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Measuring CO2 emission reduction potential using a cost approach

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Highlights

- Propose a methodology for estimating environmental efficiency with endogenous material flow coefficients
- Apply proposed methodology to study pollution cost under two dimensions: energy use reduction and improvement in the energy mix, as revealed by the aggregate emission coefficient
- Illustrate the methodology for a balanced sample of 33 OECD and BRICS countries over the period 2001 to 2019
- Provide interesting insights on available options to decarbonize energy systems and/or to scale down the energy intensity of the economy of the selected 33 countries,
- Show that a total cost minimization problem is equivalent to an average cost minimization one
- Suggest a linearization solution for cost minimization models with two dimensions (input quantity and price)

Abstract

Departing from traditional approaches based on treating carbon dioxide (CO2) emissions as a bad output, thus relying on the weak disposability assumption, CO2 emissions are considered in this paper as a cost to minimize. We extend the Coelli et al. (2007) pollution cost approach preserving the materials balance condition by considering that peers are evaluated, besides their energy use, on their carbon intensity per total energy consumption. The proposed methodology is applied to estimate the extent to which a selection of 33 OECD and BRICS countries can reduce their CO2 emissions given their Gross Domestic Product and population over the period 2001-2019. Our results indicate that the period mean reduction potential for CO2 emissions of 53% (i.e., an efficiency level of 47%) can be decomposed into a 36% reduction in the energy intensity and a 27% decrease in the carbon intensity of energy (i.e., efficiency of 64% and respectively, 73%).

Keywords: Carbon dioxide emissions, Emission-generating technologies, Pollution cost, Energy use, Activity model, Data envelopment analysis (DEA).

JEL codes: Q52, Q40, D24, C61

1. Introduction

Global warming and climate change primarily caused by the direct release of carbon dioxide (CO2) and other greenhouse gases (GHG) into the atmosphere are one of the world's most crucial and pressing challenges. The need for urgent actions for a liveable world was recalled during the 27th Conference of the Parties of the United Nations Framework Convention on Climate Change (COP 27) held in Sharm el-Sheikh (Egypt, November 2022) by the United Nations Secretary-General António Guterres. He warned that our planet is on a "highway to climate hell" and that concrete and appropriate actions should be taken in this decade.

A crucial point for effective climate policies consists in identifying the underlying main drivers behind emissions. Indeed, they provide policymakers with information on available pathways for reducing emissions and mitigating the environmental impact of human activities. To this end, an interesting starting point is the Kaya (1989;1990) identity which expresses emissions (CO2) as a function of Population (*POP*), Gross Domestic Product (*GDP*), and Energy (*E*) as follows.

$$CO2 = \left(\frac{CO_2}{E}\right)\left(\frac{E}{GDP}\right)\left(\frac{GDP}{POP}\right)POP \Leftrightarrow CO2 = e\left(\frac{E}{GDP}\right)\left(\frac{GDP}{POP}\right)POP \qquad (1).$$

Emissions are therefore driven by four main factors: the carbon intensity of energy or the CO2 emission coefficient (CO2 */E* or *e*), the energy intensity of the economy (*E/GDP*), *GDP* per capita (*GDP/POP*), and population (*POP*). Given the level of population, one of the challenges faced by many developed countries is the following: how to scale down emissions at the maximum possible while preserving (or limiting the decline of) economic activity? Following Kaya, a possible solution already followed by many developed countries (at various speeds, however) consists of decarbonizing energy systems (i.e., to act on CO2*/E*). That is, they explore opportunities to increase the share of renewables and nuclear in the energy mix and reduce the share of fossil fuels such as coal, petroleoum and gas. Another path consists in reducing the energy intensity of the economy (*E/GDP*), promoting for example energy-efficient renovation of dwellings and energy-efficient machines.

In this paper, we explore the extent to which a selection of 33 OECD and BRICS countries could reduce their CO2 emissions given their level of *GDP* and population over the period 2001-2019. Considering Kaya's equation, this amounts to identifying the possible decreases in both *E* and *e*. We use a nonparametric production frontier framework, based on activity models to estimate such decreases. Indeed, when it comes to the estimation of possible reduction of resources without reducing production, nonparametric frameworks present the advantage of dispensing with the need to define a functional form for the production technology and its associated pollution cost frontier. Based on linear programming techniques to construct a surface that envelops observations, these methods derive an efficient cost frontier. The distance of a given observation to this frontier gives an estimation of feasible reductions. We observe that to estimate the level of feasible CO2 emissions reduction while considering the current *GDP* and/or Population, most studies in the nonparametric production frontier literature treat emissions either as undesirable outputs and use the framework of Färe et al. (1989), Färe and Grosskopf (2004) or as by-products using the framework of Murty et al. (2012), Murty and Russell (2018).

In the first approach, the production technology modelling assumes weak disposability of undesirable outputs and the null-jointness of both desirable and undesirable outputs. Intuitively this means that the undesirable output cannot be reduced without affecting the production of desirable outputs. Using this framework, Lin et al. (2013) measure environmental efficiency in 63 countries over the period 1981-2005 and test if the adoption of the Kyoto Protocol had a positive effect on environmental efficiency. Environmental efficiency is computed using a directional output distance function which allows to simultaneously expand *GDP* (desirable output) and contract CO2 (undesirable output). They find that high-income countries achieved the highest progress in their average environmental efficiency whereas lower-middle-income low-income countries recorded negative growth in their average environmental efficiency. In the same vein, Zhou et al. (2006) proposed slacks-based efficiency measures to study the CO2 emissions of thirty OECD countries from 1998 to 2002. Lozano and Gutierrez (2008), using a selection of developed countries from 1990 to 2004 estimate, among other things, the minimum level of GHG emissions compatible with given levels of population, *GDP*, etc. Also following this approach, Boussemart et al. (2017) estimated, for 119 countries on all continents, shadow carbon prices via a robust nonparametric framework. Their empirical results revealed that the global shadow carbon price increases by about 2.24% per year although there are significant regional disparities over the period analysed. They found a substantial sigma convergence process in carbon prices across countries over the period 1990-2007. However, following the global financial crisis, a divergence movement prevails. Finally, they analysed the relationship between shadow carbon prices and the implementation of the Kyoto Protocol.

In the second approach (i.e., the by-production approach), two sub-technologies are considered: one generating the desired outputs and the other generating the by-products. Operational and environmental efficiencies can be estimated with regard to the specific corresponding sub-technology frontier. This approach also adequately enables suitable trade-offs among outputs (desired and undesired) and inputs. Within this framework, Ray et al. (2018) use data on an unbalanced panel of 64 countries from 1986 to 2011 to compute for each country opportunity costs of a targeted reduction in CO2 emission (by-product) in terms of amounts of reduction in *GDP* (desired output). Extending the by-production approach, Boussemart et al. (2020) analyse the country's performance across the economic, environmental, and social pillars. By linking together three sub-technologies, they model a global technology that combines the objectives of these three pillars. The practicality of their approach was illustrated by the analysis of policy trade-offs between economic, environmental, and social objectives.

In our analysis, following Coelli et al. (2007), we approach the problem of pollution-generating technologies (i.e., the joint production of undesired and desired outputs) as a cost-minimization problem. Unlike the undesirable outputs framework (Färe et al., 1989), the environmental efficiency measure introduced by Coelli et al. (2007) ensures consistency with the materials balance principle (MBP). Specifically, the approach of Coelli et al. (2007) does not incorporate undesirables as inputs or outputs in the production technology modeling but utilizes material flow coefficients to identify the input mix that results in the minimal material inflow required to produce a certain bundle of desirable outputs. Their material inflow minimization problem mimics the well-known cost minimization problem in economics. Lauwers (2009) provides further arguments to justify the adaptation of traditional frontier-based models to comply with the MBP. Hoang and Coelli (2011) extend the initial efficiency-based frame to estimate the inter-temporal TFP performance of farms. The nutrient total factor productivity index (NTFP) also respects the material balance principle. More recently, Rødseth (2016) complemented this eco-efficiency estimation via the cost-polluting approach by considering that additional inputs could be used to control pollutant emissions. By adding the minimization of these pollution control inputs to the initial model of Coelli et al. (2007), they showed that the new efficiency measure allows for an improved ranking of the environmental performance of the evaluated DMUs.

