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Evaluating Horizontal Mergers in Swedish District Courts Using Plant Capacity Concepts: With a Focus on Nonconvexity*

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Abstract

This contribution investigates the effects of horizontal mergers and acquisitions on the plant capacity utilisation of Swedish district courts over the period 2000-2017. More specifically, we empirically illustrate the decomposition of input-oriented and output-oriented plant capacity utilisation concepts. Moreover, we also explore the impact of convexity on input-oriented and output-oriented measures of plant capacity in the short-run in an attempt to discover the potential rationale behind the merger wave. To the best of our knowledge, we are the first to assess horizontal mergers by employing plant capacity utilisation concepts. The results indicate that the horizontal mergers improve plant capacity utilisation. Furthermore, the nonconvex frontier method provides a more conservative estimate of plant capacity changes of this merger wave.

Keywords: Data Envelopment Analysis; Free Disposal Hull; Plant capacity utilisation; Horizontal mergers and acquisitions

JEL code: D24

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1 Introduction

The regulation of horizontal mergers and acquisitions (HM&As) is a central pillar of modern competition regulation around the world. This issue is now receiving increasing attention due to concerns about rising concentration and market power in traditional sectors and new technologies. HM&As are mergers between industries that produce or perform the same or similar products and services, which has significant policy implications for the degree of competition in the relevant industries. Horizontal mergers have two obvious objectives: one is to achieve economies of scale and economies of scope, and the other is to increase the degree of concentration in the industry (e.g., De Loecker, Eeckhout, and Unger (2020) and the ensuing book of Eeckhout (2021)). Indeed, horizontal mergers may achieve social welfare gains by cost reductions, but they also can expand the market power of companies by consolidating industries yielding a dead weight loss. In particular, horizontal mergers (especially by large firms) tend to undermine competition and may create highly monopolistic situations. Since greater cost savings contribute to HM&As being approved by regulators, HM&A participants have a clear incentive to overstate cost savings potential. In this regard, it is essential to provide constructive suggestions to companies and regulators by adopting a conservative approach to the assessment of horizontal mergers.

As a matter of fact, a large majority of empirical studies have been conducted to examine the effects of HM&As based on applied production analysis. The industrial organization literature distinguishes between direct price comparisons before and after the mergers, event studies for stock market listed firms assessing shareholder value, and merger simulations using pre-merger market information calibrating some noncooperative oligopoly models (see, e.g. Belleflamme and Peitz (2010, Section 15.4) for a broad overview). This industrial organization literature also acknowledges that technical and cost inefficiencies may contribute to cost savings of horizontal HM&As (see, e.g., Caves (2007)).

In this contribution our empirical evaluation tool is based on deterministic nonparametric frontier methods (sometimes denoted as Data Envelopment Analysis (DEA)) providing inner approximations of the production boundaries while maintaining minimal axioms on what is feasible (see Ray (2004)). In this deterministic nonparametric production frontier literature, various strands of literature analyse the potential ex ante and ex post efficiency gains of HM&As. We limit ourselves to studies analysing courts.

Efficiency studies on court performance include the following. Gorman and Ruggiero (2009) evaluate input-oriented technical and scale efficiency for prosecutorial offices in US judicial districts, reporting that the majority of offices are overstaffed for the measured outputs and exhibit significant decreasing returns to scale. Castro and Guccio (2018) employ a nonparametric frontier method to analyse the 165 Italian judicial counties, finding that technical efficiency is the main sources of poor performance. Similar studies on courts are found in Castro and Guccio (2018) and Peyrache and Zago (2016). Moreover, Silva (2018) appears to be the first to quantify efficiency of Portuguese courts by using three approaches that represent links between inputs and outputs: independent assessments, ratios, and differences. In some of these models outputs are generated by output-specific inputs rather than by all inputs jointly.

Other studies measure the productivity growth of courts. Falavigna, Ippoliti, and Ramello (2018) apply nonparametric technical efficiency scores and the Malmquist index to better understand the impact of a specific policy on the productivity of the Italian tax courts, and decompose the index into efficiency change and technology change, emphasizing that technology does not fully replace the productive role of the judges and that a suitable policy should be created to improve productivity. Mattsson, Månsson, Andersson, and Bonander (2018) incorporate the potential heterogeneity between the output variables by a weighting based on differences in spent resources between the 14 categories, employ the super-efficiency model to eliminate outliers, and calculate the output-oriented Malmquist productivity index by applying a nonparametric frontier technology to evaluate the efficiency and productivity of Swedish district courts from 2011 to 2015: they obtain a 1.7% average productivity decline per year. Blank and van Heezik (2020) employ a parametric non-frontier cost function model to time series data for the Dutch judicial sector from 1980 to 2016 to assess productivity development, obtaining a sharp decline in productivity over the period despite different legislative interventions and technology advancements.

Research on gains and efficiency of horizontal mergers using deterministic nonparametric frontier methods in courts includes the following examples. Mattsson and Tidanå (2019) utilize a nonparametric frontier decomposition method to identify the potential ex ante merger gains, showing that some mergers have no potential for efficiency gains, while others can yield significant merger gains. Agrell, Mattsson, and Månsson (2020) employ a nonparametric frontier method to measure the ex post efficiency of horizontal mergers for the Swedish district courts, showing that the merged courts are more efficient than the non-merged ones. Moreover, Chen, Kerstens, and Zhu (2021) are the first to combine traditional convex (C) with nonconvex (NC) nonparametric frontier methods to calculate efficiency before and after the Swedish district courts mergers, suggesting that mergers improve efficiency mainly via scale efficiency under NC, but technical efficiency under C. In addition, under C most observations are subject to decreasing returns to scale, while under NC one could have selected among increasing returns to scale observations for the mergers.

While there is more research on the efficiency of courts using various kinds (see the survey by Voigt (2016)), studies computing plant capacity utilisation before and after horizontal mergers are unknown to us: neither in general, nor for courts. Capacity utilisation is a key determinant of corporate profitability and a major indicator of macroeconomic performance. In determining whether to expand their production facilities, companies rely heavily on observed capacity utilisation rates at the enterprise level. Moreover, horizontal mergers can also be interpreted as an external expansion for the acquiring units to obtain more production capacity in the short run.

The oldest output-oriented plant capacity utilisation (PCU) concept has been introduced in the literature by Fare, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) for the single output case and the multiple output case, respectively. Their definition boils down to a ratio of two output-oriented efficiency measures: one with given observed inputs, and another one for unlimited variable inputs. These output-oriented efficiency measures are evaluated relative to nonparametric deterministic technologies based on observed inputs and outputs.

Kerstens, Sadeghi, and Van de Woestyne (2019) state that this output-oriented PCU is impractical: the amounts of variable inputs required to obtain maximum capacity output may not be available at either the firm or the industry level. To solve this attainability issue, Kerstens, Sadeghi, and Van de Woestyne (2019) define a new type of attainable output-oriented plant capacity utilization that essentially limits the availability of variable inputs. This solution is conceptually appealing, but empirically the problem is to specify a realistic level of attainability for the variable inputs. Therefore, we abstain from applying this capacity notion in this contribution.

An input-oriented PCU notion has been defined by Cesaroni, Kerstens, and Van de Woestyne (2017) as a ratio of two variable input-oriented efficiency measures: one with current output levels, and one with zero outputs to mark the setup of production. Again, both these efficiency measures are measured relative to nonparametric deterministic technologies based on observed inputs and outputs. This notion lends itself to a comparison with the traditional and widely used economic concepts based on cost functions.

The axiom of convexity is known to cause a potential impact on technology-based empirical analyses (see, e.g., Tone and Sahoo (2003)). Walden and Tomberlin (2010) first propose an empirical illustration of the effects of C on the output-oriented PCU notion. Afterwards, Cesaroni, Kerstens, and Van de Woestyne (2017) conduct an empirical comparison between output-oriented and input-oriented PCU notions, showing that C indeed has a powerful influence on both concepts. Kerstens, Sadeghi, and Van de Woestyne (2019) offer the most extensive empirical study today illustrating the impact of C on a variety of both PCU and cost-based capacity concepts.

