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Ilke Aydogan
IESEG School of Management, Univ. Lille, CNRS, UMR 9221 - LEM - Lille Economie´ Management, F-59000 Lille, France; and iRisk Research Center on Risk and Uncertainty (i.aydogan@ieseg.fr)

Loïc Berger
CNRS, Univ. Lille, IESEG School of Management, UMR 9221 - LEM - Lille Economie´ Management, F-59000 Lille, France; iRisk Research Center on Risk and Uncertainty; RFF-CMCC European Institute on Economics and the Environment (EIEE), and Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (l.berger@ieseg.fr)

Vincent Théroude
Université de Lorraine, Université de Strasbourg, CNRS, BETA, 54000, Nancy, France (vincent.theroude@univ-lorraine.fr).

IÉSEG School of Management Lille Catholic University 3, rue de la Digue F-59000 Lille Tel: 33(0)3 20 54 58 92 www.ieseg.fr

More Ambiguous or More Complex? An Investigation of Individual Preferences under Model Uncertainty*

Ilke Aydogan† Loïc Berger‡ Vincent Théroude§

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Abstract

This paper explores the drivers of individual preferences under uncertainty. We propose a characterization of the situations of model uncertainty such as the ones introduced by Ellsberg (1961) by building on the *more ambiguous* relations of Jewitt and Mukerji (2017) and Izhakian (2020) and on two new *more complex* relations. Reconsidering existing data sets from the recent literature and combining them with new experimental evidence, we show that uncertainty preferences can be driven by considerations regarding both the degree of complexity and ambiguity that a situation entails.

Keywords: Ambiguity, model uncertainty, complexity, Ellsberg paradox
JEL Classification: D81

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†IESEG School of Management, Univ. Lille, CNRS, UMR 9221 - LEM - Lille Économie Management, F-59000 Lille, France; and iRisk Research Center on Risk and Uncertainty, France (i.aydogan@ieseg.fr).

‡CNRS, Univ. Lille, IESEG School of Management, UMR 9221 - LEM - Lille Économie Management, F-59000 Lille, France; iRisk Research Center on Risk and Uncertainty, France; RFF-CMCC European Institute on Economics and the Environment (EIEE), and Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (loic.berger@cnrs.fr).

§Université de Lorraine, Université de Strasbourg, CNRS, BETA, 54000, Nancy, France (vincent.theroude@univ-lorraine.fr).

1 Introduction

More often than not, the ex-ante information that a decision-maker (DM) has at her disposal is insufficient to single out a unique probability model (or distribution) quantifying the uncertainty over the relevant states of the world. Consequently, most decisions are made under Knightian *uncertainty* (i.e., situations where probabilities are unknown, see Knight 1921). In such situations, it is often convenient to decompose uncertainty into distinct *layers* of analysis, among which *risk* and *model ambiguity* (Hansen, 2014; Marinacci, 2015; Hansen and Marinacci, 2016; Aydogan et al., 2023). The layer of *risk* characterizes the uncertainty *within* a given probability model induced by a specific data-generating mechanism. As such, it features an aleatory type of uncertainty typically represented by an objective probability measure. In contrast, the layer of *model ambiguity* characterizes the uncertainty *across* different potential probability models. It thus features an epistemic type of uncertainty, which can only be quantified by subjective probabilities.

Ellsberg's (1961) classical thought experiments are useful to illustrate the distinction between the layers of risk and model ambiguity. In the two-color problem, a DM bets on a draw from one of two following urns: (a) a known urn containing 50 red and 50 black balls; and (b) an unknown urn containing 100 balls, each of which is either red or black. In this setup, each single urn composition gives rise to a specific probability distribution or risk. Accordingly, the known urn is an instance featuring only the layer of risk. In contrast, the unknown Ellsberg urn features both the layers of model ambiguity and risk. In particular, when the unknown urn contains 100 balls, the DM faces uncertainty both *across* the 101 possible urn compositions and *within* each of those compositions. The widespread finding of ambiguity aversion highlighted in the Ellsberg paradox indicates behavioural differences towards the layers of model ambiguity and risk.[1]