Our contributions are threefold. First, we extend Coelli et al. (2007) and propose a model which endogenizes the material flow coefficients which in our application are the CO2 emission coefficients (e). This provides countries with two leverages to minimize emissions: energy consumption reduction and CO2 emission coefficient reduction. Note that Eder (2022) proposes another generalization of Coelli et al. (2007). Their framework allows the estimation of environmental efficiency scores when the material flow coefficients are heterogeneous across decision-making units and non-discretionary. Contrary to our approach, the model in Eder (2022), does not consider material flow coefficients as a lever to decrease pollution: they are non-discretionary (uncontrollable). There is no direct comparison (for potential decrease) of material flow coefficients across all DMUs and the heterogeneity of material flow coefficients is considered by restricting the set of possible reference units of a given DMU (weight restrictions). Moreover, our generalization leans on fewer sets of assumptions.

Second, we adapt the methodology proposed by Boussemart et al. (2022) in which DMUs minimize their costs while considering both input quantities and input prices to estimate our extended Coelli et al. (2007) model with endogenous emission coefficients.

Third, we use a balanced sample of 33 OECD and BRICS countries, observed from 2001 to 2019, to explore their energy efficiency and existing room for decarbonization (transition towards renewables and low-carbon energies). Our analysis, as it provides interesting insights on available options to decarbonize energy systems and/or to scale down the energy intensity of the economy of the selected 33 countries, is also particularly crucial in the current energy crisis derived from Russia's war on

Ukraine which highlighted the urgency for economies dependent on imported fossil fuels to improve their security by improving both their energy intensity and their energy mix.

The remainder of the paper unfolds as follows. In the next section, we describe our cost approach for modeling pollution with endogenous emission coefficients and present a methodology for its estimation. Section 3 presents the data used, the adaptation made to our general model to stick to the available data, and the obtained results. Section 4 makes some concluding remarks.

2. Methodology: a cost approach for modelling pollution

Our work aiming at enhancing the comprenhension of the different levers of action to reduce the CO2 emissions is based on the previous work of Coelli et al. (2007). This methodology relies on activity models such as data envelopement analysis (DEA) and it treats pollution as a cost that is to be minimized. In line with standard approaches, their model assumes that emission coefficients are given. The first sub-section deals with this general approach. The extension proposed consists in allowing for endogeneous emission coefficients. This hypothesis makes sense for specific models, such as the one studied here, where the energy is considered at an aggregate level, implying a possible different mix of energy sources amongst DMUs and thus, possibly different emission coefficients. Thus,we propose to evaluate, besides the efficiency of the energy use, the efficiency of the emission coefficient. The adaptation of the existing model to encompass this assumption is done in the second sub-section.

2.1 Traditional production technology with exogenous emission coefficients

Consider in the following the case of a decision-making unit (DMU) producing an output quantity vector $\mathbf{Y} \in \mathbb{R}_+^M$, with $M = \{1, \ldots, m, \ldots, M\}$ out of a vector of input quantities $\mathbf{X} \in \mathbb{R}_+^I$, with $I = \{1, \ldots, i, \ldots, I\}$. The corresponding technology can then be defined by:

$$T = \left\{ (\mathbf{X}; \mathbf{Y}) \in \mathbb{R}_+^I \times \mathbb{R}_+^M : \mathbf{X} \text{ can produce } \mathbf{Y} \right\} \qquad (2).$$

Assumptions regarding technology are standard and refer to no free lunch, boundedness, closure, free disposability, and convexity (see Banker et al., 1984). If we observe N DMUs with corresponding production plans $(X_n; Y_n), \forall n = 1, \ldots, N$, then the technology ($T$) could be estimated by Data Envelopment Analysis (DEA) under constant returns to scale (CRS) as follows:

$$T_{DEA} = \left\{ \begin{array}{l} (\mathbf{X};\mathbf{Y}): \\ \quad X_i \geq \sum_{n=1}^{N} \mu_n X_{i,n}, \forall i \in I, \\ \quad Y_m \leq \sum_{n=1}^{N} \mu_n Y_{m,n}, \forall m \in M, \\ \quad \mu_n \geq 0, \forall n \in N \end{array} \right\} \qquad (3).$$

In what follows, we associate to the input quantity vector ($\mathbf{X}$), a vector of emission coefficients $\mathbf{e} \in \mathbb{R}_+^I$ leading to a total level of pollution (or emissions) defined as $P = \mathbf{X}\mathbf{e}^T$.[1] Coelli et al. (2007), show that pollution is minimized when aggregate emission related to inputs use is minimized. Thus, minimizing the pollution level for a given DMU *a* mimics the classical cost minimization problem, i.e. $P = C(\mathbf{Y}, \mathbf{e})$, with C defined in equation (4) below. As shown by Coelli et al. (2007), the main advantage of this method, compared to the undesirable output approach (such as Färe and al. 1989 and Färe and Grosskopf, 2004 to cite only a few) is that in a cost minimization problem, the material balance principle is always respected. For the technology defined in equation (2) we define the emission (cost) function following Coelli et al. (2007):

$$C(\mathbf{Y}, \mathbf{e}) = \min_{\mathbf{X}} \left\{ \mathbf{X}\mathbf{e}^T : (\mathbf{X}, \mathbf{Y}) \in T_{DEA} \right\} \qquad (4).$$

The above emission minimization problem (equation 4) implicitly assumes that DMUs face identical emission coefficients which reduces the problem to an input quantity allocation. Indeed, under this approach, DMUs seek only the optimal input quantities given identical emission coefficients. Thus, DMUs' strategy to reduce emissions is based uniquely on an “input quantity effect”, while completely ignoring the possibility to act on their emission coefficients, and in fine, on their energy mixes. Related to traditional cost minimization and allocative efficiency models, the limitation of analyses based on exogenous input prices has been emphasized several times in the literature (Camanho and Dyson(2008), Portela and Thanalousis (2014), Ayouba et al. (2019), Boussemart et al. (2022)).

We propose to extend existing pollution cost models, i.e. Coelli et al. (2007) based on exogenous emission coefficients by considering that DMUs when choosing different input quantity mixes end up having different emission coefficients for those mixes. In other words, the emission coefficients will be endogenized in our approach.

[1] In the same way, we can assume $\mathbf{e}_y \in \mathbb{R}_+^R$ a vector of technical coefficients associated with the output. From this perspective, the methodology developed here, based on the cost minimization problem, can be extended to include both input and output emission coefficients in a profit maximization approach.

2.2 Proposition of a pollution cost approach with endogenous emission coefficients

2.2.1 General setting

The standard frame of analysis does not consider the differences in the emission coefficients and therefore cannot estimate DMUs efficiency in this dimension: emission coefficients are considered as given for each DMU and are not compared among DMUs. Indeed, for a very detailed nomenclature grid of a polluting input such as energy comprising different types of fossil fuels (coal, petroleum, natural gas, oil shales, bitumens, tar sands, heavy oils, etc.), one can consider that the emission factor is identical for each type of fuel according to physical-chemical laws. Therefore, it would not be relevant to compare DMUs along this dimension.

However, at a more aggregate level, when confronted with an aggregate mix of polluting inputs (e.g. energy = fossils + nuclear + renewables), the resulting weighted average emission coefficient can be different among DMUs. This leads to a form of heterogeneity among the DMUs both from the perspective of the input quantities used, but also from the perspective of their respective mixes as revealed by the potentially different weighted emissions factors.

Consequently, DMUs dispose of two leverages to reduce their pollution cost: change the quantity of polluting inputs and/or, choose a different mix of polluting inputs resulting in different emissions coefficients. From a practical point of view, this approach has the advantage to give DMUs more room for action compared to the traditional approach.