However, most researchers ignore the potential impact of C on the cost function, which is due to its property in the outputs that is often ignored. More specifically, the cost function is non-decreasing and C in the outputs when technology is C, otherwise the cost function is NC in the outputs (see Jacobsen (1970)). Empirical studies rarely put this property to the test. An exception is the recent study of Kerstens and Sadeghi (2021) documenting a huge impact of C on empirical cost estimates. Furthermore, Kerstens, Sadeghi, and Van de Woestyne (2019) compare four PCU concepts (input-oriented versus output-oriented, short-run versus long-run) with a series of cost-based capacity utilisation measures. Two key conclusions are generated. First, input-oriented PCU tends to be more naturally compared to cost-based capacity concepts than output-oriented PCU notions. Second, C makes an obvious difference for both technical and economic capacity notions. Note that cost-based capacity utilisation measures are not options for our analysis: this requires input price information, which is lacking in our data set.

Therefore, this contribution focuses on the development of PCU measures using C and NC frontier technologies for Swedish district courts. More specifically, this contribution intends to solve the following two issues. First, what are the effects of horizontal mergers on the short-run input-oriented and output-oriented PCU under C and NC technologies? For this purpose, we respectively adopt the C and NC nonparametric frontier methods to calculate the various orientations of PCU under the short-run scenario for Swedish district courts. Second, what is the difference in short-run input-oriented and output-oriented PCU between the pre-merger and post-merger observations under C and NC measures? This question is addressed by comparing the efficiency and PCU between the pre-merger and post-merger observations under C and NC measures.

The remainder of the contribution proceeds as follows. In Section 2, we introduce a brief literature review on plant capacity. We recall some elements of our methodology in the Section 3, such as production technologies, efficiency measures, and plant capacity utilisation concepts. Section 4 illustrates the empirical application of Swedish district courts, and the final Section 5 offers the conclusions.

2 Plant Capacity Utilisation Notions: A Brief Literature Review

In the economics literature, it is common to distinguish between on the one hand technical or engineering capacity concepts based solely on quantity information on inputs and outputs, and economic capacity concepts that in addition require some price information depending on the specific value function involved. Starting from the technical or engineering capacity concepts, Johansen (1968) is probably the first to introduce such an approach by the introduction of the notion of PCU using a single output production function. The notion of output-oriented PCU has been informally described as "the maximum quantity of output that can be produced per unit of time with the available equipment without limiting the variable factors of production"(Johansen (1968, p. 362)). This notion has been generalised to a multi-output frontier framework by utilizing a combinations of two output efficiency measures. More specifically, Fare, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) define the output-oriented PCU concept as a ratio of two output-oriented efficiency measures evaluated relative to nonparametric deterministic technologies based on observed inputs and outputs for the single output case and the multiple output case, respectively.

Most existing research on output-oriented PCU focuses on fisheries and hospitals (see, e.g., Kerstens and Shen (2021) for a selective review). For example, Färe, Grosskopf, and Valdmanis (1989) find that the hospitals in their sample do not operate at full capacity. Dividing the sample into urban and non-urban status (whereby urban proxies for competition), they find that urban hospitals over-utilize doctors and other staff, whereas these inputs are underutilized in non-urban hospitals. For instance, Vestergaard, Squires, and Kirkley (2003) estimate capacity utilization for vessels in the the multi-species Danish Gill-net fleet.

More recently, this output-oriented PCU has been extended to account for good and bad outputs jointly. Yang and Fukuyama (2018) propose an output-oriented directional distance function to measure the regional production potential of Chinese provinces based on a generalized capacity utilization indicator. Yang, Fukuyama, and Song (2019) adopt a capacity utilisation indicator based on a directional distance function to investigate the capacity utilization of Chinese manufacturing industries. Fukuyama, Liu, Song, and Yang (2021) measure PCU with the unrestricted capacity directional distance function by focusing on the 48 largest iron and steel enterprises in China.

Kerstens, Sadeghi, and Van de Woestyne (2019) argue and empirically illustrate that the traditional output-oriented PCU is unrealistic, since the amounts of variable inputs needed to reach maximum capacity output may not be available at either the firm or industry level. Hence, in response to this co-called attainability issue (pointed out by Johansen (1968)), Kerstens, Sadeghi, and Van de Woestyne (2019) define a new attainable output-oriented PCU that limits the availability of variable inputs and leads to more realistic output capacity targets. As stated earlier, this solution is hard to implement because of the need to specify a realistic level of attainability for the variable inputs. Therefore, we ignore this capacity notion in the remainder.

Cesaroni, Kerstens, and Van de Woestyne (2017) propose a new input-oriented PCU measure and illustrate both input-oriented and output-oriented decompositions of technical efficiency integrating these concepts of capacity utilisation. The empirical illustration using specialized Canadian dairy farms highlights the similarities and differences between input-oriented and output-oriented plant capacity, and confirms that C has a significant impact on the estimation of plant capacities.

Cesaroni, Kerstens, and Van de Woestyne (2019) propose new long-run output-oriented and input-oriented PCU concepts, which allow for changes in all input dimensions simultaneously rather than allowing for changes in the variable input dimensions solely. The earlier PCU concepts only focusing on changes in the variable inputs can then be considered as short-run PCU concepts.

Kerstens and Shen (2021) empirically adopt these various short-run and long-run input-oriented and output-oriented PCU measures to measure hospital capacity in the Hubei province in China during the recent COVID epidemic. This empirical study indicates that the input-oriented concept of long-run PCU is best associated with observed mortality, despite the limited sample size. This may lead empirical researchers to reconsider their choice of PCUconcept. Furthermore, these authors forcefully argue against the seminal articles of Fare, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) and the tendency in some of the early output-oriented PCU empirical applications to impose constant returns to scale rather than variable returns to scale. The latter variable returns to scale assumption is applied in the new input-oriented and long-run PCU concepts discussed earlier.

Turning to economic capacity concepts, we draw on Nelson (1989) and distinguish at least three approaches proposed in the literature for defining the concept of cost-based capacities, aiming to isolate short-run over- or under-utilization of existing fixed inputs. The first definition focuses on the outputs that are available at the short-run minimum average total cost given existing input prices (see Hickman (1964)). The second definition centers on output that is tangent to the short-run and long-run average total cost curves (see Segerson and Squires (1990)). There are two variations of this concept of tangency point, depending on what the decision variables are. One concept assumes that the output is constant and identifies the optimal variable and fixed inputs. The other concept assumes that fixed inputs cannot be adjusted, but outputs, output prices, and fixed input prices can be adjusted. The final definition of economic capacity assumes that output is determined by the minimum of the long-run average total cost (see Klein (1960)). Empirical applications of economic capacity concepts in relation to some other PCU concepts is found in Kerstens, Sadeghi, and Van de Woestyne (2019).

These different engineering and economic capacity concepts have been integrated into a static efficiency decomposition in De Borger, Kerstens, Prior, and Van de Woestyne (2012). The output-oriented PCU notion has been integrated into a decomposition of the Malmquist productivity index in De Borger and Kerstens (2000).

3 Methodology

3.1 Production Technologies

In this section, we introduce some basic definitions and define the court production technology. In accordance with the theory of axiomatic production, homogeneously observed units determine the production possibility set. Consider a set of K observations $\boldsymbol{A} = \{(x_1, y_1), ..., (x_K, y_K)\} \in \mathbb{R}_+^{M+N}$. A production technology describes all available possibilities to transform input vectors $\boldsymbol{x} = (x_1, \ldots, x_m) \in \mathbb{R}_+^M$ into output vectors $\boldsymbol{y} = (y_1, \ldots, y_n) \in \mathbb{R}_+^N$. The production possibility set or technology $\boldsymbol{S}$ summarizes the set of all feasible input and output vectors:

$$\boldsymbol{S} = \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}_+^{M+N} : \boldsymbol{x} \text{ can produce at least } \boldsymbol{y}\}. \tag{1}$$

Drawing upon Färe, Grosskopf, and Lovell (1994, p.44-45), some regularity conditions on the data for inputs and outputs should be imposed: (1) Each producer utilizes non-negative amounts of each input to produce non-negative amounts of each output. (2) There is an aggregate utilization of positive amounts of every output, and an aggregate utilization of positive amount of every input. (3) Each producer adopts a positive amount of at least one input to produce a positive amount of at least one output.