Ellsberg's examples have been widely used to motivate the development of various ambiguity theories (e.g., Segal, 1987; Gilboa and Schmeidler, 1989; Schmeidler, 1989; Tversky and Kahneman, 1992; Ghirardato et al., 2004; Klibanoff et al., 2005; Maccheroni et al., 2006; Seo, 2009, see also Gilboa and Marinacci, 2013 and Machina and Siniscalchi, 2014 for recent surveys). Urns of different size have also been used to empirically study ambiguity preferences and calibrate ambiguity attitudes (e.g., Cubitt et al., 2018; Berger and Bosetti, 2020; see Trautmann and Van De Kuilen, 2015 for a survey). Yet the characteristics of Ellsberg urns with different sizes

[1] We typically observe a preference for betting on the known urn over the unknown one (i.e., ambiguity aversion) in the two-color Ellsberg experiment in the domain of gain. Remark however that this may not be the case for events with small likelihoods and in the loss domain, where ambiguity-seeking behaviours are also common (see Trautmann and Van De Kuilen, 2015; Baillon and Bleichrodt, 2014; Kocher et al., 2018).

and their implications for decisions have largely been overlooked (with a notable exception of the recent study of Filiz-Ozbay et al., 2021). Because the size of an ambiguous urn (i.e., its total number of balls) implicitly specifies the set of probability models to consider, which, in turn, characterizes the degrees of ambiguity and complexity of the uncertain situation, it may be critical in determining the DM's preferences.

In this paper, we characterize situations of model ambiguity, as in Ellsberg-type ambiguous urns, using two types of relations. *(a)* The first relation is the *more ambiguous* relation, whose alternative definitions have recently been proposed by Jewitt and Mukerji (2017) and Izhakian (2020). Jewitt and Mukerji's (2017) definition relies on specific classes of ambiguity preferences. Accordingly, we here consider two distinct classes: the α-maxmin expected utility and the smooth ambiguity preferences. For these families, the degree of ambiguity depends, respectively, on the range and spread of expected utilities induced by different probability models. Alternatively, the more ambiguous definition of Izhakian (2020) relies on a measure of expected volatility, derived from the expected utility with uncertain probabilities framework of Izhakian (2017). *(b)* The second type of relation we consider is the *more complex* relation. We propose two characterizations that enable us to rank different situations of model ambiguity according to their degrees of complexity. Whereas our first characterization associates the degree of complexity with the cardinality of the set of potential models to consider, our second characterization relies on a partial ordering of these sets based on their coarseness.

The *more ambiguous* and *more complex* relations may be used to derive behavioural predictions and analyze preferences over various uncertain situations. To illustrate their empirical implications, we examine different data sets in the light of our characterizations. We start by re-considering the experimental data collected by Filiz-Ozbay et al. (2021) to study preferences for the size of an ambiguous urn. To complement this data, we also present the results of a new Ellsberg experiment. Overall, the results from these experiments indicate that: (1) the preference for risk over Ellsberg ambiguity depends on the underlying set of probability models considered; (2) there exists a tendency to prefer larger-sized Ellsberg ambiguous urns over smaller ones, except when the urn is extremely small and contains only one ball; and (3) the heterogeneity observed in size preferences can be explained by a combination of both attitudes towards the degree of ambiguity and complexity of the uncertain situation. In a final step, we re-analyze the rich data of Chew et al. (2017) to show that our characterization is not limited to the situations falling under the classical Ellsberg paradigm but can be used to analyze a wide range of uncertain situations, in which the DM can postulate a set of distributions to quantify the uncertainty she is facing. Our analysis further demonstrates the important role of complexity and ambiguity in explaining preferences under uncertainty.