In the recent literature, Boussemart et al. (2022) have proposed a methodology for standard cost minimization in which DMUs minimize their costs while considering both input quantities and input prices as optimization levers.[2] This methodology can easily be adapted to study pollution emissions as:

$$C\left(\mathbf{Y}\right)=\min_{\mathbf{X},\mathbf{e}\geq 0}\left\{\mathbf{X}\mathbf{e}^{T}:\left(\mathbf{X},\mathbf{Y}\right)\in T_{DEA}\right\},\forall\mathbf{Y}\in\mathbb{R}_{+}^{M} \qquad (5).$$

Coelli et al. (2017) assume that the cost minimization problem is realized under constant returns to scale (CRS), allowing for a rescale of input and output quantities for a given set of emission coefficients. Moreover, the model necessary to assess the countries' performance in CO2 emissions in line with the Kaya equation (1) also implicitly assumes constant returns to scale as the different elements composing it are based on ratios, e.g. *E/GDP*. However, in Boussemart et al. (2022), their model was developed

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[2] In their approach and contrary to the standard approach, DMUs are evaluated not only with regard to their input quantities, while considering prices as given (which can be an acceptable hypothesis for perfectly competitive input markest) but also with regard to their observed prices (which can be related to the assumption that input markets are not perfectly competitive).

under variable returns to scale (VRS). Indeed, this more general assumption prevents the rescale of all corresponding variables, including prices, which would have been irrelevant in this specific dimension (prices). In our current approach, a more accurate framework should allow for a rescale in the quantity dimensions (input and output), but not in the emission coefficient one. This issue is solved by showing that a total cost minimization problem in CRS is equivalent to an average cost minimization one in VRS. This is done in the following subsection.

2.2.2 Dealing with returns to scale when emissions coefficients are directly considered

We show that the standard cost minimization problem under CRS assumption (equation 4), i.e. as in Coelli et al. (2007), is equivalent to the cost minimization problem under VRS assumption when all output and input components of the technology are rescaled with regard to one fixed component. In what follows, we will assume that the output $Y_{m'}$ is fixed, with $m' \in M', M' \subset M$ and $|M'| = 1$. In terms of notations, we introduce the rescaled input and output quantity vectors as:

$$\mathbf{x} = \frac{1}{Y_{m'}}\mathbf{X}, \text{and } \mathbf{y} = \frac{1}{Y_{m'}}\mathbf{Y} \qquad (6).$$

These rescaled vectors are generally referred to as the input and output intensities relative to the chosen fixed output. With these notations, the associated VRS cost function is defined as:

$$C^{VRS}(\mathbf{y},\mathbf{e}) = \min_{\mathbf{x}}\left\{\mathbf{xe}^T : (\mathbf{x},\mathbf{y}) \in T_{DEA}\right\}, \forall \mathbf{y} \in \mathbb{R}_+^M \qquad (7).$$

The above equation (7) corresponds to the minimization of the average cost for producing one unit of the fixed output.

Proposition 1.

The total pollution cost minimization problem in equation (4) under constant returns to scale is equivalent to an average cost minimization problem under variable returns to scale if all inputs and vectors are rescaled with regard to a fixed component as defined in equation (7).

$$C^{CRS}(\mathbf{Y},\mathbf{e}) \equiv C^{VRS}(\mathbf{y},\mathbf{e}) \qquad (8).$$

The proof for the above Proposition is available in the Appendix.

Thanks to these clarifications and returning to equation (5), we are now able to give the VRS version of the average cost minimization problem dealing with endogenous emission coefficients. This consists in optimizing the cost value with respect to both input intensity ($\mathbf{x}$) and emission coefficient ($\mathbf{e}$) vectors:

$$C^{VRS}(\mathbf{y}) = \min_{\mathbf{x},\mathbf{e}\geq 0}\left\{\mathbf{xe}^T : (\mathbf{x},\mathbf{y}) \in T_{DEA}\right\}, \forall \mathbf{y} \in \mathbb{R}_+^M \quad (9).$$

Equation (9) naturally leads to solving a non-linear problem NLP1, because the cost minimization solution results from the product between the optimal input quantity and the optimal emission coefficient.

$$\begin{aligned}
&\underset{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\lambda}}{Min}\, C^{VRS}(y_a) = \sum_i \left(\alpha_i x_{i,a}\right)\left(\beta_i e_{i,a}\right) \\
&\sum_n \left(\lambda_n x_{i,n}\right) \leq \alpha_i x_{i,a}, \forall i \in I \\
&\sum_n \left(\lambda_n e_{i,n}\right) \leq \beta_i e_{i,a}, \forall i \in I \\
&\sum_n \left(\lambda_n y_{m,n}\right) \geq y_{m,a}, \forall m \in M, m \neq m' \\
&\sum_n \lambda_n = 1 \\
&\lambda_n \geq 0,\ \forall n \in N
\end{aligned} \qquad \textbf{NLP1}$$

The problem faced by such non-linear problems is that the solution found is only a local one and, in most cases, proving that it is also a global one might be a complex task. In the following empirical section, we adapt the linearization method introduced by Banker and Maindiratta (1986) which is particularly well suited to our analysis of cost minimization expressed in units of CO2 emissions.

3. Data, estimation strategy, and results

3.1 Data and variables

For our analysis of CO2 emissions performance, we have selected 33 countries[3] (27 OECD countries + 6 BRICS countries) over the period 2001-2019. In line with the Kaya (1990) identity introduced in equation (1), we have retrieved the corresponding variables: *GDP*, total population (*POP*), total energy consumption (*E*), and total CO2 emissions (CO2). Based on these variables, we deduce the energy intensity (*E/GDP*) and the emission coefficient (*e*=CO2/*E*). The data used come from the World Bank and the US Energy Information Administration (see Table 1). *GDP* is measured at constant 2011 prices expressed in international US $ respecting the purchasing power parity (PPP). CO2 emissions are

[3] Australia, Austria, Belgium, Brazil, Canada, China, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, India, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Republic of Korea, Russian Federation, Slovakia, South Africa, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

expressed in millions of tons. Total energy consumption, expressed in quadrillion Btu, aggregates fossil (coal, gas, and oil), nuclear and renewable energies.

Table 1: Definition, measure unit, and source of the variables

Variables	Définition	Unit	Source
GDP	*GDP*, PPP constant 2011 international $	106 US $ 2011	https://data.worldbank.org/indicator
POP	Total population	10^6	https://data.worldbank.org/indicator
E	Energy consumption	10^{15} Btu	https://www.eia.gov/international/data/worl
CO2	CO2 emissions	10^6 Tons	https://www.eia.gov/international/data/world
E/GDP	Energy intensity	10^3 Btu/$	
e	Emission coefficient	mg/Btu	

Over the last two decades and relative to the world total, these 33 countries respectively account for 76% of *GDP*, 60% of the population, 79.5% of energy consumption, and more than 80% of CO2 emissions. Analysis of the trends in the selected variables indicates that *GDP* per capita grew at an annual rate of 2.40% due to a faster increase in *GDP* than in population. At the same time, energy intensity (*E/GDP*) decreased at a rate of -1.17% per year due to a differential in growth in favor of *GDP* compared to overall energy consumption. Finally, the CO2 emission coefficient ($e = CO2/E$) improved slightly over the period, with a trend decrease of -0.10%.