This technology can be represented by the input correspondence $L : \mathbb{R}_+^N \rightarrow 2^{\mathbb{R}_+^M}$ where $L(\boldsymbol{y})$ is the set of all input vectors that yield at least the output vector $\boldsymbol{y}$: $L(\boldsymbol{y}) = \{\boldsymbol{x} : (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{S}\}$. In the similar vein, the output vector associated with $\boldsymbol{S}$ denotes all output vector $\boldsymbol{y} \in \mathbb{R}_+^N$ that can be produced from a given input vector $\boldsymbol{x} \in \mathbb{R}_+^M$: $P(\boldsymbol{x}) = \{\boldsymbol{y} : (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{S}\}$.

In the above definition, technology $\boldsymbol{S}$ satisfies some combinations of the following standard assumptions:
($\boldsymbol{S}$.1) No free lunch and possibility of inaction, i.e., $(0, 0) \in \boldsymbol{S}$, and if $(0, \boldsymbol{y}) \in \boldsymbol{S}$, then $\boldsymbol{y} = 0$.
($\boldsymbol{S}$.2) Technology $\boldsymbol{S}$ is closed of $\mathbb{R}_+^N \times \mathbb{R}_+^M$.
($\boldsymbol{S}$.3) Strong disposability on inputs and outputs, i.e., $(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{S}$ and $(\boldsymbol{x}^{'}, \boldsymbol{y}^{'}) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, then $(\boldsymbol{x}^{'}, -\boldsymbol{y}^{'}) \geq (\boldsymbol{x}, -\boldsymbol{y}) \Rightarrow (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{S}$.
($\boldsymbol{S}$.4) Technology $\boldsymbol{S}$ is convex.

Note that not all of these axioms are included in the empirical analysis simultaneously. Moreover, we do not add a specific return to scale assumption: this corresponds to a flexible or variable return to scale assumption.

It is common to partition the input vector into a fixed and variable part $\boldsymbol{x} = (\boldsymbol{x}^f, \boldsymbol{x}^v)$, with $\boldsymbol{x}^v \in \mathbb{R}_+^{M_v}$ and $\boldsymbol{x}^f \in \mathbb{R}_+^{M_f}$ with $M = M_v + M_f$. Similarly, we define a short-run technology $\boldsymbol{S}^f = \{(\boldsymbol{x}^f, \boldsymbol{y}) :$ there exists some $\boldsymbol{x}^v$, such that $(\boldsymbol{x}^f, \boldsymbol{x}^v)$ can produce at least $\boldsymbol{y}\}$ and the corresponding input vector $L^f(\boldsymbol{y}) = \left\{\boldsymbol{x}^f : (\boldsymbol{x}^f, \boldsymbol{y}) \in \boldsymbol{S}^f\right\}$ and output set $P^f(\boldsymbol{x}^f) = \left\{\boldsymbol{y} : (\boldsymbol{x}^f, \boldsymbol{y}) \in \boldsymbol{S}^f\right\}$.

This partitioning of inputs between fixed and variable inputs leads us to sharpen the regularity conditions on the input and output data. According to Fare, Grosskopf, and Kokkelenberg (1989, p. 659–660), the following conditions apply as well: (4) Each fixed input is used by some producer, and each producer uses some fixed inputs. Furthermore, the following conditions also apply: (5) Each variable input is used by some producer; each producer uses some variable inputs. Failure to comply with these regularity conditions leads to infeasibilities of the corresponding mathematical programming problems.

Last but not least, $L(0) = \{\boldsymbol{x} : (\boldsymbol{x}, 0) \in \boldsymbol{S}\}$ is the input set compatible with a zero output level. Cesaroni, Kerstens, and Van de Woestyne (2019) present more details on these specific technology definitions, and their Figures 1 to 4 clarify the various technology definitions.

3.2 Efficiency Measures

Efficiency measures provide an equivalent representation of production technologies and focus on positioning observations relative to the boundary of the production possibility set. When an operating unit is at the boundary of technology, then it is technically efficient. However, if an operating unit is situated below the boundary, then it is technically inefficient. In this contribution, we define the radial input efficiency measure as follows:

$$DF_i\left(\boldsymbol{x},\boldsymbol{y}\right) = \min\left\{\lambda \ : \ \lambda \geq 0,\ \lambda\boldsymbol{x} \in L(\boldsymbol{y})\right\}, \tag{2}$$

where λ indicates the possible proportional reduction in inputs for a given level of outputs. This ratio measure is smaller than or equal to unity ($0 < DF_i\left(\boldsymbol{x},\boldsymbol{y}\right) \leq 1$), whereby efficient production on the isoquant of $L(\boldsymbol{y})$ is represented by unity and $1 - DF_i\left(\boldsymbol{x},\boldsymbol{y}\right)$ indicates the amount of inefficiency. An inefficient unit is found below the boundary of the input set $0 < DF_i\left(\boldsymbol{x},\boldsymbol{y}\right) < 1$.

By analogy, the radial output efficiency measure can be defined as follows:

$$DF_o\left(\boldsymbol{x},\boldsymbol{y}\right) = \max\left\{\theta \ : \ \theta \geq 0,\ \theta\boldsymbol{y} \in P(\boldsymbol{x})\right\}, \tag{3}$$

where θ is a measure of technical efficiency indicating the maximum proportional expansion of outputs that can be achieved at a given level of inputs. Note that this ratio is larger than or equal to unity ($DF_o(\boldsymbol{x},\boldsymbol{y}) \geq 1$), with efficient production on the boundary of the output set $P(\boldsymbol{x})$ represented by unity. An inefficient unit is situated in the interior of the production possibility set ($DF_o(\boldsymbol{x},\boldsymbol{y}) > 1$).

In the similar way, we can denote the radial output efficiency measure of the output set $P^f(\boldsymbol{x}^f)$ by $DF_o^f(\boldsymbol{x}^f,\boldsymbol{y})$. Then, this efficiency measure can be defined as follows:

$$DF_o^f\left(\boldsymbol{x}^f,\boldsymbol{y}\right) = \max\left\{\theta \ : \ \theta \geq 0,\ \theta\boldsymbol{y} \in P^f(\boldsymbol{x}^f)\right\}. \tag{4}$$

Furthermore, the following definitions are required. First, a sub-vector input efficiency measure reducing only the variable inputs can be defined as follows:

$$DF_i^{SR}(\boldsymbol{x}^f,\boldsymbol{x}^v,\boldsymbol{y}) = \min\left\{\lambda : \lambda \geq 0, (\boldsymbol{x}^f,\lambda\boldsymbol{x}^v) \in L(\boldsymbol{y})\right\}. \tag{5}$$

In the similar vein, a sub-vector input efficiency measure reducing variable inputs evaluated relative to this input level with a zero output set can be expressed as follows:

$$DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0) = \min \left\{ \lambda : \lambda \geq 0, (\boldsymbol{x}^f, \lambda \boldsymbol{x}^v) \in L(0) \right\}. \tag{6}$$

After introducing all of the efficiency measures specified to define the different PCU concepts, we now provide the mathematical definition of the technologies used to estimate plant capacities. Assuming data from K observations $\{k = 1, 2, \ldots, K\}$ comprising of an array of inputs and outputs $(\boldsymbol{x}_K, \boldsymbol{y}_K) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, the following unified mathematical representation of C and NC nonparametric frontier technologies is achievable under the variable returns to scale assumption, as follows:

$$\boldsymbol{S}^\Lambda = \left\{ (\boldsymbol{x}, \boldsymbol{y}) |\ \boldsymbol{x} \geq \sum_{k=1}^{K} z_k\ \boldsymbol{x}_k,\ \boldsymbol{y} \leq \sum_{k=1}^{K} z_k\ \boldsymbol{y}_k\ z_k \in \Lambda \right\}, \tag{7}$$

where

$$\begin{aligned} \text{(i) } \Lambda \ &\equiv\ \Lambda^{\mathrm{C}} = \left\{ z |\ \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}; \\ \text{(ii) } \Lambda \ &\equiv\ \Lambda^{\mathrm{NC}} = \left\{ z |\ \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0,1\} \right\}. \end{aligned}$$

The C axiom is represented by the activity vector ($\boldsymbol{z}$) of real numbers adding up to unity. NC is represented by the same sum constraint, with each vector element being a binary integer. The C technology adheres to axioms ($\boldsymbol{S}$.1) (excluding inaction) through ($\boldsymbol{S}$.4), whereas the NC technology adheres to axioms ($\boldsymbol{S}$.1) to ($\boldsymbol{S}$.3). It is now useful to qualify the efficiency measures by differentiating between C and NC nonparametric frontier technologies.