This paper contributes to a recent line of research that has studied the role of complexity in decision-making under risk (Puri, 2018; Oprea, 2022) and ambiguity (Armantier and Treich, 2016; Kovářík et al., 2016; Aydogan et al., 2023). These latter studies, for example, argue that ambiguity aversion may not be the only underlying factor behind the patterns typically observed in Ellsberg-type experiments and that a separate notion of *complexity aversion* could play an important and separate role. However, to the best of our knowledge, no formal way to measure complexity under ambiguous situations has so far been proposed.[2] In consequence, the relative role of complexity and ambiguity in determining preferences under uncertainty have remained unexplored. This paper attempts to fill this gap in the literature by proposing tractable notions of complexity and by providing experimental evidence highlighting the distinct treatment of complexity and ambiguity.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework to analyze uncertain situations using the more ambiguous and more complex relations and presents testable predictions. Section 3 reports the results of various experiments and makes the links with the possible behavioural patterns highlighted. There, we first focus on experimental data under the classical Ellsberg paradigm before extending the analysis to situations of partial ambiguity. We further discuss the implications of our characterization of model ambiguity and conclude in Section 4.

2 Theoretical framework

In this section, we present different approaches that can be used to characterize uncertain situations. These approaches are either based on the degree of ambiguity or complexity of the situations. We then derive behavioural predictions, which will be later tested using different experimental data sets (see Section 3).

2.1 Setup

Let S denote a finite set of states and C a set of consequences. In what follows, we focus on monetary consequences, so that C is assumed to be an interval of the real line. An act is a function $f : S \to C$ mapping states into consequences. The collection of all acts is denoted by $\mathcal{F}$. For the sake of exposition, we focus on a subset of acts that are called *bets*. Formally, a bet on an event A $\subseteq S$ is a binary act such that $f(s) = x$ for $s \in$ A and $f(s) = y$ for $s \notin$ A, with $x > y$.

[2] Under risk, an attempt is made by Puri (2018).

In words, a bet yields the best consequence x if A obtains and y otherwise. In this paper, we focus on bets on the color of a ball drawn from an urn. All the situations we consider entail a standard two-color Ellsberg (1961) setting, in which the color of the balls in the urn can be either red or black. Let R denote a situation of risk represented by Ellsberg's original known urn and Ei denote a situation of model ambiguity represented by Ellsberg's unknown urn containing a finite number $i \in \mathbb{N}_{>0}$ of balls, each of which can be either red or black. Each bet a_k that we consider results in a binary consequence $c \in \{x, y\}$ with $x > y$ depending on which state of the world $s_k \in \{\text{red}, \text{black}\}$ is realized in situation $k \in \{R, Ei\}$ for some i. We consider a DM who has a (reflexive, transitive, complete, monotone, continuous, and non-trivial) preference relation $\succsim$ over acts. As usual, $\sim$ denotes indifference and $\succ$ strict preference. In consequence, a $\succsim$ b means that the DM either strictly prefers act a to act b or is indifferent between the two. In line with Wald (1950), such an Ellsberg-type setting implicitly assumes that the DM knows that states are generated by a probability model that is presumed to belong to a (finite) collection M, which is taken as a datum of the decision problem. We use the following *symmetry* assumption to ease the derivation of our predictions.

Symmetry: For each act a_k, the DM is indifferent to the color on which to bet.

Such a symmetry assumption has been extensively used in the theoretical ambiguity literature. It has also been widely supported empirically (see e.g., Abdellaoui et al., 2011; Chew et al., 2017; Epstein and Halevy, 2019; Aydogan et al., 2023).

2.2 Decomposing uncertainty into layers

Following Hansen (2014); Marinacci (2015), and Hansen and Marinacci (2016), we decompose the uncertainty featured in Ellsberg-type settings into the *layers* of *risk* and *model ambiguity*.[3] A risk is described by a unique, objective probability measure characterizing the intrinsic randomness that states feature. For example, it corresponds to a bet on the known two-color urn, which may be expressed as the binary lottery $x_p y$, yielding x with an objective probability p and y otherwise.

Ambiguous Ellsberg urns, on the contrary, are characterized by a multiplicity of possible probability distributions. In principle, it is possible to posit a set M of potential models m_p describing the likelihood of the different states. While each model is itself a risk that may be characterized by $m_p \equiv x_p y$, the uncertainty about

[3]Note that, in more general situations, a third layer of uncertainty, known as *model misspecification* (uncertainty about whether or not the correct model lies among the set of models considered) is also present (see Aydogan et al., 2023). However, because all the situations in the standard Ellsberg (1961) setting can be analyzed in terms of risk and model ambiguity only, model misspecification plays no role in this paper so that it can be ignored.

the correct probability model to consider among the collection $M = \{m_p\}$ can no longer be quantified objectively, and is therefore said to have an epistemic nature. For example, if one forms a probability measure over the possible urn compositions, such a measure would be *subjective*, reflecting the degree of belief that one has in each possible model.

