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Value efficiency and its decomposition into direct price and quantity effects

Jean-Philippe Boussemart
Univ. Lille, CNRS, IESEG School of Management, UMR 9221 –LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
jp.boussemart@ieseg.fr

Hervé Leleu
IESEG School of Management, CNRS, Univ. Lille, UMR 9221 –LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
h.leleu@ieseg.fr

Raluca Parvulescu
IESEG School of Management, UMR CNRS 9221 –LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
r.parvulescu@ieseg.fr

IÉSEG School of Management Lille Catholic University 3, rue de la Digue F-59000 Lille Tel: 33(0)3 20 54 58 92
www.ieseg.fr

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Jean-Philippe Boussemart[1]
Univ. Lille, CNRS, IESEG School of Management, UMR 9221 – LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
jp.boussemart@ieseg.fr

Hervé Leleu
IESEG School of Management, CNRS, Univ. Lille, UMR 9221 – LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
h.leleu@ieseg.fr

Raluca Parvulescu
IESEG School of Management, UMR CNRS 9221 – LEM, F-59000, France
3, rue de la Digue, 59000 Lille, France
r.parvulescu@ieseg.fr

[1] Corresponding author

Abstract

Based on the idea that in real markets firms have some freedom to set their output prices and negotiate input unit costs, this paper introduces a new approach for value efficiency decomposition as the product of direct price and quantity effects. Our framework relies on the axiomatization of a value transformation set on which quantity, price and value distance functions can be defined. The methodology developed allows for various degrees of dependency between quantity and price as well as for different degrees of freedom in price setting. The value efficiency decomposition can encompass all traditional measures such as cost, revenue, profit and profitability efficiencies. An application on French cattle farms illustrates the appeal of our approach for practitioners.

Highlights

- Calculation of a direct price effect based on a comparison of prices among similar firms
- Value decomposition respects the product test
- Possibility to consider various degrees of dependency between the quantity and the price
- Generalization of traditional quantity and more recent price distance functions
- Extensions of this methodology include cost, revenue, profit and profitability measures

Keywords: Data envelopment analysis, value transformation set, value efficiency, direct price efficiency.

JEL Classification: D24; C43

Introduction

The overall financial performance of a business can be approached by various measures widely developed in the accounting or economic literature. Most of the time, performance is gauged by comparing the revenue or the turnover (R) to production costs (C) generated by the firm's activity. This difference between revenue and cost can be calculated either as a subtraction such as profit $(\Pi = R - C)$, or as a ratio such as the profit rate also called profitability $\left(\pi = R/C\right)$. Other measures are closely related to these two prior basic indicators such as the profit to cost ratio $\left(\Pi/C\right)$ or the popular return to asset ratio $\left(ROA = \Pi/A = \frac{R-C}{A} = \left(1-\pi^{-1}\right) R/A\right)$ where A is assets.

All these financial performance indicators are computed from variables expressed in value terms and thus combine two sources of change: a quantity effect and a price effect. More precisely, it is common to decompose profitability into a quantity effect measured by a productivity ratio index such as Total Factor Productivity (TFP) and a price effect resulting from a ratio index between output and input prices, named Price Recovery (PR). The firm's overall financial performance is therefore the result of an achievement in terms of productivity associated with a return in terms of margin or price recovery. The former measures the capacity of a firm to generate quantities of outputs per unit of inputs, the latter measures the ability to generate a mark-up on sales per euro of expenditure on inputs. Hence, from a strategic business perspective, prices play a key role in evaluating overall financial performance, a role that is complementary to that played by quantity choices. A selling price advantage can indeed be the result of a differentiation strategy stemming from marketing and product development skills. A competitive advantage in terms of input price can originate from the bargaining power of a company benefiting from economies of scale and/or developing a vertical integration policy. For that reason, an analytical framework integrating both price and quantity dimensions seems necessary to better analyze the firm's profitability and its quantity and price components.

Economic and statistical literatures have established analytical tools studying joint price and quantity effects on cost, revenue or profitability. Among the pioneering works developed both at the firm level and at the industry or macroeconomic levels, one can cite Davis (1955), Kendrick (1961), Kendrick and Creamer (1965), Vincent (1961), CERC (1980), Templé (1971), Courbis and Templé (1975). On this subject, Grifell-Tatjé and Lovell (2015) conducted a comprehensive and accurate literature review. The works listed by the two authors all insist on the fact that economic progress is not only synonymous with quantitative productivity gains. The latter's distribution among the different stakeholders through price advantages is also an essential part of the story. In other words, growth and distribution of productivity gains are the two inseparable sides of a same coin. In this respect, the two authors insist on the necessity to consider both dimensions when evaluating financial performance. Another significant contribution of Griffel Tatjé and Lovell (2015) is that they have succeeded in building a bridge between the pioneering works of the 1950s and 1960s relating to the distribution of productivity gains through price changes and the current approaches to productive efficiency.

The latter type of literature is devoted to business performance evaluation based on indicators (efficiency scores) that involve estimating a best practice production frontier (benchmark). For an observed practice, the gap to the frontier (efficiency score) is evaluated with a distance function defined from an axiomatic describing the production possibility set or the underlying technology. Numerous textbooks and academic papers have been dedicated to the assessment of productive performance. Among many others, one can cite Coelli, Prasada Rao and Battese (2005) as an introductory text on measuring efficiency scores through empirical and theoretical productivity indexes based on distance functions. Other textbooks, starting from basic models, develop more in depth theoretical and empirical advances on the subject such as Färe et al. (1985), Charnes et al. (1994), Fried, Lovell et Schmidt (2008) to name just a few.

Nevertheless, within benchmarking analysis general frame, the different concepts of efficiency, whether technical, allocative, cost, revenue, or profit efficiency, do not consider a possible endogenous price determination via pricing choices that would complement productivity growth objectives. Technical efficiency only gauges the potential increase of output quantities (reduction in input quantities) for a given level of input quantities (output quantities). Although, Farrell (1957) introduced cost and allocative efficiencies by considering prices, these concepts measure the producer's aptitude to allocate quantities of resources according to their own respective prices which are considered as given and/or exogenous. Thus, no price comparison between producers is offered.

To circumvent this drawback, Tone (2002) presented a new form of allocative and cost efficiencies based on a “value” production technology. Sahoo, Mehdiloozad and Tone (2014) followed this direction and relaxed the price taker assumption. They recommended using a “value” directional distance function based on a technology set covering all possibilities of costs and revenues. However, despite the undeniable contribution of these latest measures, they are still based on a reallocation behavior at the firm level for a given system of relative prices.

More recently, Ayouba et al (2019) identified a concept that could be qualified as an indirect price advantage. For this, they compare two productive efficiency scores: a traditional one calculated with input and output quantities and a second one based on the respective cost and revenue values. The ratio of the two scores representing a sort of “indirect price advantage” has a useful economic interpretation as the profitability efficiency resulting from a more advantageous price environment. This new indicator completes the picture provided by the allocative efficiency as it highlights producers’ opportunities arising from a better price environment.

Camanho and Dyson (2008) also departed from the assumption of fixed prices in the cost efficiency estimation reflecting only management skills in input quantities but ignoring price differences among producers. To overcome the drawbacks of cost and allocative efficiencies, they proposed a model aiming to measure both quantity and price components of cost efficiency. Only recently, Portela and Thanassoulis (2014) analyzed cost efficiency with both input prices and quantities as decision variables. They included a price effect within the decomposition of cost efficiency scores. However, by considering prices and quantities as completely independent from one another, they naturally obtain the lowest prices in each input dimension as the optimal solution. To avoid this, they have to use exogenously fixed price limits, which represent a mitigated solution. The main issue in this approach is, in our opinion, that price efficiency determination is not related to the evaluated DMU.

The main contribution of the present paper is to introduce a decomposition of value efficiency into a direct quantity effect and a direct price effect based on an axiomatic approach of the value transformation set. From an operational point of view, one can identify the sources of possible increase in profitability for each input and output variables. They are to be found both in an improved quantity management and in a search for new price opportunities.

Contrasting to the previous indirect price advantage which was deduced from the comparison of a value indicator with a quantity one, our value decomposition allows us to obtain a direct quantity effect and a direct price effect. Moreover, these two components satisfy the product test meaning that the value efficiency is strictly equal to their product. This contribution is worth underlining since the theoretical decompositions of profit efficiency proposed in the recent literature satisfy this property only under very restrictive assumptions. For example, Grifell-Tatjé and Lovell (2015) have shown that the combination of a Malmquist productivity index and a Könus price recovery index could at best only approximate the profitability change. Later, they established that these theoretical indexes need to be weighted by other effects of output/input mixes for both price and quantity variables in order to obtain that the product test is satisfied (Grifell-Tatjé, Lovell, 2020).