3.3 Short-Run Plant Capacity Utilisation

We recall the definition of short-run output-oriented PCU measure (see Fare, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989)). The definition of the output-oriented measure of PCU ($PCU_o^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y})$) can be defined by requiring solving an output efficiency measure relative to a standard technology and the same technology without restrictions on the available variable inputs as follows:

$$PCU_o^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}) = \frac{DF_o(\boldsymbol{x}, \boldsymbol{y})}{DF_o^f(\boldsymbol{x}^f, \boldsymbol{y})}, \tag{8}$$

where $DF_o(\boldsymbol{x}, \boldsymbol{y})$ and $DF_o^f(\boldsymbol{x}^f, \boldsymbol{y})$ are output efficiency measures relative to technologies including or excluding the variable inputs. By taking ratios of efficiency measures, one eliminates any existing inefficiencies and in this sense it gives rise to a clean concept of output-oriented PCU free of any eventual technical inefficiency.

Notice that $0 < PCU_o^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}) \leq 1$ since $1 \leq DF_o(\boldsymbol{x}, \boldsymbol{y}) \leq DF_o^f(\boldsymbol{x}^f, \boldsymbol{y})$. Therefore, the output-oriented PCU has an upper limit of unity, but no lower limit. This output-oriented PCU compares the maximum amount of output for a given input with the maximum amount of output for a potentially infinite number of variable inputs in the sample, when it is less than unity. Moreover, the last efficiency measure provides a reliable estimate of the maximum amount of output, i.e., the sample also contains the maximum plant that combines the highest level of variable inputs with the highest level of output.

In what follows, the short-run output-oriented decomposition can be defined as follows:

$$DF_o(\boldsymbol{x}, \boldsymbol{y}) = DF_o^f(\boldsymbol{x}^f, \boldsymbol{y}) \times PCU_o^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}). \tag{9}$$

Thus, the traditional output-oriented efficiency measure $DF_o(\boldsymbol{x}, \boldsymbol{y})$ can be decomposed into a biased plant capacity measure $DF_o^f(\boldsymbol{x}^f, \boldsymbol{y})$ and an unbiased PCU $PCU_o^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y})$.

Moreover, drawing on Cesaroni, Kerstens, and Van de Woestyne (2017), the input-oriented PCU measure $PCU_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y})$ can be defined as follows:

$$PCU_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}) = \frac{DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y})}{DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0)}, \tag{10}$$

where $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y})$ and $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0)$ are sub-vector input efficiency measures reducing only the variable inputs relative to the standard technology and the technology with a zero output level, respectively.

Notice that $PCU_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}) \geq 1$, since $0 < DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0) \leq DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y}) \leq 1$. Therefore, input-oriented PCU has a lower limit of unity, but no upper limit. This input-oriented PCU compares the minimum variable input for a given amount of output with the minimum variable input for the level of output at which production begins, so it is greater than one. Notice that $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0)$ provides a reliable estimate of the minimum amount of variable inputs compatible with the start-up of production to the extent that the sample also contains the smallest plants combining the lowest levels of variable inputs with zero or low level of outputs.

Thus, the short-run input-oriented decomposition can be denoted as follows:

$$DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y}) = DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0) \times PCU_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y}). \tag{11}$$

Thus, the traditional sub-vector input-oriented efficiency measure $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, \boldsymbol{y})$ can be decomposed into a biased plant capacity $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0)$ and an unbiased PCU $PCU_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}, \boldsymbol{y})$.

In summary, the short-run output-oriented PCU measure evaluates capacity by comparing the frontier output at a given observation to the maximum frontier output for unconstrained variable inputs, whereas the short-run input-oriented PCU measure evaluates capacity by comparing the minimum variable input at the frontier to the minimum variable input at the frontier point that produces zero outputs. In other words, the output-oriented PCU measure compares output to the greatest level of outputs available, whereas the input-oriented PCU measure compares variable inputs to the quantity of variable inputs compatible with zero outputs. The combination of short-run output-oriented and input-oriented biased and unbiased plant capacity utilization concepts requires computing a total of four efficiency measures: we refer the reader to Appendix A for all details.

In terms of attainability, the concepts of output- and input-oriented PCU diverge. Johansen (1968, p. 362) argues that the short-run output-oriented PCU concept is unattainable because the extra variable inputs required to achieve the maximum plant capacity outputs may not be available at the firm and/or industry level. Kerstens, Sadeghi, and Van de Woestyne (2019) empirically demonstrate that the quantity of variable inputs required to achieve full plant capacity outputs can be unrealistic.

In contrast, a short-run input-oriented concept of PCU is always possible since the number of variable inputs available can always be reduced to achieve a set of variable inputs compatible with a zero level of outputs. Because of the inaction axiom, it is typically possible to reduce variable inputs to achieve a zero output level. Inaction implies the ability to halt production. Moreover, creating zero outputs does not always imply that no inputs are utilized. Large industrial plants where maintenance operations inhibit production are examples generating zero outputs while variable inputs are nevertheless positive.

Finally, when comparing C and NC results, there are some cases in which the PCU concepts can be ordered a priori. First, we present the results for the biased notions of plant capacity.

Proposition 3.1. *Following Kerstens, Sadeghi, and Van de Woestyne (2019, p.704), it* *is straightforward to establish the following relations for the biased plant capacity concepts between C and NC measures in the short-run case:*

- *For the biased input-oriented plant capacity, we have:* $DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0|\, C) \leq DF_i^{SR}(\boldsymbol{x}^f, \boldsymbol{x}^v, 0|\, NC)$.
- *For the biased output-oriented plant capacity, we have:* $DF_o^f(\boldsymbol{x}^f, \boldsymbol{y}|\, C) \geq DF_o^f(\boldsymbol{x}^f, \boldsymbol{y}|\, NC)$.

Then, we also do the same for the unbiased PCU concepts.

Proposition 3.2. *Following Kerstens, Sadeghi, and Van de Woestyne (2019, p.704), it is straightforward to establish the following relations for the unbiased plant capacity concepts between C and NC measures in the short-run case:*

- *For the short-run input-oriented PCU, we have:* $PCU_i^{SR}(\boldsymbol{x}, \boldsymbol{x}^f, \boldsymbol{y}|C) \gtreqless PCU_i^{SR}(\boldsymbol{x}, \boldsymbol{x}^f, \boldsymbol{y}|NC)$;
- *For the short-run output-oriented PCU, we have:* $PCU_o^{SR}(\boldsymbol{x}, \boldsymbol{x}^f, \boldsymbol{y}|C) \gtreqless PCU_o^{SR}(\boldsymbol{x}, \boldsymbol{x}^f, \boldsymbol{y}|NC)$;

Thus, we cannot a priori sign the relations between C and NC PCU notions.

4 Empirical Application

Our empirical analysis proceeds in three steps. The first step discusses the descriptive statistics of the inputs and outputs during the period 2000-2017. The second step comments upon the calculation of PCU under C and NC technologies. Finally, the third step focuses on the PCU comparison between the pre-merger and post-merger observations.