In what follows, we present two types of relation that can be used to characterize the ambiguous bets under this two-layer decomposition. These relations, in turn, are used to highlight potential patterns of preferences over Ellsberg urns of different sizes.

2.3 The more ambiguous relation

Two approaches have recently been proposed in the literature to order different ambiguous situations by their degree of ambiguity. The first approach is proposed by Jewitt and Mukerji (2017) and the second by Izhakian (2020).

2.3.1 Jewitt and Mukerji (2017)

Jewitt and Mukerji (2017) propose a notion of *more ambiguous* that allows for establishing a partial ordering among ambiguous situations within a given class of preferences. Because their more ambiguous relation (which they call more ambiguous I) is based on the notion of more ambiguity averse than an ambiguity neutral preference, it is first necessary to adopt a normalization for *ambiguity neutrality*.

Ambiguity neutrality Following Ghirardato and Marinacci (2002) and Gilboa and Marinacci (2013), we consider subjective expected utility (SEU) as a benchmark for ambiguity neutrality. Under SEU, it is assumed that the DM has a subjective prior probability measure $\mu : 2^M \to [0, 1]$ quantifying the epistemic uncertainty in the layer of model uncertainty. The symmetry condition implies that the subjective probability distribution μ over the set of probability models is symmetric. The two-layer version of SEU that has been axiomatized by Cerreia-Vioglio et al. (2013) takes the form:

$$V_{\text{SEU}}(a_k) = \sum_{m_p \in M} \Big(\sum_{s \in S} u\left(a_k(s)\right)\ m_p(s) \Big)\, \mu(m_p). \tag{1}$$

In this expression, u is a von Neumann-Morgernstern utility function, translating economic consequences (measured in monetary terms) into utility levels. This function captures risk attitudes. Model ambiguity is addressed using the subjective prior probability distribution μ that quantifies the DM's belief about the correct urn composition (and thus about p). Under this framework, the layers of risk and model ambiguity are implicitly treated in the same way (Marinacci, 2015).

In order to compare bets on different urns, we furthermore assume that the DM's beliefs are fixed and belong to a single family. As argued in Jewitt and Mukerji (2017), fixing beliefs is natural and necessary in such a framework.[4]. On the other hand, imposing beliefs to belong to the same family ensures internal consistency in the treatment of urns of different sizes. For example, irrespective of the size of the urn, the family of uniform prior distributions ensures the same treatment for all physically possible compositions of a given urn.[5] All the bets taking place in an Ellsberg-type setting are evaluated in the same way under SEU. For instance, assuming a uniform family of prior μ, we have, after normalizing $u(x) = 1$ and $u(y) = 0$:

$$V_{\mathrm{SEU}}(a_k) = \sum_p p\,\mu(p) = \frac{1}{2} \quad \forall k \in \{R, Ei\}\,. \tag{2}$$

In other words, SEU predicts the following *indifference* pattern.

Pattern 1 (SEU)

$$R \sim Ei \quad \forall i \tag{3}$$

Following Ghirardato and Marinacci (2002, p. 254), a "more ambiguity averse than" relation can then be established on the following intuition.

More (less) ambiguity averse than *If a DM prefers an unambiguous (resp. ambiguous) act to an ambiguous (resp. unambiguous) one, a more (resp. less) ambiguity averse one will do the same.*

Based on this intuition, an ambiguity averse preference can be defined in relation to ambiguity neutrality as follows.

Definition 1: Ambiguity aversion An ambiguity averse preference is defined as any preference relation $\succsim_B$ for which there is a SEU preference $\succsim_A$ "less ambiguity averse than $\succsim_B$".

[4]Remark that, under different beliefs, the same act would induce correspondingly different lotteries over outcomes under expected utility, potentially leading an act to be more risky under one belief and less risky under another.