The organization of this paper is as follows. Section 2 begins by positioning the objective of our analytical framework precisely. First, in the case of cost minimization, we present the axiomatic of value transformation sets and their associated distance functions. Second, we derive the cost efficiency decomposition into quantity and price components for each input variable and the mathematical programs necessary for estimations. Third, we compare our model to other known distance functions in the literature. Finally, we extend the analysis to the revenue and profitability maximization cases. Section 3 develops an application on French cattle farms to illustrate the contribution of our approach for practitioners. After presenting the data and defining the selected variables, a first point focuses on a real case study and comments in detail on all the different elements of profit efficiency. Then, results are presented at the sample level, illustrating the importance of price-related decisions in profitability optimization.

2. Theoretical developments for joint price and quantity technology

As mentioned above, our framework compares quantities and prices between two situations that can be identified either as temporal change, a gap between two firms or, a gap between a targeted objective and an observed result. In this research, we refer to the latter situation and depending on the objective chosen for the company, i.e., cost minimization, revenue, profit or profitability maximization, we will define the so-called "basic or observed situation" through the subscript "*o*" and the optimal situation to be reached with the superscript "*".

To model production analysis, we consider vectors of output quantities $y \in \mathbb{R}^{R+}$ and their corresponding output prices $p \in \mathbb{R}^{R+}$ with $R = \{1, \dots, r, \dots, R\}$ that can be produced from a set of vectors of input quantities $x \in \mathbb{R}^{I+}$ and their corresponding input prices $w \in \mathbb{R}^{I+}$ with $I = \{1, \dots, i, \dots, I\}$. A production plan is a quadruplet $(y, p, x, w) \in \left(\mathbb{R}^{R+} \times \mathbb{R}^{R+} \times \mathbb{R}^{I+} \times \mathbb{R}^{I+}\right)$.

For example, in the particular case of profitability or profit rate maximization, the gap between the observed and optimal situations is measured by the following value index π resulting from the product of a quantity effect and a price effect:

$$\pi = \frac{\pi_o}{\pi^*} = \frac{\left(\dfrac{R_o}{C_o}\right)}{\left(\dfrac{R^*}{C^*}\right)} = \frac{\left(\dfrac{R_o}{R^*}\right)}{\left(\dfrac{C_o}{C^*}\right)} = \left(\frac{\dfrac{p_o^T y_o}{p^{*T} y^*}}{\dfrac{w_o^T x_o}{w^{*T} x^*}}\right) = \left(\frac{\dfrac{p^{*T} y_o}{p^{*T} y^*}}{\dfrac{w^{*T} x_o}{w^{*T} x^*}}\right)\left(\frac{\dfrac{p_o^T y_o}{p^{*T} y_o}}{\dfrac{w_o^T x_o}{w^{*T} x_o}}\right) \quad (1).$$

From equation (1), one can note that the last term is the product of two large brackets. The first one measures a quantity TFP index since only the quantities of outputs and inputs change while prices are fixed. Similarly, the second bracket measures a price recovery index. Here, input and output price effects are identified as Laspeyres indexes based on observed quantities while input and output quantity effects based on optimal prices are related to Paasche indexes. Alternatively, we could have decomposed the profitability index between Paasche price effects weighing price changes by the optimal quantities and Laspeyres quantity effects weighing quantity changes by the observed prices (equation (2)). Finally, profitability is also equal to the product of the geometric means of these Laspeyres and Paasche quantity or price components according to a Fisher-type decomposition (cf. equation (3)).

$$\pi = \frac{\pi_o}{\pi^*} = \left(\frac{\dfrac{p_o^T y_o}{p^{*T} y^*}}{\dfrac{w_o^T x_o}{w^{*T} x^*}}\right) = \left(\frac{\dfrac{p_o^T y^*}{p^{*T} y^*}}{\dfrac{w_o^T x^*}{w^{*T} x^*}}\right)\left(\frac{\dfrac{p_o^T y_o}{p_o^T y^*}}{\dfrac{w_o^T x_o}{w_o^T x^*}}\right) \quad (2),$$

$$\pi = \frac{\pi_o}{\pi^*} = \left[\left(\frac{\dfrac{p_o^T y_o}{p^{*T} y_o}}{\dfrac{w_0^T x_o}{w^{*T} x_o}}\right)\left(\frac{\dfrac{p_o^T y^*}{p^{*T} y^*}}{\dfrac{w_o^T x^*}{w^{*T} x^*}}\right)\right]^{1/2}\left[\left(\frac{\dfrac{p^{*T} y_o}{p^{*T} y^*}}{\dfrac{w^{*T} x_o}{w^{*T} x^*}}\right)\left(\frac{\dfrac{p_o^T y_o}{p_o^T y^*}}{\dfrac{w_o^T x_o}{w_o^T x^*}}\right)\right]^{1/2} \quad (3).$$

Since such indexes need to be calculated under a hypothesis of optimal behavior, it is necessary first to define them theoretically thanks to a value transformation set which considers both the quantities and the prices of outputs and inputs characterizing the different production plans to evaluate.

We define the value transformation set $V(y,p,x,w)$ as the set of feasible production plans (y,p,x,w) where the output quantity vector y sold at corresponding vector output price p can be produced by input quantity vector x bought at input price vector w.[1] Mathematically, $V(y,p,x,w)$ is defined as:

$$V(y,p,x,w) = \{(y,p,x,w) : x \text{ purchased at price } w \text{ can produce } y \text{ sold at price } p\} \quad (4).$$

In particular, if we observe a population of N DMUs with corresponding production plans $(y_n, p_n, x_n, w_n)\ \forall n = 1,\dots,N$, then these productions plans are obviously feasible and:

$$(y_n, p_n, x_n, w_n) \in V(y,p,x,w)\ \forall n = 1,\dots,N \quad (5).$$

Two specific subsets can be defined from $V(y,p,x,w)$:

$$L(y,p) = \{(x,w) : (y,p,x,w) \in V(y,p,x,w)\} \quad (6),$$

$$P(x,w) = \{(y,p) : (y,p,x,w) \in V(y,p,x,w)\} \quad (7).$$

Thus, the first equation (6) can be used for cost minimization approach, while the second one, (7) is useful for revenue maximization. The following three subsections deal with cost minimization and we will extend our analysis to encompass revenue, profit and profitability optimization in subsection 2.4.

2.1 A unified cost minimization approach

The input set $L(y,p)$ defined in equation (6) is more restrictive than the traditional input requirement set $L(y)$ since it includes all feasible input price and quantity couples that allow to produce an output price and quantity couple. Compared to a traditional framework, prices are not exogenous and clearly represent choice variables for a DMU.

Following the axiomatic approach of Shephard (1953, 1970), we add structure to value transformation subsets by imposing several axioms. As the different sets are interrelated, they inherit their properties from each other. Here, we start from $L(y,p)$.

A1 $L(0,p) = \mathbb{R}^{I+} \times \mathbb{R}^{I+}, (0,w) \notin L(y,p)$ for $y > 0$.

A2.i $(x,w) \in L(y,p)$ and $x' \geq x$ imply $(x',w) \in L(y,p)$.

[1] In what follows, for simplicity of notations, we drop the *R* dimension for the output vectors and the *I* dimension for the input vectors.

A2.ii $(x,w)\in L(y,p)$ and $w'\geq w$ imply $(x,w')\in L(y,p)$.

A3.i $y_2\geq y_1\geq 0\in$ imply $L(y_2,p)\subset L(y_1,p)$.

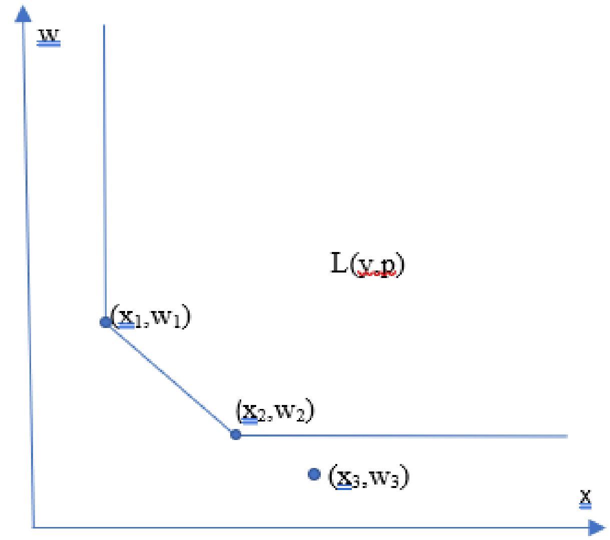
A3.ii $p_2\geq p_1\geq 0\in$ imply $L(y,p_2)\subset L(y,p_1)$.

A4 $(x_1,w_1),(x_2,w_2)\in L(y,p)$ and $0\leq\alpha\leq 1$ imply $\left(\alpha x_1+(1-\alpha)x_2),\alpha w_1+(1-\alpha)w_2\right)\in L(y,p)$.