4.1 Sample Data: Descriptive Statistics

We now illustrate the introduced PCU measures on an unbalanced sample of Swedish district courts used in Agrell, Mattsson, and Månsson (2020) and Mattsson and Tidanå (2019) [1]. From the initial data set containing yearly data related to 102 courts in the period 2000-2017,

[1] We are grateful to Pontus Mattsson for making these data available for our research contribution.

all available records are selected: this results in 1836 observations. There are 749 observations that have either missing data (because these were merged before), or are merged during this period: these are not considered. Thus, in this application the technology contains 1087 observations (= 1836 - 749).

Following Agrell, Mattsson, and Månsson (2020) and Mattsson and Tidanå (2019), the production of Swedish district courts is specified as generating three outputs ((i) criminal cases, (ii) civil cases, and (iii) petitionary matters) from four inputs ((i) judges, (ii) law clerks, (iii) other personnel, and (iv) court area). Agrell, Mattsson, and Månsson (2020, p. 662) discuss how these three output categories are generated using an aggregation procedure for self-reported time consumption starting with fourteen output categories. Bogetoft and Wittrup (2021) recently explore the whole issue of case weighting to assess the workload of the court system. For more details about these inputs and outputs, the reader can consult Chen, Kerstens, and Zhu (2021).

For the short-run input-oriented PCU measure, court area is regarded as a fixed capital input. All other three inputs are regarded as variable inputs. Descriptive statistics on inputs and outputs about all observations among all years, merging years and non-merging years are reported in Table 1.

Table 1: Descriptive Statistics Over All Years 2000-2017.

Sample	Statistics	Outputs			Inputs			
		Civil Cases	Matters	Criminal Cases	Judges	Law Clerks	Personnel	Court Area
All years (n=1084)	Average	995.13	460.39	1407.27	10.905	10.493	20.745	3519.74
	Stand.Dev	1297.83	619.95	1555.08	13.027	12.226	26.142	4172.70
	Min	0.710	0.440	0.000	0.040	0.000	0.000	0.000
	Max	9089.66	6495.18	11000.39	111.28	93.30	210.45	35360.00
Merging years (n=700)	Average	755.35	417.90	1051.43	8.76	7.82	17.23	3014.91
	Stand.Dev	1130.83	660.45	1299.74	12.56	9.71	25.96	4138.95
	Min	0.710	0.440	0.000	0.040	0.000	0.000	0.000
	Max	9089.66	6495.18	11000.39	111.28	93.30	210.45	35360.00
Non-merging years (n=384)	Average	1432.22	537.86	2055.94	14.82	15.37	27.15	4440.01
	Stand.Dev	1460.26	530.44	1761.16	12.96	14.62	25.26	4080.91
	Min	149.280	55.960	226.640	2.000	2.111	4.871	900.000
	Max	7329.56	3737.07	7269.05	70.30	78.64	133.95	25513.00

From the inputs and outputs data of Table 1, observe from the minimum values that there exist observations with zero inputs for the three variable input dimensions and a zero output for criminal cases. Since the Swedish district courts are in the process of merging, zero inputs and zero outputs are technically possible as long as the regularity conditions are respected. While a single zero variable input is not a problem in the presence of other non-zero variable inputs, likewise a zero output is not a problem in the presence of some other non-zero outputs. However, the presence of three zero variable inputs implies that the existence of a solution for the input-oriented efficiency measures is no longer guaranteed: three observations violate regularity condition (5) above and are therefore discarded for the input-oriented computations.[2] This yields 1084 observations over all years for the input-oriented computations and 1087 observations over all years for the output-oriented computations. In the merging years this leads to 700 and 703 observations for the input-oriented and output-oriented computations respectively.

Following Chen, Kerstens, and Zhu (2021), we adopt an intertemporal frontier whereby all observations over all time periods are assembled in a single production frontier. Indeed, Chen, Kerstens, and Zhu (2021) compute a Malmquist productivity index under constant returns to scale and convexity and these authors report with a simple t-test that this index does not differ significantly from unity (no technical change).

4.2 Plant Capacity under C and NC: Descriptive Analysis

At the sample level of Swedish district courts, we first illustrate the differences in the short-run input-oriented and output-oriented efficiency estimates, biased plant capacity utilisation results, and unbiased PCU results for C and NC technologies. The descriptive statistics for these concepts are shown in Table 2. The first line reports the number of efficient observations. Thereafter, we report the geometric average, the standard deviation, the minimum, and maximum values of the efficiency, biased plant capacity and unbiased plant capacity.

The final two lines report the results for the Li-test statistic. This Li-test statistic tests for the eventual significance of differences between two kernel-based estimates of density functions (see Li (1996)). The null hypothesis states that both density functions are almost equal. The alternative hypothesis maintains that the density functions are significantly different. This Li-test statistic has been refined most recently by Li, Maasoumi, and Racine (2009).[3]

Table 2 reports these descriptive statistics for both the input-oriented and the output-oriented efficiency and PCU measures in the columns 4 to 6 and the columns 7 to 9, respectively. The first horizontal part contains the sample level results that are our central focus. The second and third horizontal parts report results for the merging and the non-merging years.

[2]Including such observations in the sample leads to infeasibilities for these observations in the mathematical programming problems.

[3]We use the Matlab code developed by P.J. Kerstens based on Li, Maasoumi, and Racine (2009) and found at: `https://github.com/kepiej/DEAUtils`.

Table 2: Efficiency Results and Capacity Utilisation: Descriptive Statistics

Sample	Technology	Statistics	Input-oriented			Output-oriented		
			Efficiency	Biased PCU	Unbiased PCU	Efficiency	Biased PCU	Unbiased PCU
			$DF_i^{SR}(\boldsymbol{x}^v,\boldsymbol{x}^f,\boldsymbol{y})$	$DF_i^{SR}(\boldsymbol{x}^v,\boldsymbol{x}^f,0)$	$PCU_i^{SR}(\boldsymbol{x}^f,\boldsymbol{x},\boldsymbol{y})$	$DF_o(\boldsymbol{x},\boldsymbol{y})$	$DF_o^f(\boldsymbol{x}^f,\boldsymbol{y})$	$PCU_o^{SR}(\boldsymbol{x},\boldsymbol{x}^f,\boldsymbol{y})$
All years	Convex	# Eff.Obs.	57 (5.26%)	3 (0.28%)	4 (0.37%)	55 (5.06%)	19 (1.75%)	37 (3.40%)
		Average	0.7519	0.0133	194.22	1.3778	2.6527	0.6055
		Stand.Dev	0.1314	0.0580	259.69	0.5421	1.5329	0.1956
		Min	0.2563	0.0004	1.0000	1.0000	1.0000	0.1670
		Max	1.0000	1.0000	2394.29	16.0493	17.1796	1.0000
	Nonconvex	# Eff.Obs.	761 (70.20%)	3 (0.28%)	4 (0.37%)	749 (68.91%)	145 (13.34%)	171 (15.73%)
		Average	0.9728	0.0136	224.47	1.0563	1.8065	0.7020
		Stand.Dev	0.0592	0.0581	266.77	0.4556	1.0892	0.2427
		Min	0.3750	0.0004	1.0000	1.0000	1.0000	0.1564
		Max	1.0000	1.0000	2341.81	15.3939	15.515	1.0000
	Li-test† (Input vs. Output)		210.03***(C)	764.98***(C)	762.29***(C)	0.0641 (NC)	766.43***(NC)	763.64***(NC)
	Li-test† (C vs. NC)		441.54***	-1.0096	16.086***	432.77***	85.43***	56.40***
Merging years	Convex	# Eff.Obs.	34 (4.86%)	3 (0.43%)	4 (0.57%)	32 (4.55%)	12 (1.71%)	22 (3.13%)
		Average	0.7250	0.0177	147.68	1.4480	2.9997	0.5610
		Stand.Dev	0.1348	0.0718	236.45	0.6490	1.7005	0.1879
		Min	0.2563	0.0004	1.0000	1.0000	1.0000	0.1670
		Max	1.0000	1.0000	2394.29	16.0493	17.1796	1.0000
	Nonconvex	# Eff.Obs.	489 (69.86%)	3 (0.43%)	4 (0.57%)	477 (67.85%)	70 (9.96%)	79 (11.24%)
		Average	0.9693	0.0180	174.04	1.0741	2.0101	0.6532
		Stand.Dev	0.0659	0.0719	234.64	0.5642	1.2368	0.2474
		Min	0.3750	0.0004	1.0000	1.0000	1.0000	0.1564
		Max	1.0000	1.0000	2341.81	15.39	15.52	1.0000
	Li-test† (Input vs. Output)		155.45***(C)	181.28***(C)	489.62***(C)	0.0435 (NC)	489.28***(NC)	490.80***(NC)
	Li-test† (C vs. NC)		289.922***	-1.178	15.966***	281.32***	53.99***	23.01***
Non-merging years	Convex	# Eff.Obs.	23 (5.99%)	0 (0.00%)	0 (0.00%)	23 (5.99%)	7 (1.82%)	15 (3.91%)
		Average	0.8008	0.0055	279.06	1.2493	2.0175	0.6868
		Stand.Dev	0.1094	0.0042	278.37	0.1884	0.8587	0.1830
		Min	0.4806	0.0006	27.22	1.0000	1.0000	0.2384
		Max	1.0000	0.0199	1370.11	2.0504	6.3387	1.0000
	Nonconvex	# Eff.Obs.	272 (70.83%)	0 (0.00%)	0 (0.00%)	272 (70.83%)	75 (19.53%)	92 (23.96%)
		Average	0.9791	0.0056	316.40	1.0237	1.4337	0.7914
		Stand.Dev	0.0440	0.0042	296.03	0.0599	0.5873	0.2061
		Min	0.7242	0.0006	46.11	1.0000	1.0000	0.2784
		Max	1.0000	0.0199	1600.28	1.4579	4.3929	1.0000
	Li-test† (Input vs. Output)		71.11***(C)	73.20***(C)	274.73***(C)	-0.141 (NC)	284.48***(NC)	275.22***(NC)
	Li-test† (C vs. NC)		153.629***	-1.389	8.606***	152.05***	41.33***	35.39***