Table 2. Descriptive statistics of the variables

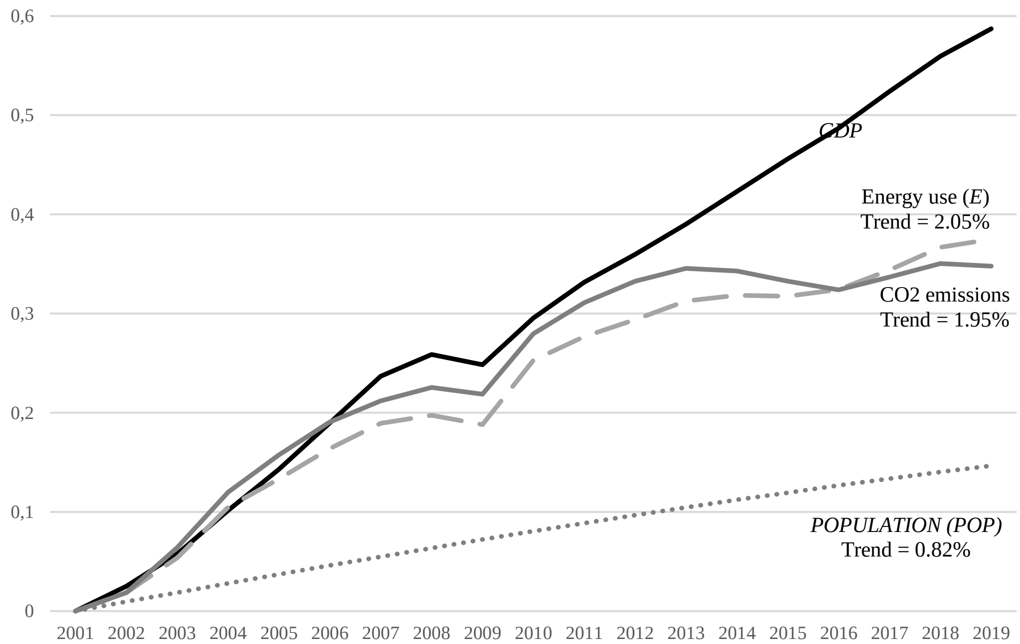
	Average 2001-2019	**Total 33 countries**
GDP	Level	74 593 497
	% of Total world	75.99%
	Trend 2001-2019 (%)	3.22%
POP	Level	4 191
	% of Total world	60.19%
	Trend 2001-2019 (%)	0.82%
E	Level	410
	% of Total world	79.51%
	Trend 2001-2019 %	2.05%
CO2	Level	25 245
	% of Total world	80.18%
	Trend 2001-2019 (%)	1.95%
GDP/POP	Level	17 799
	Trend 2001-2019 (%)	2.40%
E/*GDP*	Level	5.50

	Trend 2001-2019 (%)	-1.17%
e	Level	61.55
	Trend 2001-2019 (%)	-0.10%

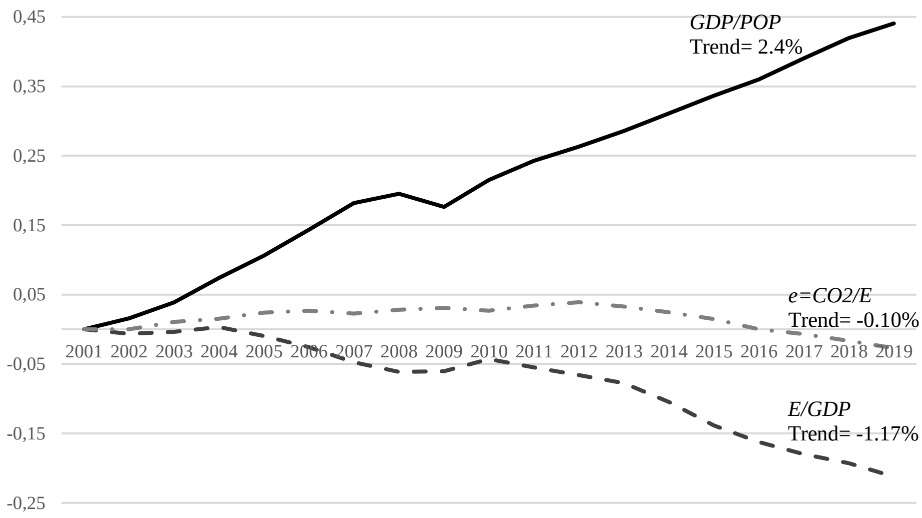
Figure 1.a illustrates the indices (and growth rates) of the main variables expressed in level (first four lines in Table 2). Figure 1.b complements this analysis by the evolutions of the same variables expressed this time with regard to the population, *GDP*, and energy consumption respectively (the latter 3 rows in Table 2). Note that the Kaya identity in equation (1) is obtained based on these three ratios. The analysis of these two figures allows us to conclude that the strong increase in *GDP* per capita accompanied by a more moderate increase in the world population are the main causes of CO2 emissions. The respective decreases in energy intensity and emission coefficients (by changing the energy mix in favor of non-carbon resources) are not sufficient to reduce CO2 pollution.

Figure 1.Illustrations of the evolutions of the indices (and their trends) for the main variables in Kaya identity

a. Main variables in level (1= 2001, semi-logarithmic scale)



b. Evolution (and growth rates) for ratios used in the Kaya identity (1= 2001, semi-logarithmic scale)



Algebraically, the relationships between the related growth rates of the variables in Kaya's identity (trend 2001-2019 in %) can be expressed as follows:

$$CO2 = e\left(\frac{E}{GDP}\right)\left(\frac{GDP}{POP}\right)POP$$

$$\Rightarrow$$

$$\frac{dCO2}{CO2} = \frac{de}{e} + \frac{d\left(\frac{E}{GDP}\right)}{\left(\frac{E}{GDP}\right)} + \frac{d\left(\frac{GDP}{POP}\right)}{\left(\frac{GDP}{POP}\right)} + \frac{dPOP}{POP}$$

$$1.95\% = -0.10\% - 1.17\% + 2.40\% + 0.82\%$$

3.2 Estimation strategy

According to the availability of data presented above and starting from Kaya's (1990) identity expressed in equation (1), we adapt the technology set accordingly. The total cost to minimize is expressed in terms of CO2 emissions as:

$$CO2 = e\left(\frac{E}{GDP}\right)\left(\frac{GDP}{POP}\right)POP$$

Rearranging this equation leads to the following equivalent formulation:

$$\frac{CO2}{GDP}=e\left(\frac{E}{GDP}\right)\left(\frac{POP}{POP}\right)=e\left(\frac{E}{GDP}\right) \qquad (10).$$

Consequently, the average cost of pollution $\frac{CO2}{GDP}$ is expressed in terms of CO2 emissions per unit of output quantity given by the *GDP* produced over the year: $\mathbf{Y}=(GDP)$. The polluting input quantity is given by the total quantity of energy denoted *E* needed to produce the *GDP*. Consequently, the input vector can be reduced to the scalar $\mathbf{X}=(E)$. Similarly, the emission coefficient vector is denoted by the scalar $\mathbf{e}=(e)=\left(\frac{CO2}{E}\right)$. Rescaled output and input variables with regard to the fixed output (*GDP*) are then equal to: $y=\left(\frac{GDP}{GDP}\right)=(1)$ and $x=\left(\frac{E}{GDP}\right)$. The rescaled output vector corresponds naturally to the unit whereas, in the rescaled input vector (*x*) we have the quantity of energy used to produce one unit of *GDP* called energy intensity.

In this specific case of a single aggregate input (such as the sum of energies all expressed in the same unit), the average cost of CO2 per unit of *GDP* is simply expressed by the multiplication of two scalars $\frac{CO2}{GDP}=e\left(\frac{E}{GDP}\right)$. Consequently, the minimization of CO2 emissions per unit of *GDP* can be solved through the following nonlinear program NLP2 below. Note that this program is the adaptation of the previous NLP1 where, in the vein of Banker and Maindiratta (1986), we deal with geometric convexity instead of linear convexity. Moreover, this solution also has the advantage to address more general production situations which involve non-concavity in some regions and a non-convex production possibility set.

$$\begin{aligned}
&\underset{\boldsymbol{\sigma},\boldsymbol{\omega},\boldsymbol{\tau}}{Min}\left(\frac{CO2_a}{GDP_a}\right)=\sigma\left(\frac{E_a}{GDP_a}\right)\omega e_a\\
&\prod_n\left(\frac{E_n}{GDP_n}\right)^{\tau_n}\leq\sigma\left(\frac{E_a}{GDP_a}\right)\\
&\prod_n(e_n)^{\tau_n}\leq\omega e_a\\
&\sum_n\tau_n=1\\
&\tau_n\geq 0,\forall n\in N
\end{aligned} \qquad \textbf{NLP2.}$$

The linearization of the objective function in NLP2 is possible by a simple logarithmic transformation: $\min_{\sigma,\omega,\tau}\left[\ln\sigma+\ln\left(\frac{E_a}{GDP_a}\right)+\ln\omega+\ln e_a\right]\Leftrightarrow\min_{\sigma,\omega,\tau}\left[\ln\sigma+\ln\omega\right]$.

