82.67%	4.00%
$\Pi_{SR,MU}$	31.08%	63.51%***	5.41%	31.58%	65.79%***	2.63%
$\Pi_{SR,MM}$	34.67%	61.33%***	4.00%	34.21%	61.84%***	3.95%
$\Pi_{CR,MU}$	32.90%	59.21%***	7.89%	25.97%	66.24%**	7.79%
$\Pi_{MU,MM}$	14.47%	77.64%	7.89%	19.23%	71.80%	8.97%

Notes: Star signs indicate rejection of the null hypothesis H_0: Proportion($\Pi_{i,j} > 0$)=Proportion($\Pi_{i,j} < 0$) based on two-sided sign-tests.

C Explaining Ellsberg ambiguity

As discussed in Section 7, the relation between the presentation of uncertainty in multiple stages and Ellsberg ambiguity has been the topic of several recent studies (e.g., Halevy, 2007; Abdellaoui, Klibanoff, and Placido, 2015; Chew, Miao, and Zhong, 2017; Berger and Bosetti, 2020). In this appendix, we explore the relative role of stages and layers of uncertainty in explaining Ellsberg ambiguity attitudes in our student sample.

Table C.1 presents the Spearman's rank correlation coefficients between the Ellsberg ambiguity premium ($\Pi_{SR,EE}$) and the relative premia for compound risk ($\Pi_{SR,CR}$) and the epistemic layers of uncertainty ($\Pi_{CR,MM}$). We observe that Ellsberg ambiguity attitudes correlate with both attitudes towards compound risk and the epistemic layers of uncertainty. Notably, the correlation with the epistemic layers is higher than that with compound risk when $P = 0$ and lower when $P = 25$. The discrepancy between the two treatments is possibly due to the fact that the compoundness in the treatment with $P = 25$ is more salient than with $P = 0$, as the latter can be seen as encompassing only one *stage of uncertainty*.

We also regress the Ellsberg ambiguity premium on the three components obtained in our principle component analysis (see Section 5.3) to understand the relative role of each of them in explaining attitudes towards Ellsberg ambiguity. As emphasized earlier, the first component can be taken as an index capturing the general attitude towards multiple stages

Table C.1: SPEARMAN CORRELATIONS OF THE ELLSBERG AMBIGUITY PREMIUM WITH OTHER RELATIVE PREMIA

$P = 0$		$P = 25$	
	$\Pi_{SR,EE}$		$\Pi_{SR,EE}$
$\Pi_{SR,CR0}$	0.236^{*}	$\Pi_{SR,CR25}$	0.526^{***}
$\Pi_{CR0,MM0}$	0.529^{***}	$\Pi_{CR25,MM25}$	0.374^{***}

Notes: *** p-value<0.001 ** p-value<0.01 * p-value<0.05

of uncertainty, whereas the second and the third components can be taken as indices for the marginal attitudes towards the layers of model uncertainty and model misspecification. The results of the regressions are shown in Table C.2. In both treatments $P = 0$ and $P = 25$, we find significantly positive coefficients for the first two components (p<0.001). However the last coefficient, capturing the effect of the layer of model misspecification, is not significant (p>0.1). This result suggests that, in line with our conjecture, Ellsberg-type settings may not provide the relevant complexity for model misspecification to be a central concern.

Table C.2: ELLSBERG AMBIGUITY ATTITUDE

	$P = 0$	$P = 25$
1st comp. (attitude towards stages)	1.391^{***}	1.412^{***}
2nd comp. (attitude towards model uncertainty)	0.878^{***}	1.288^{***}
3rd comp. (attitude towards model misspecification)	0.430	0.333
R^2	0.517	0.617
No. of observations	114	113

Notes: This table shows the results of the OLS regressions with Ellsberg ambiguity attitude ($\Pi_{SR,EE}$) as the dependent variable. The independent variables are the components of the principal component analysis defined in Table 3. The regressions include constants but these are not displayed for brevity's sake. *** p-value<0.001 ** p-value<0.01 * p-value<0.05.

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