A1 states that it is possible to produce a null output quantity from any nonnegative inputs, but it is not possible to produce positive output from a null input vector. A2.i implies the strong disposability of inputs quantity. It is always feasible to produce the same level of output sold at the same price with greater inputs quantity at a given input price. A2.ii deals with the strong disposability of input prices. While we assume that a minimum price can exist for any quantity of input, it is however possible to buy the same quantity of input at a higher price and therefore produce the same level of output at the given output price. A3.i implies the disposability of output quantity. It states that she who can do more can do the less, or formally, if an input vector (x,w) yields an output vector (y_2,p) then it can also yield any output vector (y_1,p) with $y_1\leq y_2$. A3.ii involves strong disposability for output price meaning that if an input vector (x,w) yields an output vector (y,p_2) then it can also yield any output vector (y,p_1) with $p_1\leq p_2$. A4 is a convexity assumption defined on input quantities and prices. While convexity in quantities traditionally reflects the law of diminishing marginal rate of substitution between inputs, this convexity between quantity and price echoes to the law of demand which states that quantity purchased varies inversely with price. Convexity is also a very useful mathematical assumption whenever the dimension of the input/output space is high by allowing combinations of observed DMUs with various levels of activities. However, it can be rejected in special circumstances and a free disposal hull (FDH) technology can be used instead by ignoring A4. Finally, other properties such as returns to scale can be added or more mathematical properties to ensure that input sets are bounded, closed and well-defined.

Axioms A1 to A4 are illustrated in Figure 1. We consider only one input and production plans are represented in an input quantity/price space (x,w). Here, $\left(x_1,w_1\right)$ and $\left(x_2,w_2\right)$ are two feasible production plans from observed DMUs. $\left(x_3,w_3\right)$ is not observed and is not a feasible production plan. It is outside the set $L(y,p)$. The horizontal prolongation at $\left(x_2,w_2\right)$ represents quantity disposability while the horizontal continuation at $\left(x_1,w_1\right)$ indicates price disposability. For any DMU, we consider that buying the same quantity of input at a higher price is feasible while buying it at a lower price is not feasible if it is not observed. The segment between $\left(x_1,w_1\right)$ and $\left(x_2,w_2\right)$ illustrates the convexity assumption A4. It is worth noticing that $\left(x_3,w_3\right)$ is infeasible even though it uses more input quantity compared to $\left(x_1\right)$ and $\left(x_2\right)$. It simply means that a price as low as $\left(w_3\right)$ is neither observed, nor considered as feasible for a quantity (x_3). This contrasts with a traditional input requirement set $L(y)$ for which (x_3) would have been feasible by the disposability assumption on quantity.

Figure 1. Illustration of $L(y,p)$ with the assumption A4.



In (6), the definition of $L(y,p)$ assumes that input quantities and their prices are possibly dependent on each other. This is common sense since they were retrieved from observed, real-life DMUs. However, in some special circumstances, this assumption can be relaxed if we consider that, for the same input, its price and quantity can be independent from one another. In that case, two sub-technologies are considered, one based on input quantities, the other based on input prices. The resulting technology is the intersection of the two sub-technologies.

$$L_X(y,p) = \{x : (y,p,x) \in V(y,p,x)\} \qquad (8),$$

$$L_W(y,p) = \{w : (y,p,w) \in V(y,p,w)\} \qquad (9),$$

$$L'(y,p) = L_X(y,p) \cap L_W(y,p) \qquad (10).$$

Axioms A.1, A2.i, A3.i and A3.ii are associated to $L_X(y,p)$. The convexity axiom A4 is replaced by a convexity in quantity only:

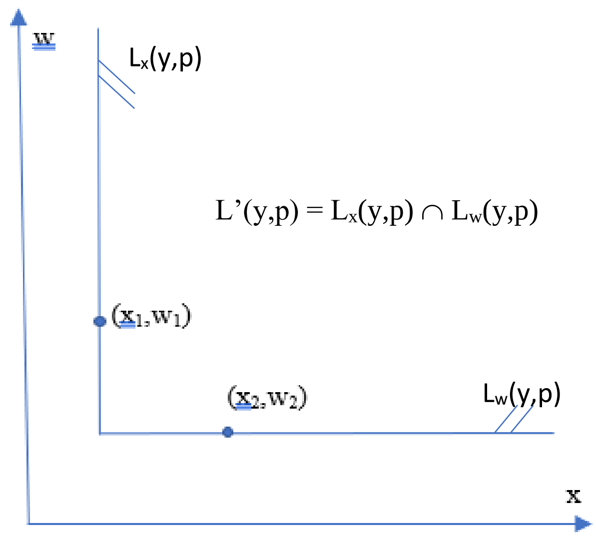
A4.i $x_1 \in L_X(y,p), x_2 \in L_X(y,p)$ and $0 \le \alpha \le 1$ imply $(\alpha x_1 + (1-\alpha)x_2)) \in L_X(y,p)$.

Axioms A.1, A2.ii, A3.i and A3.ii are associated to $L_W(y,p)$. The convexity axiom A4 is replaced by a convexity in price only:

A4.ii $w_1 \in L_W(y,p), w_2 \in L_W(y,p)$ and $0 \le \alpha \le 1$ imply $(\alpha w_1 + (1-\alpha)w_2)) \in L_W(y,p)$.

The alternative value input set $L'(y,p)$ is illustrated in Figure 2. In $L_X(y,p)$, efficient production plans minimize the necessary input quantities to produce (y,p) whatever the input prices. In parallel, in $L_W(y,p)$, the optimization targets input prices independently of their respective quantities. The intersection shows that the efficient production plan is characterized by the minimal input quantity observed associated with the minimal input price. Obviously, it is most likely that this efficient production plan is not observed but that it is constructed based on the assumption that all DMUs have access to the same input prices whatever their respective quantities. This clearly characterizes prices independence from quantities and this model can be interpreted as a theoretical standard for frictionless markets with perfect information and without any form of bargaining power.

Figure 2. Illustration of $L'(y,p)$, with assumptions A4.i and A4.ii.



Our preference clearly goes to the first approach where we assume that input quantities and their prices are somehow interrelated. In a real world, prices may convey information such as quality, supplier market power, transaction costs… Or, the traditional analysis based on $L(y)$, by considering prices as exogenous, immediately excludes these effects. An independence such as the one implied by $L'(y,p)$ is an intermediate alternative by refuting input prices heterogeneity among DMUs while allowing for heterogeneity regarding the level of activity and output price (y,p). Finally, $L(y,p)$ is more encompassing by allowing substitution between price and quantity. In Figure 1, (x_1,w_1) and (x_2,w_2) are both feasible and efficient possibly because the DMU producing with (x_1,w_1) uses a better quality input which is more expensive. Substitution between input price and quantity may reflect differences in imperfections described above. A valid theoretical objection to this approach is the following: whenever inputs have different qualities then they should be considered as distinct inputs in a traditional framework. However, in practice, differences of quality are never contained in the description of input quantities which are undistinguishable in that dimension. On the contrary, differences are often conveyed in the prices and our approach seems more appropriate in this context.

By considering a sample of K observed DMUs issued from a population N, a DEA-type value transformation set can be defined from the property that all observed DMUs are feasible and from axioms A1-A4. Under variable returns to scale, this is equivalent to:

$$L(y,p)=\left\{\begin{array}{l}(x,w):\\ \sum_{k=1}^{K}\mu_k y_{k,r}\ge y_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\mu_k p_{k,r}\ge p_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\mu_k x_{k,i}\le x_i\ \ \forall i=1,\dots,I\\ \sum_{k=1}^{K}\mu_k w_{k,i}\le w_i\ \ \forall r=1,\dots,I\\ \sum_{k=1}^{K}\mu_k=1,\ \mu_k\ge 0\ \forall k=1,\dots,K\end{array}\right\}\quad(11)$$

The alternative input set $L'(y,p)$ is modeled by considering the intersection of two input sets based on quantity and price respectively. The corresponding DEA-type production possibility set can be defined by:

$$L'(y,p)=\left\{\begin{array}{l}(x,w):\\ \sum_{k=1}^{K}\mu_k y_{k,r}\ge y_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\mu_k p_{k,r}\ge p_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\mu_k x_{k,i}\le x_i\ \ \forall i=1,\dots,I\\ \sum_{k=1}^{K}\mu_k=1,\ \mu_k\ge 0\ \forall k=1,\dots,K\\ \sum_{k=1}^{K}\lambda_k y_{k,r}\ge y_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\lambda_k p_{k,r}\ge p_r\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\lambda_k w_{k,i}\le w_i\ \ \forall r=1,\dots,I\\ \sum_{k=1}^{K}\lambda_k=1,\ \lambda_k\ge 0\ \forall k=1,\dots,K\\ \sum_{k=1}^{K}\mu_k y_{k,r}=\sum_{k=1}^{K}\lambda_k y_{k,r}\ \ \forall r=1,\dots,R\\ \sum_{k=1}^{K}\mu_k p_{k,r}=\sum_{k=1}^{K}\lambda_k p_{k,r}\ \ \forall r=1,\dots,R\end{array}\right\}\quad(12).$$

In (12), two sets of activity variables (μ, λ) are used. For a given level of output quantities and prices $\left(\sum_{k=1}^{K} \mu_k y_k, \sum_{k=1}^{K} \mu_k p_k\right) = \left(\sum_{k=1}^{K} \lambda_k y_k, \sum_{k=1}^{K} \lambda_k p_k\right)$, all feasible production plans are defined independently for input quantities and, respectively, input prices. It is worth to note that the use of $L'(y,p)$ instead of $L(y,p)$ does not lead to the somewhat trivial result of taking the "best prices" in the sample since $L_W(y,p)$ is conditional on the level of output quantities and prices. This contrasts with the approach proposed by Portela and Thanassoulis (2014).