Finally, the logarithmic transformation of the first two constraints leads to the final log-linear program:

$$\begin{cases}\min_{\sigma,\omega,\tau}\left[\ln\sigma+\ln\omega\right]\\ \sum_n \tau_n \ln\left(\frac{E_n}{GDP_n}\right)\le \ln\sigma+\ln\left(\frac{E}{GDP}\right)_a\\ \sum_n \tau_n \ln e_n \le \ln\omega+\ln e_a\\ \sum_n \tau_n \ln\left(\frac{GDP_n}{GDP_n}\right)\ge \ln\left(\frac{GDP_a}{GDP_a}\right)\Leftrightarrow 0\ge 0\\ \sum_n \tau_n =1,\ \forall n\in N\\ \tau_n\ge 0\end{cases} \qquad \textbf{LP3.}$$

In the above LP3, we retrieve, for the energy intensity and for the emission coefficient a specific score, measured by σ and respectively ω which give the potential changes in the input quantity and emission coefficient to reach the optimal average cost level.

A country is deemed efficient whenever the product of the two scores is equal to the unit, $\ln\sigma+\ln\omega=0\Leftrightarrow\sigma\omega=1$.

Note that in this program the goal is cost minimization instead of minimizing each specific score. Consequently, the two coefficients are not restricted meaning that a minimisation of the cost can lead to several situations:

- A country can decrease both its energy intensity and its emission coefficient to become efficient.
- A country can decrease its energy intensity and, at the same time, increase its emission coefficient.
- A country can increase its input energy intensity while reducing its emission coefficient.

According to Kaya's initial equation, the change in the amount of CO2 emissions can be achieved not only through simultaneous variations in the emission coefficient and energy intensity but also over a change in the standard of living expressed by the *GDP* per capita for a given level of population. In this analysis, we will consider that environmental policies of countries focus first on the impacts in terms of pollution caused by quantities and mixes of energy. While economic activity and demography are more likely to be a matter of economic and social policies. In this context, we consider that *GDP* and population levels are exogenous variables not monitored by energy choices. In more precise terms, this means that the minimum emission cost per unit of *GDP* is obtained with a given level of *GDP* per capita. As a result, the observed $\frac{GDP}{POP}$ is treated as a control variable and a specific constraint must be added to LP3. Thus, the final linear program that will be estimated corresponds to:

$$\begin{cases} \min\limits_{\sigma,\omega,\tau} \left[\ln\sigma + \ln\omega\right] \\ \sum\limits_n \tau_n \ln\left(\dfrac{E_n}{GDP_n}\right) \le \ln\sigma + \ln\left(\dfrac{E_a}{GDP_a}\right) \\ \sum\limits_n \tau_n \ln e_n \le \ln\omega + \ln e_a \\ \sum\limits_n \tau_n \ln\left(\dfrac{GDP_n}{POP_n}\right) = \ln\left(\dfrac{GDP_a}{POP_a}\right) \\ \sum\limits_n \tau_n = 1,\ \forall n \in N \\ \tau_n \ge 0 \end{cases} \qquad \textbf{LP4.}$$

3.2 Results and discussion

The efficiency score analysis shows that the CO2 emission reduction potential would be significant if the different countries could adopt the characteristics of the best practices in terms of energy. According to Figure 2, the average CO2 score for the 33 countries oscillates between 44% and 50% depending on the year, i.e., a reduction potential varying between 56% and 50%. The efficiency in terms of pollution cost is first explained by the energy intensity (*E/GDP*), for which the efficiency score varies between 62% and 66%, while the efficiency score for the emission coefficient resulting from the energy mix is higher (between 70% and 76%).

Figure 2. Efficiency scores in % (sample geometric mean)

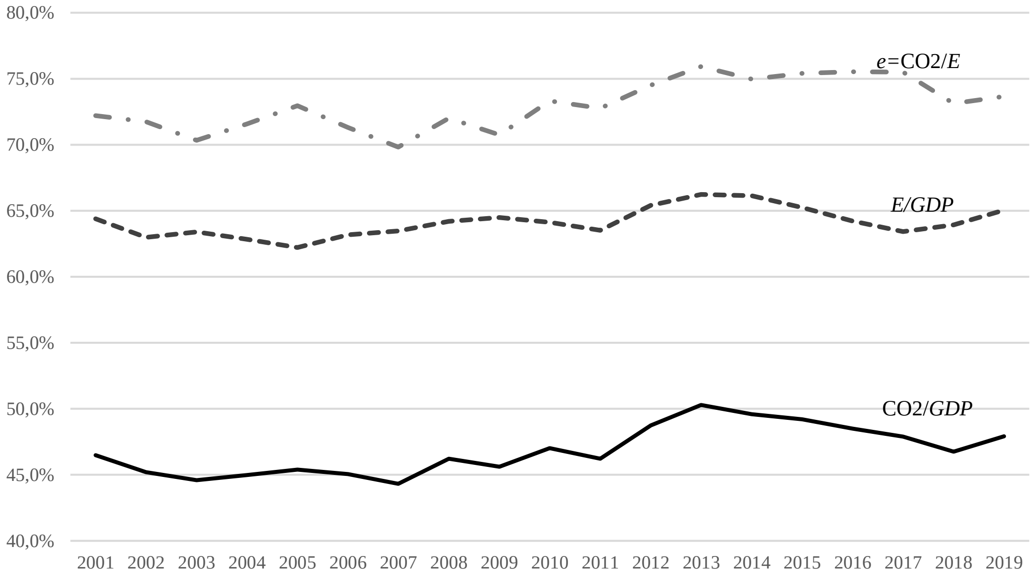
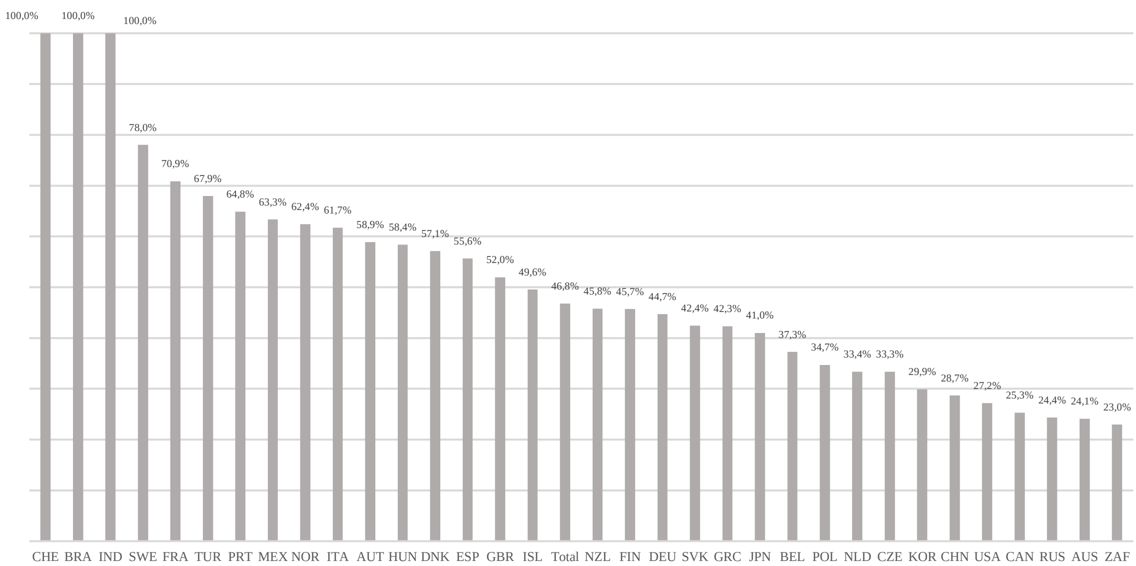


Figure 3 shows that in terms of the efficiency for the CO2 emissions per unit of *GDP*, Switzerland, Brazil, and India are the most efficient countries, followed by Sweden, France, Turkey, Portugal, Mexico, Norway, and Italy. At the other end of the scale, the least efficient countries are South Africa, Australia, Russia, Canada, the USA, China, and the Republic of Korea.