† Li-test: critical values at 1% level=2.33 (***); 5% level=1.64(**); 10% level=1.28(*).

The empirical analysis at the sample level yields the following conclusions. First, contrasting C and NC technologies, we find for the radial input-oriented efficiency measure 5.26% efficient observations under C and 70.20% under NC. For the input-oriented biased plant capacity, we obtain 0.28% efficient observations under C and exactly the same percentage under NC. Also, for the input-oriented unbiased PCU, one finds 0.37% efficient observations under C and also 0.37% under NC. Second, the geometric average reveals for the input-oriented radial efficiency measure an amount of about 75.19% under C and 97.28% under NC. For the input-oriented biased plant capacity, we find an efficiency magnitude of 1.33% under C and of 1.36% under NC. This means that the observations on average only need about 1% of their current variable input levels to start up production at a zero level. Thirdly, for the input-oriented unbiased PCU, we obtain a capacity utilisation percentage of 194.22% under C and of 224.47% under NC. The latter means that to generate current output levels on average the courts use about double and more than double the amounts of variable inputs compatible with zero outputs.

Turning to the short-run output-oriented measures under C and NC technologies, first for the output-oriented radial efficiency measure we observe 5.06% efficient observations under C and 68.91% under NC. For the output-oriented biased plant capacity, we find 1.75% efficient observations under C and 13.34% under NC. For the unbiased PCU, we obtain 3.40% efficient observations under C and 15.73% under NC. Second, the geometric average indicates for the output-oriented efficiency yields a magnitude of about 137.78% under C and 105.63% under NC. For the biased plant capacity, the output-oriented efficiency magnitude becomes 265.27% under C and 180.65% under NC. This implies that courts can more than double their outputs or increase these by about 80% when provided with unlimited additional variable inputs. Finally, for the output-oriented unbiased PCU, we observe a capacity utilisation of 60.55% under C and 70.20% under NC. The latter results imply that current output levels amount to on average about sixty to seventy percent of the maximum outputs that can be produced from unlimited variable inputs.

Moreover, the bottom two lines of each horizontal part report the results of the Li-test statistic. Starting with the first horizontal part focusing on all years, the results confirm that efficiency, the biased plant capacity, and the unbiased PCU under C between the input-orientation and output-orientation all differ significantly at the 1% significance level. In addition, the biased plant capacity and the unbiased PCU under NC between the input-orientation and the output-orientation all differ significantly at the 1% significance level, but the efficiency under NC between the input-orientation and the output-orientation has the same distribution. Furthermore, the bottom line reporting the results of the Li-test statistic confirm that efficiency and the unbiased PCU under both input-orientation and output-orientation between C and NC all differ significantly at the 1% significance level. However, the biased plant capacity under an input-orientation between C and NC has the same distribution, while that under an output-orientation between C and NC is significantly different at the 1% significance level.

Furthermore, the empirical analysis at the level of the merging years generates the following conclusions. First, for the radial input-oriented efficiency measure among the 700 merging year observations, there are 4.86% efficient observations under C and 69.89% under NC, there appear 0.43% efficient observations under both C and NC for the biased plant capacity, and there occur 0.57% efficient observations under both C and NC for the unbiased PCU. Second, the geometric average reveals an input-oriented radial efficiency measure of about 72.50% under C and 96.93% under NC. For the input-oriented biased plant capacity, we observe an efficiency magnitude of 1.77% under C and of 1.80% under NC. This implies that on average the courts only need about 2% of their current variable input levels to launch production at a zero level. For the input-oriented unbiased PCU, a capacity utilisation of 147.68% under C and of 174.04% under NC is observed. The latter numbers imply that to generate current output levels the average courts employ less than double the amounts of variable inputs that are compatible with zero output levels.

In a similar vein, among the 703 merging year observations for the radial output-oriented efficiency measure we observe 4.55% efficient observations under C and 67.85% under NC. For the biased plant capacity we find 1.71% efficient observations under C and 9.96% efficient observations under NC. For the output-oriented unbiased PCU measure we see that 3.13% observations under C and 11.24% observations under NC are efficient. Furthermore, the geometric average reveals for the radial output-oriented efficiency an amount of 144.8% under C and 107.41% under NC. For the biased plant capacity, the output-oriented efficiency reaches a magnitude of 299.97% under C and a 201.01% under NC. Thus, courts can almost triple or at least double their outputs under C and NC, respectively. Finally, for the unbiased output-oriented PCU, one notes a capacity utilisation of 56.10% under C and a 65.32% under NC. Thus, current outputs are on average about fifty-five to sixty-five percent of the maximum outputs producible with no limits on variable input availability. Hence, these merging year observations lead very much to the same conclusions as those at the sample level: under NC more observations are efficient, and the NC results are probably more credible and realistic.

Thereafter, the empirical analysis at the level of the non-merging years allows making the following conclusions. First, among the 384 non-merging year observations for the radial input-oriented efficiency measure there are 5.99% efficient observations under C and 70.83% under NC, and no efficient observations at all under C and NC for the input-oriented biased plant capacity, and the input-oriented unbiased PCU. Second, the geometric average reveals an input-oriented radial efficiency of about 80.08% under C and 97.91% under NC. For the input-oriented biased plant capacity, we measure an efficiency magnitude of a 0.55% under C and 0.56% under NC. On average, this means that the courts only need less than 1% of their current variable input levels to launch production at a zero level. For the input-oriented unbiased PCU, one observes a capacity utilisation of 279.06% under C and of 316.40% under NC. These figures imply that to generate current output levels the courts on average employ about triple the amounts of variable inputs that are compatible with zero output levels.