Based on $L(y,p)$ or $L'(y,p)$, a number of distance functions can be defined in order to measure technical, price or cost efficiencies. Depending on the context, one can either include in the model all quantity and price dimensions or exclude some of them. The following distance functions are defined indiscriminately on $L(y,p)$ or $L'(y,p)$:

D.1 $D_q^i(y,p,x,w) = \min_{\beta \geq 0} \{\beta : (\beta x, w) \in L(y,p)\}$

D.2 $D_p^i(y,p,x,w) = \min_{\theta \geq 0} \{\theta : (x, \theta w) \in L(y,p)\}$

D.3 $D_{q,p}^i(y,p,x,w) = \min_{\rho \geq 0} \{\rho : (\rho x, \rho w) \in L(y,p)\}$

D.4 $D_{uq}^i(y,x) = D_q^i(y,0,x,\infty) = \min_{\beta \geq 0} \{\beta : (\beta x, \infty) \in L(y,0)\}$

D.5 $D_{up}^i(y,w) = D_p^i(y,0,\infty,w) = \min_{\theta \geq 0} \{\theta : (\infty, \theta w) \in L(y,0)\}$

D.6 $C(y,p) = D_c^i(y,p) = \min_{w,x \geq 0} \{w^T x : (x,w) \in L(y,p)\}$

Distance functions D.1 to D.6 are computed from optimization programs. D.1 to D.5 are solutions to linear programs while D.6 is estimated through a non-linear optimization program. D.1 is a quantity measure, but which is conditional on prices. D.2 is a price measure conditional on quantities. D.3 is a new type of efficiency. It is a combined "quantity/price" radial efficiency that ensures for efficient DMUs that no other DMU could use less inputs quantity at a lower price. D.4 and D.5 define distance functions unconditional on prices and, on quantities respectively. Finally, D.6 defines cost functions. Contrary to the traditional cost function, input prices are no longer exogenous. Output prices can be included or not depending on the context. Input quantity and price can be considered either jointly or independently, following the choice of $L(y,p)$ or $L'(y,p)$. For a given DMU o with a corresponding production plan (y_o, p_o, x_o, w_o), the following program computes the minimal cost function $C(y_o, p_o)$ based on $L(y,p)$:

$$
\begin{aligned}
& C\left(y_o, p_o\right) = D_c^i\left(y_o, p_o\right) = \min_{\tilde{w},\tilde{x},\mu} \sum_{i=1}^{I} \tilde{w}_i \, \tilde{x}_i \\
& \sum_{k=1}^{K} \mu_k y_{k,r} \geq y_{o,r} \quad \forall r = 1,\ldots,R \\
& \sum_{k=1}^{K} \mu_k p_{k,r} \geq p_{o,r} \quad \forall r = 1,\ldots,R \\
& \sum_{k=1}^{K} \mu_k x_{k,i} \leq \tilde{x}_i \quad \forall i = 1,\ldots,I \qquad\qquad (13). \\
& \sum_{k=1}^{K} \mu_k w_{k,i} \leq \tilde{w}_i \quad \forall i = 1,\ldots,I \\
& \sum_{k=1}^{K} \mu_k = 1 \\
& \mu_k \geq 0 \;\forall k = 1,\ldots,K \\
& \tilde{x}_i \,, \tilde{w}_i \geq 0 \;\forall i = 1,\ldots,I
\end{aligned}
$$

In (13), the non-linearity appears only in the objective function while constraints are obviously linear. Therefore, traditional solvers are able to solve the program easily.

2.2 Various decompositions of cost efficiency

Two types of decompositions are introduced and discussed here. Firstly, we show that cost efficiency, based on the optimal cost level implied by direction function D.6 and obtained through program (13) can be decomposed as the product of a quantity and a price effect. In the second paragraph, we show that these decompositions can be pursued further at the input level. Thus, for each input considered, we can determine their respective quantity and price effects. The latter is especially relevant for practitioners.

a) Decomposition of cost efficiency into quantity and price components

$C\left(y, p\right)$ developed in (13), is a basis for defining a value efficiency by computing the minimal cost to achieve a level of activity conditional on output selling price. Naturally, the cost value efficiency (CVE) is defined as the ratio of minimal cost to observed cost:

$$CVE\left(y_o, p_o, x_o, w_o\right) = \frac{C^*}{C} = \frac{C(y_o, p_o)}{w_o^T x_o} = \frac{w^{*T} x^*}{w_o^T x_o} \quad (14).$$

Quantities and prices are jointly optimized in the objective of minimizing total cost. Contrary to the efficiency measures defined through D.1 to D.5, where mixes of input quantities and prices are kept constant (they are contracted radially), the cost value efficiency in (14) allows variations in mixes through reallocations of input quantities and prices. At the optimal solution of (13), the benchmark for the evaluated DMU$_o$ can show lower or higher inputs quantities and/or prices.

In traditional economic literature on indices, value decomposes in a quantity and a price component. Since D.6 is the most comprehensive efficiency measure obtained by optimizing both input quantities and their prices, it is useful to decompose it into quantity and price efficiency measures. All the necessary information can be extracted from (13):

A Laspeyres input quantity effect is defined as: $E_X^{La} = \dfrac{w_o^T x^*}{w_o^T x_o}$

A Paasche input quantity effect is defined as: $E_X^{Pa} = \dfrac{w^{*T} x^*}{w^{*T} x_o}$

A Laspeyres price effect is defined as: $E_W^{La} = \dfrac{w^{*T} x_o}{w_o^T x_o}$

A Paasche price effect is defined as: $E_W^{Pa} = \dfrac{w^{*T} x^*}{w_o^T x^*}$

Quantity and price efficiency measures are then computed as Fisher indexes: $E_X^F = \sqrt{E_X^{La} E_X^{Pa}}$ and $E_W^F = \sqrt{E_W^{La} E_W^{Pa}}$. We can readily verify that:

$$CVE\left(y_o, p_o, x_o, w_o\right) = E_X^F \times E_W^F \quad (15).$$

To sum up, we have defined a value efficiency measure based on a value transformation technology with endogenous prices and quantities. This value efficiency is the result of the combination of a direct quantity effect and a direct price effect.

b) Decomposition of quantity and price effects by input variables

A significant advantage of our approach is that is also enables to derive specific effects by input variables. Indeed, D.6 can also be written as:

$$\text{D.6} \;\; C\left(y, p\right) = D_c^i\left(y, p, x, w\right) = \min_{\beta,\theta \geq 0}\left\{\sum_{i=1}^{I} \theta_i w_i \beta_i x_i : \left(\beta \odot x, \theta \odot w\right) \in L(y, p)\right\} \quad (16).$$

In (16), specific input scores are introduced in the spirit of a Färe-Lovell measure. The corresponding optimization program is given by:

$$
\begin{aligned}
& C(y_0,p_0)=D_c^i\left(y_o,p_o,x_o,w_o\right)=\min_{\beta,\theta,\mu}\ \sum_{i=1}^{I}\theta_i w_{o,i}\beta_i x_{o,i} \\
& \sum_{k=1}^{K}\mu_k y_{k,r}\geq y_{o,r}\ \ \forall r=1,\ldots,R \\
& \sum_{k=1}^{K}\mu_k p_{k,r}\geq p_{o,r}\ \ \forall r=1,\ldots,R \\
& \sum_{k=1}^{K}\mu_k x_{k,i}\leq \beta_i x_{o,i}\ \ \forall i=1,\ldots,I \\
& \sum_{k=1}^{K}\mu_k w_{k,i}\leq \theta_i w_{o,i}\ \ \forall i=1,\ldots,I \\
& \sum_{k=1}^{K}\mu_k=1 \\
& \mu_k\geq 0\ \forall k=1,\ldots,K \\
& \beta_i,\ \theta_i\geq 0\ \forall i=1,\ldots,I
\end{aligned}
\qquad (17).
$$

We directly verify that $\tilde{w}_i=\theta_i w_{o,i}$ and $\tilde{x}_i=\beta_i x_{o,i}$. Thus (13) and (17) are strictly equivalent and the cost value efficiency can be rewritten as:

$$
CVE\left(y_o,p_o,x_o,w_o\right)=\frac{C^*}{C_o}=\frac{w^{*T}x^*}{w_o^T x_o^T}=\frac{\sum_{i=1}^{I}\theta_i\beta_i w_{o,i}x_{o,i}}{\sum_{i=1}^{I}w_{o,i}x_{o,i}} \qquad (18).
$$

For each input $i=1,\ldots,I$, $\beta_i \overset{>}{\underset{<}{=}} 1$ and $\theta_i \overset{>}{\underset{<}{=}} 1$ are the necessary changes in quantities and prices, respectively, to achieve the minimum cost.