Figure 3. Ranking of countries according to their CO2-to-GDP emissions scores (period geometric mean)

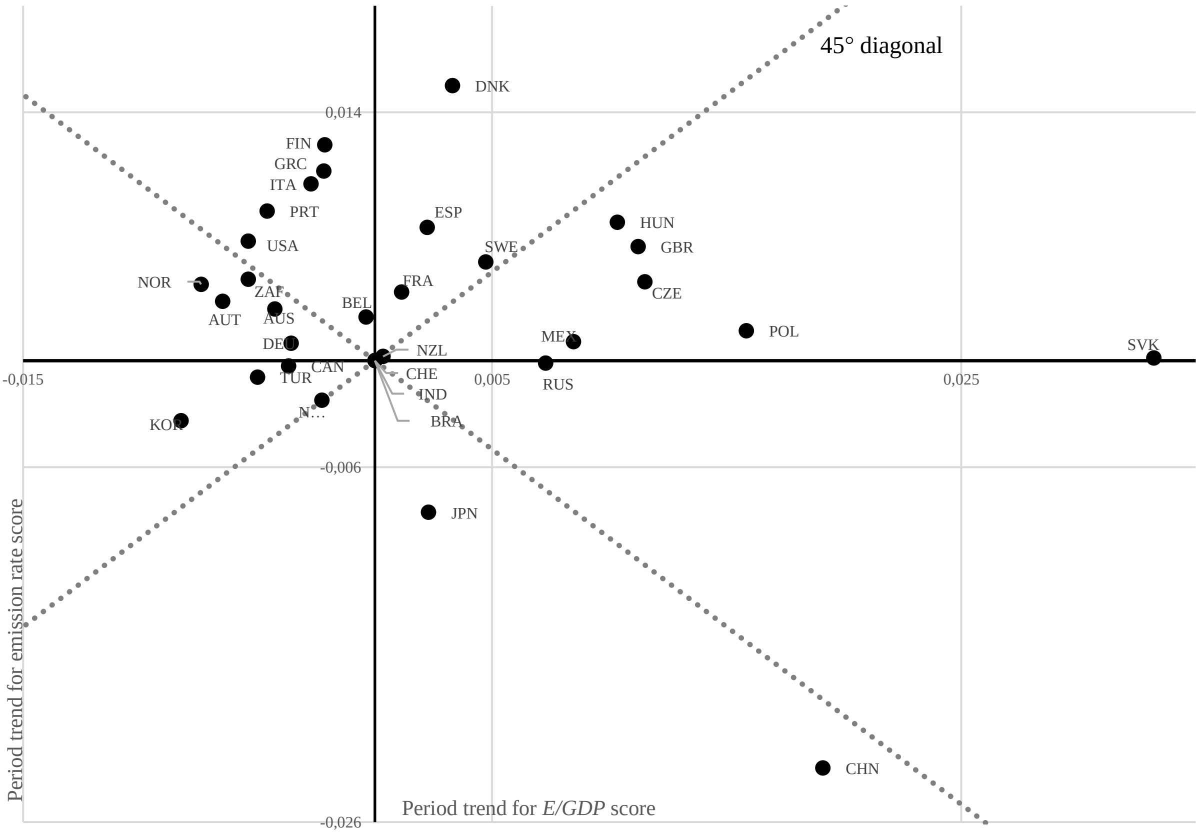


Beyond their overall CO2 efficiency scores, the different countries can be categorized according to a typology that crosses the two efficiency scores respectively energy intensity and emission coefficient related to the mix of different types of energies. Figure 4 plots the period trends for each score thus enabling a comparison of the dynamics between these two dimensions. Countries can therefore be classified along the four frames to which they belong. We notice that efficient countries in both dimensions are placed at the origin of this plot (Switzerland, Brazil, and India). Countries that have managed to increase their efficiency in both dimensions are plotted in the upper right-hand frame. With regard to the 45° diagonal (the dashed lines), for some countries situated beneath this diagonal, the increase in the efficiency of the energy-to-*GDP* ratio was stronger than the improvement observed by the emissions rate efficiency (Czechia, Great Britain, Hungary, Mexico, New Zealand, Poland and, Slovakia). The reversed situation was observed for countries in the same frame but above the 45° diagonal (Denmark, France, Spain, and Sweden).

In the left-hand, upper frame, one finds most of the countries in the sample for which the improvement in the emissions rate efficiency was associated with a decrease in the efficiency of the efficiency in the energy-to-*GDP* ratio. For countries above the 45° diagonal (Belgium, Finland, Greece, Italy, Portugal, and the United States) the increase in the efficiency of the mix was stronger than the deterioration of the efficiency of energy consumption for one unit of *GDP*. The reverse was observed for countries in this frame situated below the diagonal (Australia, Austria, Germany, Iceland, Norway, and South Africa).

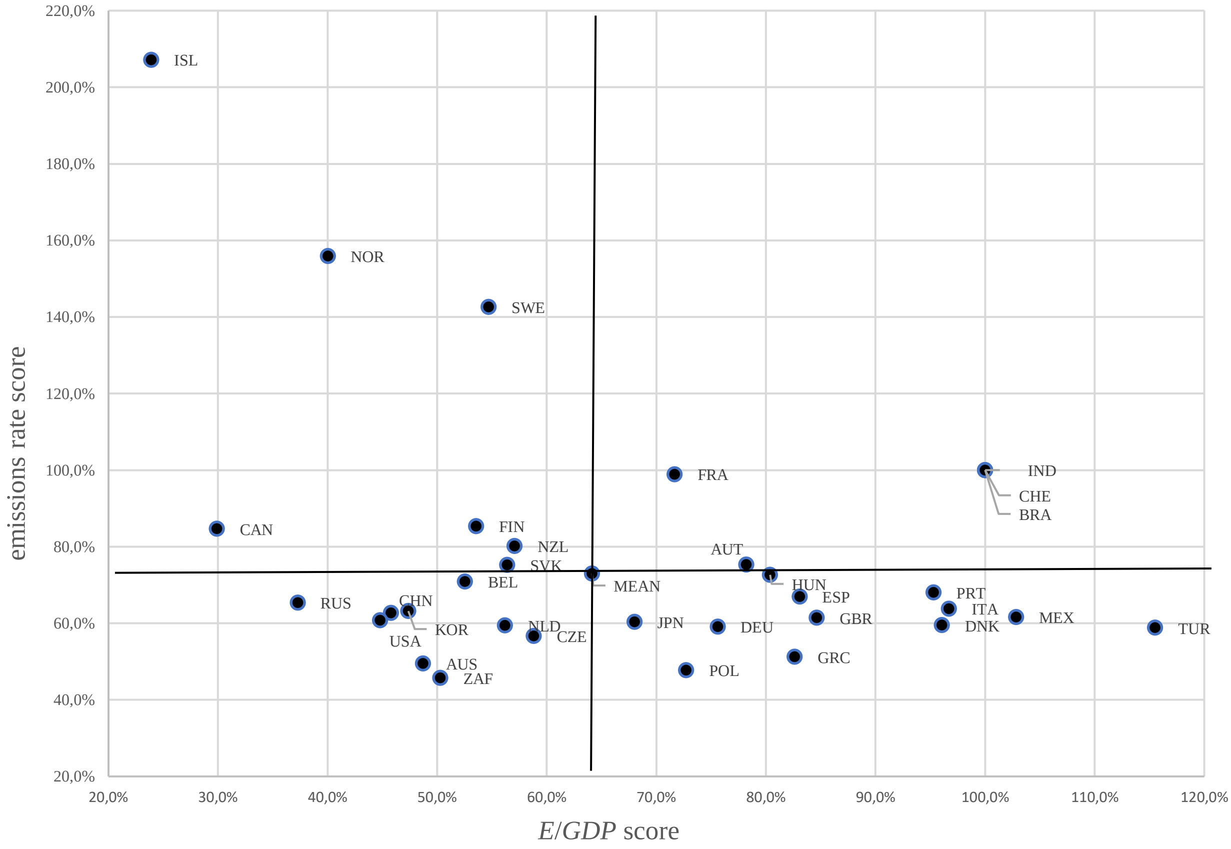
In the left-hand, bottom frame, one finds the countries for which both efficiencies observed a negative trend throughout the study period (Canada, South Korea, the Netherlands, and Turkey). For all of them, the decline in the energy mix efficiency was stronger than the decrease in the efficiency of energy use for one unit of *GDP*. Finally, in the last frame (right hand, bottom), there are three countries for which the efforts in improving their energy to *GDP* efficiency were related to a deterioration of the energy mix efficiency (China, Japan, Russia). For the first two countries, the increase in the efficiency of the first score was stronger than the decrease in the latter score. However, for Russia, the reverse was observed.