Next, among the 384 non-merging year observations, for the output-oriented radial efficiency measure there are 5.99% efficient observations under C and 70.83% under NC. For the biased plant capacity we count 1.82% efficient observations under C and 19.53% efficient observations under NC. For the output-oriented unbiased PCU measure we notice that

3.91% observations under C and 23.96% observations under NC are efficient. Furthermore, the geometric average shows for the radial output-oriented efficiency an amount of 124.93% under C and 102.37% under NC. For biased plant capacity, the output-oriented efficiency has a value of 201.75% under C and 143.47% under NC. Thus, courts can double or at least magnify their outputs about 50% under C and NC, respectively. Finally, for the unbiased output-oriented PCU, one notes a capacity utilisation of 68.68% under C and 79.14% under NC. Thus, current outputs are on average about seventy to eighty percent of the maximum outputs that can be produced without limits on variable inputs. Overall, these non-merging year observations imply about the same conclusions as those at the sample level and the merging year observations: under NC more observations are efficient, and the NC results tend to fit the data better.

Last but not least, the bottom lines of the second and third horizontal parts reporting the results of the Li-test statistic confirm that among the merging and non-merging year observations all input-oriented vs. output-oriented and all C vs. NC results differ significantly at the 1% significance level, except for the following two cases. First, the NC radial efficiency measure has an identical distribution under the input-orientation and the output-orientation. Second, the input-oriented biased plant capacity yields a similar distribution under C and NC.

4.3 Plant Capacity under C and NC: Comparing Pre-merger and Post-merger Observations

In addition to the empirical analysis at the sample level and at the level of merging years and non-merging years above, we now dig deeper in detail by focusing on the comparison between pre-merger and post-merger observations solely. In this subsection, we conduct a comparative analysis and statistical tests on the efficiency, biased plant capacity, and unbiased PCU results between pre-merger and post-merger observations.

Descriptive statistics are reported in Table 3: this table is structured very similarly to Table 2. This empirical analysis allows us to infer the following conclusions. First, for the input-oriented efficiency measure, the geometric average reveals an about 70.23% efficiency under C and a 97.43% efficiency under NC among the pre-merger observations, and an about 74.10% under C and 97.10% under NC among the post-merger observations. For the input-oriented biased plant capacity, one observes a 8.18% efficiency under C and a 8.27% efficiency under NC among the pre-merger observations, and a 0.54% efficiency under C and a 0.55%

Table 3: Efficiency and Capacity Utilisation: Comparing Pre- and Post-Merger Observations

Technology	Sample	Statistics	Input-oriented			Output-oriented		
			Efficiency	Biased PCU	Unbiased PCU	Efficiency	Biased PCU	Unbiased PCU
Convex	Pre-merger obs.	Average	0.7023	0.0818	76.61	1.7726	3.8421	0.5346
		Stand.Dev	0.1218	0.1546	83.21	1.1412	1.9377	0.1869
		Min	0.4512	0.0037	1.1317	1.0992	1.4939	0.2615
		Max	0.9329	0.6667	473.91	7.6770	8.8968	0.9448
	Post-merger obs.	Average	0.7410	0.0054	198.19	1.3410	2.2944	0.6260
		Stand.Dev	0.0953	0.0038	177.30	0.1628	0.6610	0.1592
		Min	0.5813	0.0009	39.97	1.0470	1.0825	0.3001
		Max	0.9658	0.0203	1070.90	1.6613	3.6457	0.9673
Nonconvex	Pre-merger obs.	Average	0.9743	0.0827	95.74	1.2789	2.2969	0.6447
		Stand.Dev	0.0443	0.1546	87.83	1.2861	1.4661	0.1716
		Min	0.8333	0.0038	1.3333	1.0000	1.0372	0.2849
		Max	1.0000	0.6667	499.38	8.1970	8.8856	0.9643
	Post-merger obs.	Average	0.9710	0.0055	247.61	1.0490	1.9260	0.6643
		Stand.Dev	0.0480	0.003	187.74	0.1380	1.1040	0.2469
		Min	0.8271	0.0009	48.75	1.0000	1.0000	0.1872
		Max	1.0000	0.0205	1134.00	1.7338	5.3426	1.0000

efficiency under NC among the post-merger observations. For the input-oriented unbiased PCU, one obtains a 76.61% capacity utilisation under C and a 95.74% capacity utilisation under NC among the pre-merger observations, and a 198.19% capacity utilisation under C and a 247.61% capacity utilisation under NC among the post-merger observations.

In a similar way, for the radial output-oriented efficiency measure, the geometric average reveals an efficiency level of 177.26% under C and 127.89% under NC among the pre-merger observations, and about 134.10% under C and 104.9% under NC among the post-merger observations. For the output-oriented biased plant capacity, we observe a 384.21% efficiency under C and a 229.69% efficiency under NC among the pre-merger observations, and a 229.44% efficiency under C and a 192.60% efficiency under NC among the post-merger observations. For the output-oriented unbiased PCU, one finds a 53.46% capacity utilisation under C and a 64.47% capacity utilisation under NC among the pre-merger observations, and a 62.60% capacity utilisation under C and a 66.43% under NC among the post-merger observations.

Focusing on the differences in results under input-oriented and output-oriented C and NC measures between the pre-merger and post-merger observations, several conclusions can be deduced. Focusing on the efficiency notion and the unbiased PCU solely, we observe for the input-orientation that under C the efficiency improves and unbiased PCU more than doubles, while under NC the efficiency remains constant and unbiased PCU again more than doubles. For the output-orientation under C the inefficiency decreases and thus efficiency improves and unbiased PCU increases, while under NC the efficiency improves as well and unbiased PCU increases slightly.

Furthermore, we analyse the Li-test statistics of results between C and NC for input-oriented and output-oriented measures among the pre-merger and post-merger observations, and between the pre-merger and post-merger observations for input-oriented and output-oriented measures under C and NC.

Table 4: Li-test Results on Decompositions Among the Pre- and Post-merger Observations

			Efficiency	Biased PCU	Unbiased PCU
Input-oriented	Convex	Pre- vs. Post-	-0.336	9.621***	9.051***
	Nonconvex	Pre- vs. Post-	-0.079	11.825***	9.081***
Output-oriented	Convex	Pre- vs. Post-	1.314***	2.855***	0.973
	Nonconvex	Pre- vs. Post-	0.343	0.520	0.649
Pre-merger Obs.	Input-oriented	C vs. NC	14.100***	-1.6856	-0.2874
	Output-oriented	C vs. NC	12.483***	2.429***	1.454***
Post-merger Obs.	Input-oriented	C vs. NC	15.036***	-1.517	0.721
	Output-oriented	C vs. NC	15.252***	5.338***	1.165

Li-test: critical values at 1% level=2.33 (***); 5% level=1.64(**); 10% level=1.28(*) .

The first and second horizontal parts of Table 4 respectively report the results of Li-test statistics between pre-merger and post-merger observations for a given orientation and convexity axiom. First, under the input-orientation, the Li-test results of the pre-merger and post-merger observations under the C and NC measures do not show differences in efficiency, but the Li-test results of the pre-merger and post-merger observations of biased and unbiased PCU both under the C and NC measures show significant differences at the 1% significance level. Second, under the output-orientation, only under C do the pre-merger and post-merger efficiency and biased PCU values reveal significant differences. But, the rest of the Li-test statistics indicate that the two compared densities follow the same distribution.

The third and fourth horizontal parts of Table 4 report the results of Li-test statistics between C and NC for the pre-merger and post-merger observations under input-oriented and output-oriented measures. For the input-orientation, for both pre- and post-merger observations only the efficiency measure has a different distribution under C and NC, while biased and unbiased PCU have a common distribution. For the output-orientation, for pre-merger observations all three concepts have a different distribution under C and NC, while for the post-merger observations only efficiency and biased PCU have a different distribution, but the unbiased PCU component has a common distribution.

Focusing again on the efficiency notion and the unbiased PCU solely, we observe for the input-orientation that under C and NC the efficiency improves insignificantly and unbiased

PCU doubles in a significant manner. Under the output-orientation the efficiency improves only significantly under C (not under NC) and unbiased PCU improves in an insignificant way.

These results are somewhat related to Chen, Kerstens, and Zhu (2021) who use only an input-orientation focusing on all input dimensions rather than a sub-vector. These authors find that technical efficiency improves only under C and scale efficiency only under NC. In the current setup, for the input-orientation under both C and NC the efficiency improves insignificantly while unbiased PCU doubles significantly. This improvement in PCU can be seen in parallel to the improvement in scale efficiency.