2.3 Links with existing models in the literature

Our general framework encompasses many existing models developed in the literature. Below we show the relationships that can be established between our model and the traditional approach regarding input quantity and more recent ones dealing with input price distance functions.

a) Traditional input quantity distance function

First, traditional input distance functions arise from the unconditional quantity distance function D.4. For a given DMU*o* with corresponding production plan $\left(y_o,p_o,x_o,w_o\right)$, by setting $p_o=0$ and $w_o=\infty$ in D.1, we get the unconditional quantity distance function D.4 which is computed equivalently by:

$$
\begin{aligned}
& D_{uq}^{i}\left(y_o,0,x_o,\infty\right)=\min_{\beta,\mu}\ \beta \\
& \sum_{k=1}^{K}\mu_k y_{k,r}\geq y_{o,r}\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k p_{k,r}\geq 0\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k x_{k,i}\leq \beta x_{o,i}\ \ \forall i=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k w_{k,i}\leq \infty\ \ \forall r=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k=1 \\
& \mu_k\geq 0\ \forall k=1,\dots,K \\
& \beta\geq 0
\end{aligned}
\quad \text{or} \quad
\begin{aligned}
& D_{q}^{i}\left(y_o,x_o\right)=\min_{\beta,\mu}\ \beta \\
& \sum_{k=1}^{K}\mu_k y_{k,r}\geq y_{o,r}\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k x_{k,i}\leq \beta x_{o,i}\ \ \forall i=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k=1 \\
& \mu_k\geq 0\ \forall k=1,\dots,K \\
& \beta\geq 0
\end{aligned}
\quad (19).
$$

As shown in (19), the traditional input distance function for computing technical efficiency is just a special case of D.1.

b) Input price efficiency

D.5 follows the same line for price efficiency. The unconditional price efficiency defined by $D_{up}^{i}\left(y,w\right)=D_{p}^{i}\left(y,0,\infty,w\right)$ is computed equivalently by:

$$
\begin{aligned}
& D_{up}^{i}\left(y_o,0,\infty,w_o\right)=\min_{\beta,\mu}\ \beta \\
& \sum_{k=1}^{K}\mu_k y_{k,r}\geq y_{o,r}\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k p_{k,r}\geq 0\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k x_{k,i}\leq \infty\ \ \forall i=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k w_{k,i}\leq \beta w_{o,i}\ \ \forall i=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k=1 \\
& \mu_k\geq 0\ \forall k=1,\dots,K \\
& \beta\geq 0
\end{aligned}
\quad \text{or} \quad
\begin{aligned}
& D_{p}^{i}\left(y_0,w_0\right)=\min_{\beta,\mu}\ \beta \\
& \sum_{k=1}^{K}\mu_k y_{k,r}\geq y_{0,r}\ \ \forall r=1,\dots,R \\
& \sum_{k=1}^{K}\mu_k w_{k,i}\leq \beta w_{0,i}\ \ \forall i=1,\dots,I \\
& \sum_{k=1}^{K}\mu_k=1 \\
& \mu_k\geq 0\ \forall k=1,\dots,K \\
& \beta\geq 0
\end{aligned}
\quad (20).
$$

The right hand side of (20) is the price efficiency model used by Griffell-Tatje and Lovell (2020) in order to estimate a Könus input price index.

c) Other approaches regarding input price efficiency

Camanho and Dyson (2008) and Portela and Thanassoulis (2014) also rejected exogeneity of prices and proposed different models to measure price efficiency. Portela and Thanassoulis (2014) analyzed cost efficiency with both prices and input quantities as decision variables and showed that their models can encompass the different scenarios of price setting suggested in Camanho and Dyson (2008). We show here that our approach can be related and differentiated from the general model of Portela and Thanassoulis (P&T in the following). We present their model (3) p. 38 in our notations in (21) to facilitate the comparison.

$$
\begin{aligned}
&\min_{\theta,\beta,\mu,\lambda} \sum_{i=1}^{I} \theta_i w_{o,i} \beta_i x_{o,i} \\
&\sum_{k=1}^{K} \mu_k y_{k,r} \geq y_{o,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \mu_k x_{k,i} \leq \beta_i x_{o,i} \ \ \forall i = 1,\ldots,I \\
&\sum_{k=1}^{K} \mu_k = 1, \ \mu_k \geq 0 \ \forall k = 1,\ldots,K \qquad (21) \\
&\sum_{k=1}^{K} \lambda_{k,i} w_{k,i} \leq \theta_i w_{o,i} \ \ \forall i = 1,\ldots,I \\
&\sum_{k=1}^{K} \lambda_{k,i} = 1 \ \forall i = 1,\ldots,I, \\
&\lambda_{k,i} \geq 0 \ \forall k = 1,\ldots,K, \forall i = 1,\ldots,I, \\
&\theta_i^{\min} \leq \theta_i \leq \theta_i^{\max} \ \ \forall i = 1,\ldots,I
\end{aligned}
$$

Note that so far, our main approach introduced in (17) relies on $L(y,p)$ where we assume a dependency relationship between quantity and price for each DMU while P&T in (21) work on a set comparable to $L'(y,p)$ where optimal input quantities and prices are independent from one another. Therefore, while we use the same activity variables, they use two sets of activity variables for quantity and price respectively. Moreover, the latter set is input- but also DMU-specific (activity variables in the price dimension $\lambda_{k,i}$ are defined for each input *i*). Thus, in their program, optimal prices are naturally selected as the lowest ones in each input dimension but moreover, they may also come from different DMUs. Consequently, the optimal input prices are simply the minimal prices observed in the sample. In order to overcome this drawback, Portela and Thanassoulis (2014) introduce an upper and lower limit $(\theta_i^{\min}, \theta_i^{\max})$ on the price efficiency score. However, we argue that this is mostly an artifice since only two things can happen. Either the limits are not reached and the solution is set at the minimal observed prices or, in the alternative case, one of the limits is active and the optimal price will be set at this exogenous limit. In the second case, the optimal solution is therefore set by the researcher.

The main difference between our approach and P&T is that, in their model, the price efficiency is completely independent from any exogenous characteristic of the evaluated DMU. Starting from an economic definition of a value transformation set as we did, a model based on $L'(y,p)$ that can fit with P&T approach is given in (22). Here, input prices are entirely unrelated to input quantities as in P&T but they are linked to the level of evaluated DMU's output quantity and price. Therefore, for each level of output quantity and its price, an optimal vector of input prices is determined and is no longer set at the minimal observed prices or the exogenously fixed lower bound $\theta_i^{\min}$.

$$
\begin{aligned}
&\min_{\theta,\beta,\mu,\lambda} \sum_{i=1}^{I} \theta_i w_{o,i} \beta_i x_{o,i} \\
&\sum_{k=1}^{K} \mu_k y_{k,r} \geq y_{o,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \mu_k p_{k,r} \geq p_{o,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \mu_k x_{k,i} \leq \beta_i x_{o,i} \ \ \forall i = 1,\ldots,I \\
&\sum_{k=1}^{K} \mu_k = 1,\ \mu_k \geq 0 \ \forall k = 1,\ldots,K \\
&\sum_{k=1}^{K} \lambda_k y_{k,r} \geq y_{o,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \lambda_k p_{k,r} \geq p_{o,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \lambda_k w_{k,i} \leq \theta_i w_{o,i} \ \ \forall r = 1,\ldots,I \\
&\sum_{k=1}^{K} \lambda_k = 1,\ \lambda_k \geq 0 \ \forall k = 1,\ldots,K \\
&\sum_{k=1}^{K} \mu_k y_{k,r} = \sum_{k=1}^{K} \lambda_k y_{k,r} \ \ \forall r = 1,\ldots,R \\
&\sum_{k=1}^{K} \mu_k p_{k,r} = \sum_{k=1}^{K} \lambda_k p_{k,r} \ \ \forall r = 1,\ldots,R
\end{aligned}
\tag{22}
$$

2.4 Extension to revenue, profitability and profit

This last sub-section is devoted to the extension of the analysis to output sets and to profit optimization. From a conceptual and technical point of view, the framework developed for the inputs dimension is readily extended to definitions of $P(x,w)$, $P'(x,w)$, $D^o\left(y,p,x,w\right)$, $R\left(x,w\right)$ and $RVE\left(y,p,x,w\right)$.