[5]A uniform prior is consistent with the principle of insufficient reason (Bernoulli, 1713; Laplace, 1814), the idea of the simplest non-informative prior in Bayesian probability (Bayes, 1763), and the principle of maximum entropy (Jaynes, 1957). Alternatively, one can assume the family of binomial distributions, which results from associating the same subjective belief to each individual ball being red or black.

Next, we consider two families of preferences $\mathcal{P}$ relating each $\succsim \in \mathcal{P}$ to an ambiguity neutral element of $\mathcal{P}$. These families are the smooth ambiguity family of Klibanoff, Marinacci, and Mukerji (2005) and the α-maxmin expected utility family of Ghirardato et al. (2004).

Smooth ambiguity family Under the smooth model proposed by Klibanoff, Marinacci, and Mukerji (2005), model ambiguity is quantified using a single probability measure. However, contrary to the SEU, this approach allows for a distinct treatment of the layers of risk and model ambiguity. In particular, by letting v and u represent the DM's attitudes towards model ambiguity and risk respectively, the smooth ambiguity criterion emerges as a natural generalization of the SEU criterion as follows

$$V_{\text{smooth}}(a_k) = \sum_{m_p \in M} \phi\Big(\sum_{s \in S} u\left(a_k(s)\right)\ m_p(s)\Big)\ \mu(m_p), \tag{4}$$

where $\phi \equiv v \circ u^{-1}$. Under this framework, ambiguity aversion is characterized by a concave function ϕ, reflecting a more averse attitude towards the layer of model ambiguity than that of risk. Ambiguity aversion is thus characterized by an aversion to mean-preserving spreads in the distribution of expected utilities induced by different urn compositions.[6]

Maxmin models The second family of preference we analyze originates in the work of Gilboa and Schmeidler (1989). Their *multiple priors* approach relaxes the assumption of model ambiguity being quantified by a single probability measure μ and instead allows for the possibility of multiple priors belonging to a set C.[7] Under the α-maxmin expected utility criterion of Ghirardato et al. (2004), both the least favorable among all the (classical) subjective expected utilities determined by each prior μ in C and the most favorable one appear respectively with weights α and $1-\alpha$. The multiple priors maxmin model of Gilboa and Schmeidler (1989) naturally emerges as a special case when $\alpha = 1$, while the classical SEU criterion is recovered when the set C is singleton. When C consists of all possible prior probability measures, we recover the criterion due to Hurwicz' (1951) when $\alpha \in (0, 1)$ and to Wald (1950) when $\alpha = 1$. In what follows, we focus on Hurwicz (1951) version. The utility of the bet a_k, in that case, is

[6]Note that Nau (2006) and Ergin and Gul (2009) characterize representations that, at least in special cases, can take the same representation as (4) and share the same interpretation.

[7]This set of possible priors C incorporates both the attitude towards ambiguity and an information component: a smaller set C reflecting, for example, both better information and/or less ambiguity aversion. Note moreover that in this case, the symmetry condition translates to a symmetric set of priors C. These priors can, however, be non-symmetric or have non-symmetric support.

$$V_{\alpha-mxm}(a_k) = \alpha \min_{m_p} \Big(\sum_{s \in S} u\left(a_k(s)\right)\ m_p(s) \Big) + (1-\alpha) \max_{m_p} \Big(\sum_{s \in S} u\left(a_k(s)\right)\ m_p(s) \Big). \tag{5}$$

In this expression, α may be interpreted as an index of ambiguity attitude. For example, $\alpha = 0$ corresponds to a situation in which the DM is extremely optimistic and considers only the best possible composition of the urn, while $\alpha = 1$ corresponds to a DM being extremely pessimistic, considering only the worst possible composition.

The Jewitt and Mukerji's (2017) definition of the *more ambiguous* relation is as follows.