Figure 4. Typology of the countries with respect to the period trend for each efficiency score



We pursue the analysis by presenting the period geometric mean for each score. In Figure 5 these aggregate scores are illustrated in reference to the whole sample (geometric) mean. We first notice that Switzerland, India, Brazil, and France show above-average performance on both dimensions. For Switzerland, France, and Brazil, this efficient positioning is explained by their good profiles both in terms of productivity and choice of energy mixes in favor of renewable and nuclear (particularly for France). For India, although its emission coefficient is among the worst, its overall efficiency is explained by a relatively good position in terms of energy productivity but also by the specificity of having a very low level of *GDP* per capita. Three Northern European countries (Iceland, Sweden, and Norway) appear super-efficient in terms of emission rates, while their energy intensity scores are below average (especially for Iceland). On the other hand, Turkey, Mexico, Portugal, Italy, and Denmark derive their good performance from high scores on the energy dimension. As such, Turkey and Mexico are super-efficient in this dimension. Among the least efficient countries in terms of CO2 emissions, some accumulate bad performances both in their energy use and their emission rate: Russia, the USA, China, the Republic of Korea, Australia, and South Africa.

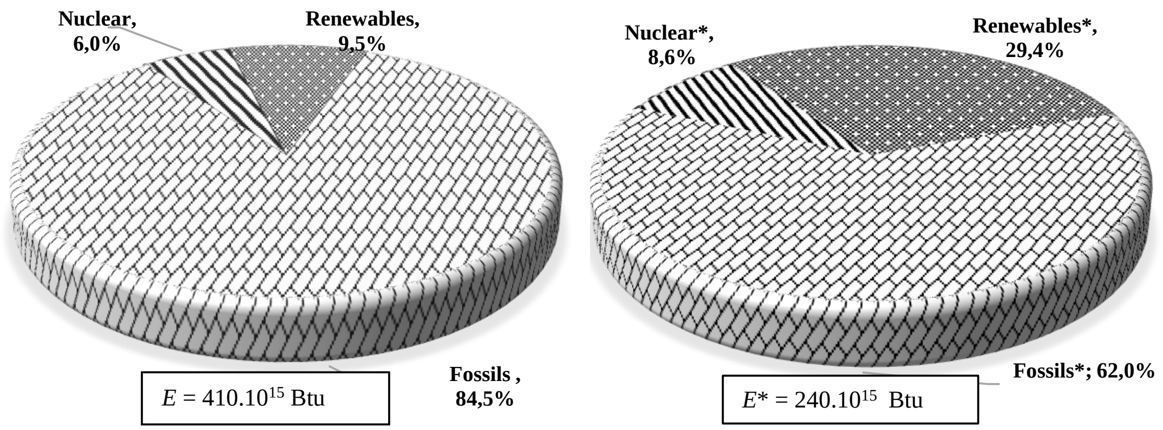
Figure 5.Typology of countries by their efficiency profile (period geometric mean)



At the sample mean, all these results lead to the conclusion that a 53% reduction target of CO2 emissions[4] would require countries to not only decrease their overall energy consumption but also change their energy mix. This is illustrated by the pie chart in Figure 6. Not surprisingly, fossil energies would have to decrease their share from 84.5% to 62% while renewable energies would have to increase from 9.5% to 29.4%. Also not surprising is the increase in the share of nuclear power from 6% to 8.6%, which also contributes to the reduction of CO2 emissions.

[4] The mean for all countries efficiency scores for the entire period is 47%, meaning that the potential reduction of CO2 emissions has been of 53% on average.

Figure 6. Observed and optimal energy mixes (sample mean)



This strong decrease in the use of fossil fuels in favor of renewable energies can be found in most countries, but even more so in the most polluting ones, with more or less pronounced nuances in the use of nuclear energy. For example, for China, the biggest polluter which accounts for 33% of the CO2 emissions of the total sample, fossil energies should reduce their weight from 91% to 71%, with nuclear power maintaining a small share of around 1% to 1.3%, while renewable energies should increase from 8% to 28% (cf. Figure 7). For the USA, the second largest polluter of the group, which alone accounts for 22% of CO2 emissions, there should also be a drastic decrease in the share of fossil fuels (from 83% to 52%) in favor of an increase in nuclear power (from 8.4% to 17.2%) and renewables (from 8.3% to 30.5%) (cf. Figure 8).

Figure 7.Observed and optimal energy mixes for China (period mean)

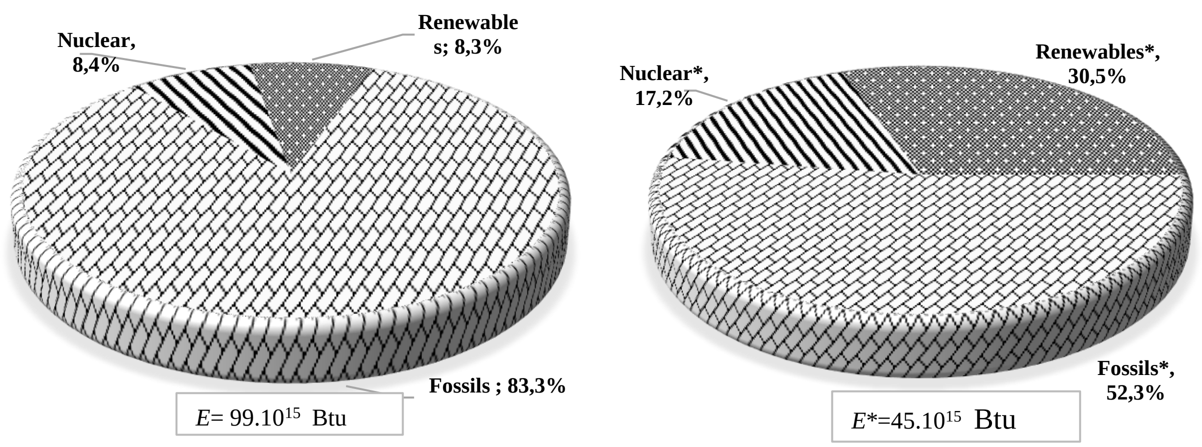
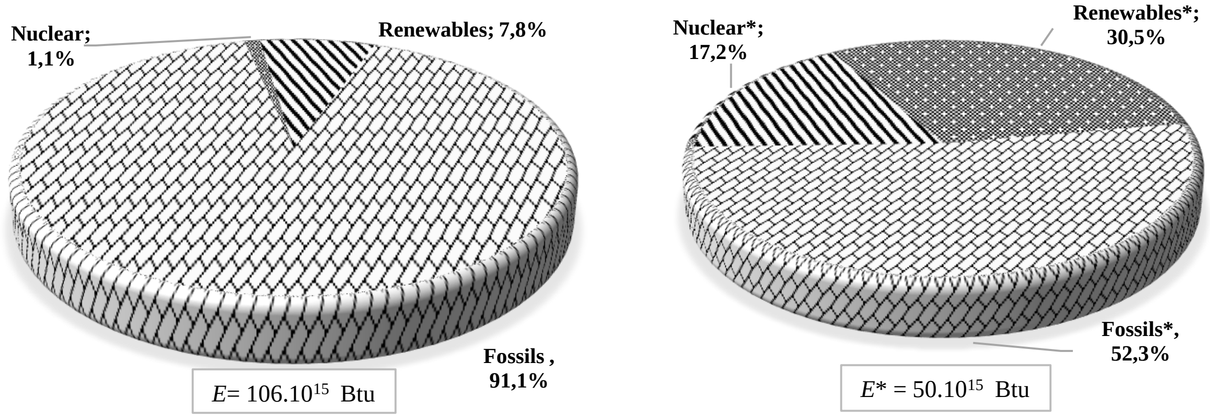