A deeper discussion regarding profitability is necessary. In the cost analysis, output quantities and their prices are considered as exogenous while, equivalently, in the revenue analysis, it is the inputs quantities and their prices which are fixed. But this is no longer the case for profitability optimization with endogenous input and output prices and respectively quantities where there are no more exogenous variables to count on. In a traditional framework, based on quantities only, input and output prices are the exogenous information for profit maximization. Here, as prices are also endogenous, the problem of profitability maximization seems irrelevant. If DMUs control their prices and quantities, no limit is given to profit optimization without an exogenous constraint. For real-world applications based on a DEA-type framework, the solution to the profitability maximization problem is not impossible but trivial. All DMUs in the sample would be compared to the same benchmark, which is necessarily the observed DMU with the highest profit rate. However, profitability maximization implies drastic changes for the DMU and is often considered as a long-term ideal standard rather than a useful benchmark in the short-run. For example, the size and the input/output mix of the DMU at profit maximizing production plan can be radically modified. It is not the case for input cost optimization where the level of output is maintained or for equivalent output revenue maximization where input quantities are given. A reasonable solution for profitability optimization is to fix the size of the DMU. The size can be gauged by the level of one or the other input or output. Whenever it makes sense to consider one fix or quasi-fix input or output, the problem of profitability maximization regains interest because it optimizes profit for each level of the fix factor considered. Since firms are heterogenous in size or in a fix factor, they will no longer be altogether compared to the maximum observed profitability. The corresponding optimization program is derived below where $\bar{x}$ is a vector of J inputs considered as fixed ($J \in I$):

$$
\begin{aligned}
&\pi^*(\bar{x}_o) = D_{\pi_{rate}}\left(y_o, p_o, x_o, \bar{x}_o, w_o\right) = \max_{\alpha,\gamma,\beta,\theta,\mu} \frac{\sum_{r=1}^{R} \gamma_r p_{o,r} \alpha_r y_{o,r}}{\sum_{i=1}^{I\setminus\{j=1,\dots,J\}} \theta_i w_{o,i} \beta_i x_{o,i} + \sum_{j=1}^{J} \theta_i w_{o,j} \bar{x}_{o,j}} \\
&\sum_{k=1}^{K} \mu_k y_{k,r} \geq \alpha_r y_{o,r} \;\; \forall r = 1,\dots,R \\
&\sum_{k=1}^{K} \mu_k p_{k,r} \geq \gamma_r p_{o,r} \;\; \forall r = 1,\dots,R \\
&\sum_{k=1}^{K} \mu_k x_{k,i} \leq \beta_i x_{o,i} \;\; \forall i = 1,\dots,I \setminus \{j = 1,\dots,J\} \\
&\sum_{k=1}^{K} \mu_k w_{k,i} \leq \theta_i w_{o,i} \;\; \forall i = 1,\dots,I \\
&\sum_{k=1}^{K} \mu_k \bar{x}_{k,j} = \bar{x}_{o,j} \;\; \forall j = 1,\dots,J \\
&\sum_{k=1}^{K} \mu_k = 1 \\
&\mu_k \geq 0 \;\forall k = 1,\dots,K \\
&\alpha_r, \gamma_r \geq 0 \;\forall r = 1,\dots,R \\
&\beta_i, \theta_i \geq 0 \;\forall i = 1,\dots,I \setminus \{j = 1,\dots,J\}
\end{aligned} \qquad (23).
$$

Obviously, the above program (23) can be adapted to obtain the optimal profit level rather than the profit rate, by replacing the objective function by:

$$\Pi(\bar{x}_o) = D_\pi\left(y_o, p_o, x_o, \bar{x}_o, w_o\right) = \max_{\alpha,\gamma,\beta,\theta,\mu} \sum_{r=1}^{R} \gamma_r p_{o,r} \alpha_r y_{o,r} - \sum_{i=1}^{I\backslash\{j=1,\ldots,J\}} \theta_i w_{o,i} \beta_i x_{o,i} - \sum_{j=1}^{J} \theta_i w_{o,j} \bar{x}_{o,j} \quad \textit{(24).}$$
$$= p^{*T} y^* - w^{*T} x^*$$

However, we believe that the analysis of profitability is a real concern for DMUs' managers and a real-world lever for action for management decision making. A main advantage that profit rate has on its (profit) level counterpart is that comparisons based on the former are independent from the DMUs' size. Obviously, a DMU can achieve a higher profit than its peers simply due its larger size despite a lower profitability. The rest of our analysis will therefore be based on the program (23) whose objective function results in the optimal profitability based on optimal input and output quantities and prices as in:

$$\pi^*(\bar{x}_0) = \frac{p^{*T} y^*}{w^{*T} x^*}$$

According to equations (1)-(3), the value profitability efficiency is computed as:

$$\pi = \pi\left(y_o, p_o, \bar{x}_o, x_o, w_o\right) = \frac{\dfrac{p_o y_o}{p^{*T} y^*}}{\dfrac{w_o^T x_o}{w^{*T} x^*}} \quad (25),$$

and can be decomposed into:

$$\pi = E_Q^F \times E_P^F \quad (26)$$

where $E_Q^F = \sqrt{E_Q^{La} E_Q^{Pa}}$ and $E_P^F = \sqrt{E_P^{La} E_P^{Pa}}$

and, $E_Q^{La} = \dfrac{\dfrac{p_o^T y_o}{p_o^T y^*}}{\dfrac{w_o^T x_o}{w_o^T x^*}}$ $E_Q^{Pa} = \dfrac{\dfrac{p^{*T} y_o}{p^{*T} y^*}}{\dfrac{w^{*T} x_o}{w^{*T} x^*}}$, $E_P^{La} = \dfrac{\dfrac{p_o^T y_o}{p^{*T} y_o}}{\dfrac{w_o^T x_o}{w^{*T} x_o}}$, $E_P^{Pa} = \dfrac{\dfrac{p_o^T y^*}{p^{*T} y^*}}{\dfrac{w_o^T x^*}{w^{*T} x^*}}$.

The decomposition of the value indicator into a direct quantity effect and a direct price effect presents the advantage of satisfying the product test. This is a notable contribution compared to previous attempts in the literature.

Equation (26) is packed with managerial information regarding the evaluated DMU. The profitability efficiency measure gives a synthetic efficiency index for DMU performance. π is equal to 1 for efficient DMUs. $\left(1-\pi\right)$ gives the % of inefficiency, i.e. the % of potential increase for profit rate. Beyond this synthetic index, managers can clearly identify the sources of improvement between quantity effects and price effects. The decision maker is now informed whether managerial efforts must focus on technical efficiency or price negotiation with clients and/or suppliers. Moreover, each source of strength and weakness can be analyzed in detail. For each output and for each input, a detailed diagnosis in terms of quantity and price is therefore available through this analytical value framework.

3. An empirical analysis of the profitability efficiency decomposition

Our empirical study is based on a sample regarding 50 breed suckler cattle farms located in the Charolais-area grasslands (Northern French Massif Central)[2] for the year 2016. Data is collected on hired and family labor, intermediate inputs (cattle feed concentrates, veterinary costs, fertilizer, pesticide, energy, ...), fixed capital depreciation (equipment, livestock buildings, storage structures, …), land allocation scheme, herd, beef live-weight produced, others products, subsidies, investments and borrowing.

After a description of the variables retained in the underlying value transformation as well as some descriptive statistics of the sample, we will first develop a case study on a particular farm to illustrate the operational scope of our model to establish a complete diagnosis of profitability efficiency. Finally, synthetic results at the sample level are presented.

3.1. Data and variables

The empirical model retains two outputs and five inputs. The total income is the sum of beef live-weight produced sales and other products (sheep or pig meat, cereals and oilseeds crops, …). The total cost includes cattle feed concentrates and other intermediate consumption, land and labor costs as well as fixed capital consumption measured by the depreciation of fixed assets (farm buildings, fixed equipment, machinery and other equipment).

For each farm, the variables beef live-weight produced, concentrate feeding stuffs, land and labor expressed in value terms are decomposed into their respective quantity and unit price components.[3] Total agricultural area is expressed in hectares (ha) assuming that the owned land is paid at an opportunity price equal to the observed cost per hectare of rented land. Finally, labor quantity aggregates family and hired employees converted in full time equivalent persons (FTE). Labor cost is the sum of salaries and social contributions of employees as well as social contributions paid for family staff.

Tables 1 displays the main structural and economic characteristics of the sample. According to figures displayed, these farms are big commercial beef cattle operations which are similar to the observed commercial beef cattle farms in the Farm Accountancy Data Network (FADN, farming type 46 specialist cattle) statistically representative of such French farm businesses (Veysset et al., 2015).

[2] We would like to thank the INRAE — Clermont-Theix team and more particularly Michel Lherm and Patrick Veysset for making the data available.