Definition 2: More ambiguous [Jewitt and Mukerji, 2017]

Let $\mathcal{P}$ be a class of preferences over a set A. Assume that a binary relation "more ambiguity averse" is given, and that each $\succsim \in \mathcal{P}$ is related to an ambiguity neutral element of $\mathcal{P}$. Given two acts f, $g \in$ A, f is a *more ambiguous* act than g if the following conditions are satisfied:

(i) if $\succsim \in \mathcal{P}$ is ambiguity neutral, then $g \sim f$;

(ii) for all $\succsim_A$, $\succsim_B \in \mathcal{P}$ such that $\succsim_A$ is an ambiguity neutral preference and $\succsim_B$ is more (less) ambiguity averse than $\succsim_A$, we have $g \succsim_B (\precsim_B) f$.

In words, an act f is more ambiguous than an act g if an ambiguity-averse DM prefers g to f, but an ambiguity-neutral DM is indifferent between the acts.

As made explicit, the more ambiguous relation arises on the back of a specific *more ambiguity averse* relation on preferences. Focusing on the two families of preferences presented above, we now use the more ambiguous definition to highlight different patterns of preferences when comparing Ellsberg-type urns of different sizes.

Under the smooth ambiguity family, bets on Ellsberg-type situations can be strictly ordered in terms of how much they are affected by ambiguity. For example, under a uniform prior μ, a bet on an Ellsberg urn with i balls in it is always more ambiguous than on an urn with j balls, as long as $i \leq j$ (see Berger, 2022). The implied pattern under ambiguity aversion is summarized as follows:

Pattern 2 (smooth ambiguity aversion)

$$Ej \succ Ei \Longleftrightarrow R \succ Ej \quad \forall i, j \in \mathbb{N}_{>0} \text{ such that } i < j. \tag{6}$$

Unsurprisingly, smooth ambiguity seeking corresponds to the reversed pattern.

Alternatively, under the maxmin family, bets on Ellsberg-type ambiguous urns are all more ambiguous than R, but cannot be (strictly) ordered according to the more ambiguous relation, as they share the same worst ($p = 0$) and best ($p = 1$) possible models. Thus, irrespective of the degree of ambiguity aversion α, we have, under the maxmin models:

Pattern 3 (α-maxmin)

$$Ei \sim Ej \quad \forall i, j \in \mathbb{N}_{>0} \text{ and } \forall \alpha \in [0, 1]. \tag{7}$$

2.3.2 Izhakian (2020)

An alternative approach for ordering acts according to their degree of ambiguity has been recently proposed by Izhakian (2020). Under this approach, the degree of ambiguity of a bet a_k may be quantified by its expected volatility of probabilities

$$\mho^2 [a_k] \equiv \sum_{s \in S} \mathrm{E}_\mu \left[m_p^{a_k}(s) \right] \mathrm{Var}_\mu \left[m_p^{a_k}(s) \right], \tag{8}$$

where $m_p^{a_k}(s)$ is the probability of being in state s under model m, and $\mathrm{E}_\mu [.]$ and $\mathrm{Var}_\mu [.]$ are the expectation and variance operators, respectively, taken using the prior probability measure μ. The measure $\mho^2$ is argued to be independent of attitudes towards risk and ambiguity, and has the advantage of being easily computable. The underlying decision-making model of this measure is the expected utility with uncertain probabilities (EUUP) of Izhakian (2017), in which preferences for ambiguity apply exclusively to probabilities of events and are therefore outcome independent. Under this framework, a more ambiguous relation is defined as follows.

Definition 3: More ambiguous [Izhakian, 2020] Given two acts f, $g \in \mathrm{A}$ under which the expected probabilities of each consequence $c \in C$ are identical, f is a *more ambiguous* act than g if and only if

$$\mho^2 [f] \geq \mho^2 [g].$$

In words, an act g whose associated probabilities are on average less volatile than an act f is deemed less ambiguous. In a framework where ambiguity aversion takes the form of aversion to mean-preserving spreads in the space of second-order distributions, such an act is moreover preferred by any ambiguity averse individual.