Figure 8. Observed and optimal energy mixes for the United States (period mean)



4. Conclusion

To limit the deterioration of the environment, it is increasingly acknowledged that past and current energy use patterns should be adapted. This confronts developed countries with the challenge of scaling down CO2 emissions from energy use at the maximum possible while preserving (or limiting the decline of) economic activity. In this article, we provide an answer to this crucial question. Specifically, we use a nonparametric pollution cost frontier framework to explore the extent to which a selection of 33 OECD and BRICS countries could reduce their CO2 emissions given their level of *GDP* and population over the period 2001-2019. We extend the model in Coelli et al. (2007), consistent with the materials balance condition, in which the problem of pollution-generating technologies (i.e., the joint production of undesired and desired outputs) is approached as a cost minimization problem. While in their traditional frame, Coelli et al. (2007) assume that emission coefficients are given, our contribution consists in endogenizing them. Indeed, when dealing with heterogeneous energy mixes, the resulting weighted coefficient emission factor can differ across countries. Therefore, in our approach, countries' performance related to their CO2 emissions has been assessed not only with respect to their energy use per unit of *GDP* but also with regard to their carbon intensity of the energy mix used.

We also provide an estimation procedure for our extended model. For this, we address two issues stemming from the cost minimization problem stated. The first one consists in showing the equivalence between a total cost minimization under constant returns to scale assumption and an average cost minimization problem under variable returns to scale. As noted above, the assumption of constant returns to scale assumption is used in Coelli et al. (2007) and it is also in line with the Kaya identity (equation 1). However, in our frame, modeling endogenous coefficient emissions in the optimization problem is ill-suited to the constant returns to scale assumption as those coefficients cannot be rescaled. The second one consists of having dealt with the nonlinearity of the resulting objective function by adapting the Banker and Maindiratta (1986) log-linearization approach.

Our results show the reduction potential for CO2 emission varies between 56% and 50% depending on the year if the different countries could adopt the characteristics of the best practices in terms of energy. This results from a reduction in energy intensity (between 34% and 38%) and a reduction in the carbon intensity of energy (between 24% to 30%). Interestingly, at the sample aggregate scale, the reduction in the carbon intensity of energy is obtained by decreasing the share of fossil energies (from 84.5% to 62%) and increasing the share of both renewable energies (9.5% to 29.4%) and nuclear energy (6% to 8.6%).

As it is increasingly acknowledged that past and current energy use patterns should be adapted to limit the deterioration of the environment, our results provide interesting insights into available current options.

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6. Appendix: proof to Proposition 1.

Let us start from the LPs that can be used to obtain each of the problems stated in equation (7). Denote LP1 the linear program used to calculate the solution to the problem stated in equation (3).

$$\begin{aligned}
&\min_{\boldsymbol{\mu},\delta} C^{CRS}{}_a = \min_{\boldsymbol{\mu},\delta} \delta\left(\mathbf{X}_{\mathrm{a}}\mathbf{e}_{\mathrm{a}}^{\mathbf{T}}\right) \\
&\sum_n \mu_n \left(X_{i,n} e_{i,n}\right) \leq \delta\left(X_{i,a} e_{i,a}\right), \forall i \in I \\
&\sum_n \mu_n Y_{m,n} \geq Y_{m,a},\ \forall m \in M, m \neq m' \\
&\sum_n \mu_n Y_{m',n} = Y_{m',a}, \\
&\mu_n \geq 0, \forall n \in N
\end{aligned} \qquad \textbf{LP1.}$$

Likewise, denote LP2 the linear program used to calculate the solution to the cost function in equation (6), using the notations in equation (5) where we denote by small letters the input and output quantity vectors rescaled with regard to the fixed output.

$$\begin{aligned}
&\min_{\boldsymbol{\lambda},\delta} C_a^{VRS} = \min_{\boldsymbol{\lambda},\delta} \delta\left(\mathbf{x}_a\mathbf{e}_a^{\mathbf{T}}\right) \\
&\sum_n \lambda_n \left(x_{i,n} e_{i,n}\right) \leq \delta\left(x_{i,a} e_{i,a}\right), \forall i \in I \\
&\sum_n \lambda_n y_{m,n} \geq y_{m,a}, \forall m \in M, m \neq m' \\
&\sum_n \lambda_n = 1 \\
&\lambda_n \geq 0, \forall n \in N
\end{aligned} \qquad \textbf{LP2.}$$

Next, in LP1, set $\mu_n Y_{m',n} = \lambda_n Y_{m',a} \Leftrightarrow \mu_n = \lambda_n \frac{Y_{m',a}}{Y_{m',n}}$. LP1 becomes:

$$\min_{\boldsymbol{\lambda},\delta} \delta\left(\mathbf{X}_{\mathrm{a}}\mathbf{e}_{\mathrm{a}}^{\mathbf{T}}\right)$$

$$\sum_n \left(\lambda_n \frac{Y_{m',a}}{Y_{m',n}}\right)\left(X_{i,n}e_{i,n}\right) \le \delta\left(X_{i,a}e_{i,a}\right), \forall i \in I$$

$$\sum_n \left(\lambda_n \frac{Y_{m',a}}{Y_{m',n}}\right) Y_{m,n} \ge Y_{m,a}, \forall m \in M, m \ne m' \qquad \Leftrightarrow$$

$$\sum_n \left(\lambda_n \frac{Y_{m',a}}{Y_{m',n}}\right) Y_{m',n} = Y_{m',a}$$

$$\lambda_n \frac{Y_{m',a}}{Y_{m',n}} \ge 0,\ \forall n \in N$$

$$\min_{\boldsymbol{\lambda},\delta} \frac{\delta \mathbf{X}_{\mathrm{a}}\mathbf{e}_{\mathrm{a}}^{\mathbf{T}}}{Y_{m',a}}$$

$$Y_{m',a} \sum_n \left(\lambda_n \frac{X_{i,n}e_{i,n}}{Y_{m',n}}\right) \le \delta X_{i,a}e_{i,a}, \forall i \in I$$

$$Y_{m',a} \sum_n \left(\lambda_n \frac{Y_{m,n}}{Y_{m',n}}\right) \ge Y_{m,a}, \forall m \in M, m \ne m' \qquad \Leftrightarrow$$

$$\sum_n \lambda_n = 1$$

$$\lambda_n \ge 0,\ \forall n \in N$$

$$\min_{\boldsymbol{\lambda},\delta} \delta\left(\mathbf{x}_a\mathbf{e}_a^{\mathbf{T}}\right)$$

$$\sum_n \lambda_n \left(x_{i,n}e_{i,n}\right) \le \delta\left(x_{i,a}e_{i,a}\right), \forall i \in I$$

$$\sum_n \lambda_n y_{m,n} \ge y_{m,a}, \forall m \in M,\ m \ne m' \qquad \Leftrightarrow \qquad \textbf{LP2.}$$

$$\sum_n \lambda_n = 1$$

$$\lambda_n \ge 0,\ \forall n \in N$$