3 It is quite common that, for certain accounting data of a business, the manager does not have clear information allowing them to separate the value in price and quantity. For these specific variables only available in value, it is still conceivable to include them as such in our model. Obviously, it is not possible to derive their respective price and quantity efficiency components. This flexibility of our analytical framework allows to adapt to real world case studies.

Table 1. Variables retained in the value transformation technology

N=50, year=2016	Variables	Mean	Relative standard deviation
Beef production (kg live weight)	y_1	54 122	51,1%
Beef selling price (€/kg)	p_1	2,20	10,0%
Beef sales (€)	R_1	118 382	49,7%
Sales of other products (€)	R_2	17 871	153,8%
Cattle feed concentrates (Kg)	x_1	58 526	127,1%
Cattle feed unit price (€/kg)	w_1	0,30	19,2%
Cattle feed expenses (€)	C_1	16 651	120,4%
Other intermediate input expenses (€)	C_2	94 020	53,6%
Utilized agricultural area (ha)	x_3	175	40,9%
Unit cost of land (€/ha)	w_3	121,10	23,6%
Land cost (€)	C_3	21 048	43,8%
Total labor (FTE)	x_4	1,9	34,0%
Unit labor cost (€/FTE)	w_4	6 872	62,3%
Labor cost (€)	C_4	14 135	90,1%
Fixed capital consumption (€)	C_5	28 402	43,6%

On average, farms achieve a turnover of € 136,000 for a total cost of € 174,000 (cf. table 2). Without the subsidies which amount to € 69,000, these farms would not make a positive profit. The total revenue including subsidies represents 122.7% of the total cost but if the only sales generated directly by the farm activities are considered then the profitability ratio does not exceed 80%.

Table 2. Revenue, cost and profitability ratio

	Variables	Mean(1)	Relative standard deviation
Total revenue (€)	R=R1+R2	136 253	48,0%
Total cost (€)	C=C1+C2+C3+C4+C5	174 257	51,7%
Aids/subsidies (€)	Subs	68 674	36,6%
Profitability without subsidies	R/C	79,31%	16,6%
Profitability including subsidies	(R+Subs)/C	122,7%	17,1%

(1) unweighted individual mean

From table 3, we note that the breeding activity is predominant with over 88% of total sales. On the cost side, more than half is allocated to other intermediate inputs. However, the biggest dispersions between farms are observed for the purchases of concentrated feeds and for the labor cost, both in terms of level (cf. table 1) and relative share (cf. table 3).

Table 3. Output revenue shares and input cost shares

	Variables	Mean(1)	Relative standard deviation
Output shares		100,0%	
Beef sales	R1/R	88,7%	16,8%
Sales of other products	R2/R	11,3%	131,8%
Input shares		100,0%	
Cattle feed expenses	C1/C	8,4%	67,9%
Other intermediate input expenses	C2/C	53,8%	10,9%
Land cost	C3/C	12,6%	25,1%
Labor cost	C4/C	8,0%	55,6%
Fixed capital consumption	C5/C	17,3%	26,8%

(1) unweighted individual mean

3.2. Results and analysis

To illustrate the relevance of our model for performing a complete diagnosis in terms of profitability efficiency, we first present the results of a farm specific case in the sample and then we present an aggregate analysis at the sector level.

a) Decomposition of profitability efficiency by specific inputs and outputs: a case study

The evaluated farm generates a turnover of 122,580 € for a total cost of 198,593 € and receives an amount of aids reaching 59,631 €.[4] The share of livestock activity is around 65%. Compared to its profitability benchmark, this farm could increase the profitability ratio (+73.6%)[5] through a growth of the turnover (+42.5%) and a reduction of the total cost (- 17.9%). The total quantity effect (+68.7%) explains most of this potential increase in profitability, while the contribution of the price effect would be of only +2.9%. The total quantity effect is shared between an increase in output (+32.7%) and a decrease in input (-21.4%). The total price effect comes from a simultaneous increase in output prices (+ 7.4%) and input prices (4.4%). This last result reveals that an objective of higher profitability does not necessarily involve lower input prices (cf. table 5).

The potential progress of total revenue can be decomposed as follows (cf. table 5). First, a possible growth in the quantity of beef production (+84.9%) then an increase in the selling price per kilo of beef (+9.5%). On the other hand, revenue related to other products should decrease (-68.8%) to allow greater specialization in beef cattle operations (92.3% instead of 65%).

Regarding the potential cost reduction, the main effort should focus on expenditures concerning other intermediate inputs (-34.8%). The cost of land would not vary significantly (+0.1%) but should be accompanied by a deep revision of its quantity and price components. Indeed, the number of hectares would slope down (-23.3%) while its price per hectare would increase (+31.6%). The payment for staff employed should only decline very slightly following a workforce preservation[6] coupled with a minor salary increase (+2.2%). Purchases of feed concentrates should marginally rise (+1.3%) according to a growth in price (+2.1%) and a negligible reduction of quantity (-0.8%). Finally, fixed capital spending is expected to go up (+13.9%).

In line with previous comments, table 4 summarizes the global diagnosis through different efficiency scores respectively related to the potential changes in quantities or prices in input/output variables. The value efficiency score (57.6%) is split into a revenue score (70.2%) and a cost score (82.1%). The sharing of revenue efficiency leads to a quantity effect of 75.4% and a price effect of 93.1%, the distribution of cost efficiency resulting in 78.6% for quantities and 104,4% for prices. To sum up, if the evaluated farm aligns with its benchmark, the profitability ratio could improve by $+73,6\%$ thanks to a rise in the TFP level $+68,7\%$ and a smaller increase in the price recovery index $+2.9\%$. Based on this analysis, we can therefore diagnose that the main problem of this agricultural business is its ability to increase the quantities produced and decrease the input quantities. Compared to a traditional analysis based only on quantities, the current case study shows that the price dimension for the input land also has an important contribution to improve

[4] After the successive reforms of the common agricultural policy, subsidies are now significantly decoupled from the farm output level. They are not included as decision variables in this model maximizing the profitability ratio.

[5] In what follows, profitability, TFP and price recovery increases are obtained as: $+73,6\% = \left(\frac{1}{\pi} - 1\right) = \left(\frac{1}{0.576} - 1\right)$, $+68,7\% = \left(\frac{1}{E_Q^F} - 1\right) = \left(\frac{1}{0.593} - 1\right)$, and $+2.9\% = \left(\frac{1}{E_P^F} - 1\right) = \left(\frac{1}{0.972} - 1\right)$ respectively.

[6] Recall that, in order to make maximization of the profitability ratio non-trivial, i.e. the best observed profitability ratio, we consider labor quantity as a quasi-fixed input. This assumption is in line with farm activity practice.

the DMU's profitability. A possible interpretation is that the efficient farms use less land but of a better quality as reflected in their higher prices.

To finish this particular case study, table 6 presents the decomposition of value profitability efficiency and its different input and output components in both the quantity and the price dimensions.

Table 4. Value profitability efficiency score and its distribution between global quantity and price effects

	Efficiency scores in %			Variations in %		
	Revenue	Cost	Profitability	Revenue	Cost	Profitability
π	70.2%	82.1%	57.6%	42.5%	-17.9%	73.6%
E_Q^F	75.4%	78.6%	59.3%	32.7%	-21.4%	68.7%
E_P^F	93.1%	104.4%	97.2%	7.4%	4.4%	2.9%

Table 5. The value profitability efficiency and its total decomposition by variables

	Efficiency Scores			Variations		
	E_Q^F	E_P^F	π	Quantity	Price	Value
Outputs	**75,4%**	**93,1%**	**70,2%**	**32,7%**	**7,4%**	**42,5%**
Beef production	54,1%	91,3%	49,4%	84,9%	9,5%	102,4%
Other products	n/a	n/a	320,8%	n/a	n/a	-68,8%
Inputs	**78,6%**	**104,4%**	**82,1%**	**-21,4%**	**4,4%**	**-17,9%**
Cattle feed concentrates	99,2%	102,1%	101,3%	-0,8%	2,1%	1,3%
Other intermediate inputs	n/a	n/a	65,2%	n/a	n/a	-34,8%
Utilized agricultural area	76,7%	131,6%	100,9%	-23,3%	31,6%	0,9%
Total labor	n/a	102,2%	102,2%	n/a	2,2%	2,2%
Fixed capital consumption	n/a	n/a	113,9%	n/a	n/a	13,9%
Total	**59,3%**	**97,2%**	**57,6%**	**68,7%**	**2,9%**	**73,6%**

b) Decomposition of profitability change by specific inputs and outputs: an analysis at the aggregate sample level

For the entire sample, the profitability score reached 77.3%.[7] This overall performance decomposes by revenue efficiency (102.5%) and cost efficiency (75.4%). These results imply that on average, farms should slightly decrease their turnover (-2.5%) and significantly reduce their cost (- 24.6%). The quantity and price components of revenue efficiency are 104.1% and 98.1%, respectively, implying a production decline (-4.4%) and an increase in output price rise (+2%). On the cost side, the quantity component indicates an efficiency of 77.9%, i.e. a decrease in the use of inputs (-22.1%) while the price efficiency is 96.7%, i.e. a decrease in input prices (-3.3%). All in all, the potential increase in the profitability ratio (+ 29.4%) is mainly explained by a quantity

[7] Geometrical mean

effect increasing the TFP (+22.8%) while the price effect only occurs for a slight growth in price recovery (+5.4%).