5 Conclusions

Inspired by other contributions utilizing the output-oriented and input-oriented PCU notions to assess capacity utilisation, we have applied the output-oriented and input-oriented PCU measures from a short-run perspective to a large unbalanced sample panel of Swedish district courts in the presence of horizontal mergers over the period 2000-2017. To the best of our knowledge, we are the first study to estimate the PCU of horizontal mergers under both C and NC nonparametric frontier specifications.

The empirical analysis at the all, merging and non-merging sample levels yields the following conclusions. First, contrasting C and NC technologies for the input-oriented measures, the percentage of efficient observations for radial efficiency is larger under NC, while the percentage of efficient observations for biased plant capacity and unbiased PCU is the same. Second, the average values of the radial efficiency and unbiased PCU under NC are significantly higher compared to C, while that of biased plant capacity under NC is slightly larger compared to C, suggesting that to generate current output levels, the courts on average utilize more amounts of variable inputs that are compatible with zero output levels under NC.

In a similar vein, contrasting C and NC technologies for the output-oriented measures, first, the percentage of efficient observations for radial efficiency, biased plant capacity and unbiased PCU is obviously higher under NC, that is, under NC more observations are efficient. Second, radial inefficiency is smaller and thus efficiency is larger under NC. The court can amplify its output by an even smaller amount under NC. Third, the current output represents on average a larger proportion of the maximum outputs that can be produced without restrictions on the availability of variable inputs under NC. Therefore, NC results are probably more credible and realistic.

Furthermore, the Li-test results confirm that among the all, merging and non-merging year observations all input-oriented vs. output-oriented and all C vs. NC results differ significantly at the 1% significance level, except for the following two cases. First, the NC radial efficiency measure has an identical distribution under the input-orientation and the output-orientation. Second, the input-oriented biased plant capacity yields a similar distribution under C and NC.

Focusing simply on the efficiency concept and the unbiased PCU of pre-merger and post-merger observations, we discover that for the input-orientation under C and NC efficiency increases insignificantly while the unbiased PCU doubles significantly. Under output-orientation, efficiency improves only greatly under C (but not under NC), while unbiased PCU improves just somewhat. In this regard, the input-oriented PCU concepts perform better than the output-oriented PCU.

Just to sketch one avenue for future work, the imperfect sample data make it not suitable for computing the long-run PCU concepts of Cesaroni, Kerstens, and Van de Woestyne (2019). Thus, one suggestion is to apply these long-run PCU notions on more suitable samples.

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Appendix A Nonparametric Frontier Estimates: Short-run Plant Capacity Utilisation

The nonparametric frontier specifications are adopted to measure the plant capacity notions. First, assume that there are K observations $((\boldsymbol{x}_K, \boldsymbol{y}_K), k = 1, 2, ..., K)$. If we impose strong disposability in inputs and outputs and variable returns to scale, then we can compute all plant capacity notions with respect to the technology (7).

For the sake of clarity, we utilize two mathematical programming models for computing the short-run output-oriented plant capacity utilisation. For an evaluated observation $(\boldsymbol{x}_p, \boldsymbol{y}_p)$, one can obtain the radial output efficiency measure $DF_o(\boldsymbol{x}_p, \boldsymbol{y}_p)$ as follows:

$$\begin{aligned} & DF_o(\boldsymbol{x}_p, \boldsymbol{y}_p) = \max_{\theta, \boldsymbol{z}} \theta \\ s.t. \ & \sum_{k=1}^{K} z_k \boldsymbol{y}_{kn} \geq \theta \boldsymbol{y}_p, n = 1, 2, ..., N \\ & \sum_{k=1}^{K} z_k \boldsymbol{x}_{km} \leq \boldsymbol{x}_p, m = 1, 2, ..., M \\ & z_k \in \Lambda. \end{aligned} \tag{A1}$$

In the similar vein, the efficiency measure $DF_o^f(\boldsymbol{x}_p^f, \boldsymbol{y}_p)$ can be calculated for observation $(\boldsymbol{x}_p, \boldsymbol{y}_p)$ as follows:

$$\begin{aligned} & DF_o^f(\boldsymbol{x}_p^f, \boldsymbol{y}_p) = \max_{\theta, \boldsymbol{z}} \theta \\ s.t. \ & \sum_{k=1}^{K} z_k \boldsymbol{y}_{kn} \geq \theta \boldsymbol{y}_p, n = 1, 2, ..., N \\ & \sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^f \leq \boldsymbol{x}_p^f, m = 1, 2, ..., M \\ & z_k \in \Lambda. \end{aligned} \tag{A2}$$

Notice that there are no input constraints on the variable inputs. If we want to add a scalar for each variable input, then we can regard each variable input as a decision variable and we can re-write the model (A2) as:

$$\begin{aligned}
&DF_o^f(\boldsymbol{x}_p^f, \boldsymbol{y}_p) = \max_{\theta, \boldsymbol{z}, \boldsymbol{x}^v} \theta \\
s.t. \ &\sum_{k=1}^{K} z_k \boldsymbol{y}_{kn} \geq \theta \boldsymbol{y}_p, n = 1, 2, ..., N \\
&\sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^f \leq \boldsymbol{x}_p^f, m = 1, 2, ..., M^f \\
&\sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^v \leq \boldsymbol{x}_p^v, m = 1, 2, ..., M^v, M^f + M^v = M \\
&z_k \in \Lambda.
\end{aligned} \tag{A3}$$

Turning to the short-run input-oriented plant capacity measure, one can compute the radial sub-vector input measure $DF_i^{SR}(\boldsymbol{x}_p^f, \boldsymbol{x}_p^v, \boldsymbol{y}_p)$ for an evaluated observation $(\boldsymbol{x}_p, \boldsymbol{y}_p)$ as follows:

$$\begin{aligned}
&DF_i^{SR}(\boldsymbol{x}_p^f, \boldsymbol{x}_p^v, \boldsymbol{y}_p) = \min_{\lambda, \boldsymbol{z}} \lambda \\
s.t. \ &\sum_{k=1}^{K} z_k \boldsymbol{y}_{kn} \geq \boldsymbol{y}_p, n = 1, 2, ..., N \\
&\sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^f \leq \boldsymbol{x}_p^f, m = 1, 2, ..., M^f \\
&\sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^v \leq \lambda \boldsymbol{x}_p^v, m = 1, 2, ..., M^v, M^f + M^v = M \\
&z_k \in \Lambda.
\end{aligned} \tag{A4}$$

The sub-vector efficiency measure $DF_i^{SR}(\boldsymbol{x}_p^f, \boldsymbol{x}_p^v, 0)$ can be obtained for observation $(\boldsymbol{x}_p, \boldsymbol{y}_p)$

by computing the following model:

$$
\begin{aligned}
& DF_i^{SR}(\boldsymbol{x}_p^f, \boldsymbol{x}_p^v, 0) = \min_{\lambda, \boldsymbol{z}} \lambda \\
s.t. \; & \sum_{k=1}^{K} z_k \boldsymbol{y}_{kn} \geq 0, n = 1, 2, ..., N \\
& \sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^f \leq \boldsymbol{x}_p^f, m = 1, 2, ..., M^f \\
& \sum_{k=1}^{K} z_k \boldsymbol{x}_{km}^v \leq \lambda \boldsymbol{x}_p^v, m = 1, 2, ..., M^v, M^f + M^v = M \\
& z_k \in \Lambda.
\end{aligned}
\tag{A5}
$$

Observe that the output levels to the right of the output constraints are all set to zero. The zero output levels are any output levels where production is initiated. If one assumes that output level at the minimum over all units, then the right-hand side would be identical for each observation and the same solution would result for the sub-vector input efficiency measure $DF_i^{SR}(\boldsymbol{x}_p^f, \boldsymbol{x}_p^v, 0)$. Therefore, the output constraints are redundant and can be deleted.