Table 7 allows a more detailed analysis of these performance indicators by output and input. It shows that the drop in turnover relates exclusively to the sale of other products (-46.6%), while beef sales expected to increase slightly (+4.2 %) both in quantity (+1.9%) and price (+ 2.2%). Finally, to improve cost efficiency, all expenditure items are concerned:

- reduction in animal feed purchases in quantity (-34.3%) and in price (-14.2%),
- reduction in the cost of other intermediate consumption (-29.4%),
- reduction of land in quantity (-12.5%) and price (-5.4%),
- reduction in labor unit cost (-15%)
- reduction in consumption of fixed capital (-7.7%)

Table 6. The value profitability efficiency and its total decomposition by variables at the sample level

	Efficiency Scores			**Variations**		
	E_Q^F	E_P^F	π	Quantity	Price	Value
Outputs	**104,6%**	**98,1%**	**102,5%**	**-4,4%**	**2,0%**	**-2,5%**
Beef production	98,1%	97,8%	96,0%	1,9%	2,2%	4,2%
Other products	187,1%	n/a	187,1%	-46,6%	n/a	-46,6%
Inputs	**77,9%**	**96,7%**	**75,4%**	**-22,1%**	**-3,3%**	**-24,6%**
Cattle feed concentrates	65,7%	85,8%	56,4%	-34,3%	-14,2%	-43,6%
Other intermediate inputs	70,6%	n/a	70,6%	-29,4%	n/a	-29,4%
Utilized agricultural area	87,5%	94,6%	82,8%	-12,5%	-5,4%	-17,2%
Total labor	n/a	85,0%	85,0%	n/a	-15,0%	-15,0%
Fixed capital consumption	92,3%	n/a	92,3%	-7,7%	n/a	-7,7%
Total	**81,5%**	**94,9%**	**77,3%**	**22,8%**	**5,4%**	**29,4%**

4. Conclusion

The present study seeks to contribute to the existing literature tackling a firm's overall financial performance. While several indicators can be advanced, we have retained here the firm profitability, defined as the ratio between the firm's sales and its costs. As the chosen indicator is a ratio whose terms are expressed in monetary terms (a value), a complete analysis must encompass two dimensions. One of them deals with productivity improvements as a result from a better management in terms of input and output quantities. The other one is the price recovery which reveals the ability to generate a mark-up of output prices with respect to inputs prices. Indeed, prices and quantities play an equally important role for operating businesses strategically. Business literature abounds in strategies meant to provide an edge over competitors in terms of both output prices, i.e. differentiation strategies resulting from skillful marketing and product development and input prices, i.e., scale and/or scope economies, vertical relations. Moreover, economic and statistical literatures have long established that growth and distribution of productivity gains are the two inseparable sides of a same coin.

However, within the general framework of benchmarking analysis, traditional efficiency measures consider prices as being determined exogenously and instead deal exclusively with quantity improvements only. Thus, this literature lacks a direct comparison of firms' output and input prices. Some attempts to fill in for this void exist. For example, the use of a value technology is a way to indirectly consider prices. An indirect price effect has even been introduced in this perspective. But these approaches all lack a direct measure for firms' price efficiencies.

To our knowledge, Portela and Thanassoulis (2014) came closest to this objective. In their cost efficiency measure both input prices and quantities are decision variables. However, while our work and theirs share the same objective, we believe that there are several major distinctions regarding the operationalization of this objective and, consequently, the results obtained with the two measures can be very different from one another. The main difference between these two approaches is related to the fact that, in our baseline model, we assume that prices and quantities are related to one another (resulting, for example, from factor/output demand functions) while in their work, P&T considered prices and quantities as completely independent from one another. The main issue in this approach is, in our opinion, that price efficiency determination is unrelated to the evaluated DMU. In an extension of our baseline cost model we propose to maintain the assumption that input prices remain independent from input quantities as in P&T, but we relate them to the level of the evaluated DMU's output quantity and price. Therefore, for each level of output quantity and its price, an optimal vector of input prices is determined which is no longer set at the minimal observed prices or some exogenously fixed bound.

Given these considerations, our models contribute to the related literature by introducing genuine price efficiency measure which can be adapted to account for both market characteristics (the degree of dependency between prices and quantities) and observed DMU activity. This new decomposition of value efficiency into a direct quantity effect and a direct price effect for each input and output variable is straightforward and, compared to previous attempts in this direction, presents the advantage of satisfying the product test.

Our contribution also has an undeniable high operational value. Indeed, a decision maker can now identify the sources of possible profitability increase levers either through improved quantity management and/or better search for new price opportunities. We have illustrated our approach with a sample of French cattle farms. While one could have expected that in this agricultural sector prices would not play any role at all, our application shows that, although their role is indeed smaller compared to that of quantities, they still represent a lever for action for profitability improvement. This is especially observed in terms of different input prices (labor, animal feed purchases and land). Concerning the outputs, our illustration shows that farmers present the same level of inefficiency in terms of produced quantities as in terms of prices. Efforts should therefore be concentrated in the direction of increasing quantities produced as well as improved selling strategy (alternative short cycle favoring customer proximity, differentiation strategy).

5. References

Ayouba, K., Boussemart, J.-P., Lefer, H-B., Leleu, H., Parvulescu, R. (2019). A measure of price advantage and its decomposition into output- and input-specific effects. *European Journal of Operational Research*, 276(2), 688–698.

Camanho A.S., Dyson R., (2008). A generalization of the Farrell cost efficiency measure applicable to non-fully competitive settings. *Omega*, 36, 147–62

Centre d’étude des revenus et des coûts, CERC (1980). Productivité globale et compte de surplus, *La Documentation française*, n°55/56 ; 218 p.

Charnes A., Cooper W.W., Lewin A.Y., Seiford L.M., (1994). *Data envelopment analysis: theory, methodology, and applications*. Springer Netherlands

Coelli T.J., Prasada Rao D.S., Battese G.E., (2005). *An introduction to efficiency and productivity analysis*, 2nd ed. Springer.

Courbis R., Templé Ph. (1975), La méthode des comptes de surplus et ses applications macroéconomiques. *Les collections de l'INSEE*, Série C, 35, Paris, 98 p.

Davis H.S., (1955). *Productivity Accounting.* University of Pennsylvania.

Farrell M.J., (19657). The measurement of productive efficiency. *Journal of Royal Statistical Society* Series A, 120, 253–381.

Färe R., Grosskopf S., Lovell C.A.K., (1985). *The measurement of efficiency of production*. Boston: Kluwer Academic.

Fried, H.O., Lovell C.A.K., Schmidt S. S., (2008). *The measurement of productive efficiency and productivity growth*. Oxford: Oxford University Press.

Grifell-Tatjé, E., Lovell, C. A. K. (2015). *Productivity accounting. The economics of business performance*. Cambridge, UK: Cambridge University Press.

Grifell-Tatjé, E., Lovell, C. A. K. (2020). Dual productivity analysis: A Konüs/Shephard approach. *European Journal of Operational Research*, 289(1), 328–337.

Kendrick J.W., (1961). *Productivity trends in the United States*. Princeton, NJ: Princeton University Press.

Kendrick J.W., Creamer D., (1965). *Measuring Company Productivity: A handbook with case studies* (89) 3rd ed. The National Industry Productivity Board.

Portela, M.C.A.S., Thanassoulis, E., (2014). Economic efficiency when prices are not fixed: Disentangling quantity and price efficiency. *Omega*, 47, 36–44.

Sahoo, B. K., Mehdiloozad, M., Tone, K. (2014). Cost, revenue and profit efficiency measurement in DEA: A directional distance function approach. *European Journal of Operational Research*, 237(3), 921-931.

Shephard R.W., (1953). Cost and Production Functions. Princeton University Press.

Shephard R.W., (1970). Theory of Cost and Production Functions. Princeton University Press.

Templé Ph., (1971). La méthode des surplus : un essai d'application aux comptes des entreprises (1959-1967). Economie et statistique, 29, 33-50.

Tone K., (2002). A strange case of the cost and allocative efficiencies in DEA. *Journal of the Operational Research Society*, 53, 1225–31.

Veysset P., Lherm M., Roulenc M., Troquier C. and Bébin D., 2015. Productivity and technical efficiency of suckler beef production systems: trends for the period 1990 to 2012. *Animal* 9, 2050–2059.

Vincent L.-A., (1961). The Main Formulae for Productivity Measurement in a National Economy or Sector. *Productivity Measurement Review*, 25, 5-17.