In an Ellsberg-type setting, it should be clear that the symmetry condition ensures that the expected probabilities of the consequences are identical: $\mathrm{E}_{\mu}\left[m_p^{a_k}(s)\right] = \left[\sum_{m_p} m_p^{a_k}(s)\,\mu\left(m_p^{a_k}\right)\right] = 0.5$ for all $k \in \{R, Ei\}$ and all $s \in \{\text{red,black}\}$. It is then easy to show that $\mho^2$, is decreasing in the number of balls present in the urn. The more ambiguous relation based on Izhakian (2020) thus predicts the following pattern under ambiguity aversion:

Pattern 4 (ambiguity aversion à la Izhakian, 2020)

$$Ej \succ Ei \Longleftrightarrow R \succ Ej \quad \forall i, j \in \mathbb{N}_{>0} \text{ such that } i < j. \tag{9}$$

Interestingly, remark that this pattern coincides with the smooth model pattern (Pattern 2 above).

2.4 The more complex relation

Alternatively, as it has previously been argued, complexity that an ambiguous situation entails might play an important role in Ellsberg-type behaviours. In what follows, we use set theory to characterize a situation by its degree of complexity. The DM's information about the likelihood of the different states is a priori modeled by the set $M = \{m_p \text{ such that } p \in I\}$, where p is the probability of winning and $I \subseteq [0, 1]$ is a set-theoretic modeling of information characterizing the chances to make a correct bet. Assuming that the DM has information about M, her acts needs to be measurable with respect to M without being allowed to condition the choices on models that do not belong to M.

In analogy to what is done in the risk literature, where complexity is typically defined according to the "number of different outcomes of a lottery" (Sonsino et al., 2002; Moffatt et al., 2015), our first characterization of complexity relates to the *number of different potential models,* measured by the cardinality of M.

Definition 4: More complex [cardinality] Given two binary acts f, $g \in \mathrm{A}$, f is a *more complex* act than g if $|M_f| \geq |M_g|$.

Under such an intuitive definition, the degree of complexity of a situation depends exclusively on the number of different potential models that the DM has to evaluate.[8] Using this relation, we can easily order the different Ellsberg-type situations according to their degree of complexity. Specifically, the situation R is always the least complex one as the set M is singleton when the urn is not ambiguous. Moreover, we can observe that there exists a monotonic relationship between the number of balls in Ei and its associated number of potential models, i.e., $|M_{Ei}| = i + 1$. Thus, if individuals dislike complexity, i.e., exhibit a preference for simpler situations over more complex ones, we will observe:

Pattern 5 (complexity aversion 1)

$$R \succ Ei \succ Ej \quad \forall i, j \in \mathbb{N}_{>0} \text{ such that } i < j. \tag{10}$$

This order is obviously reversed for someone who likes complexity.

The second definition of *more complex* is based on a partial ordering of the sets M according to the "coarser than" relation.

Definition 5: More complex [coarser] Assume that a binary relation "coarser than" is given as follows: M is coarser than M' (and M' is finer than M) if $I' \subseteq I$. Then, given two binary acts f, $g \in$ A, we say that f is *more complex* than g if M_f is coarser than M_g.

In words, this definition means that more complex information regarding the structure of ambiguity may be naturally modeled by an enlargement of the set of potential models. Such a more complex relation can, for example, be used to order Ellsberg-type ambiguous situations Ei and Ej when j is a multiple of i, as $I_{Ei} \subset I_{Ej}$ in that case, or to compare R and Ei when i is even. Note however that this more complex relation is not complete and remains, for example, silent when comparing $E2$ and $E3$.[9] Under this characterization, a complexity averse DM will exhibit the following pattern of preferences:

[8] This notion of complexity coincides with what Einhorn and Hogarth (1985) referred to as the amount of ambiguity. They write "Moreover, the amount of ambiguity is an increasing function of the number of distributions that are not ruled out (or made implausible) by one's knowledge of the situation" (Einhorn and Hogarth, 1985, p. 435).

[9] Intuitively, the complexity of a situation may also depend on the probability models contained in M. In particular, obtaining a clear ranking may not be as straightforward as suggested by the cardinality of the set M in more general situations (e.g., when considering non-binary acts). The coarseness definition captures this intuition by remaining silent unless the comparison is straightforward as in enlargement of set M.

Pattern 6 (complexity aversion 2)

$$\begin{cases} Ei \succ Ej & \forall i, j \in \mathbb{N}_{>0} \text{ such that } i \mid j \\ R \succ Ei & \forall i \text{ such that } i = 2q \text{ where } q \in \mathbb{N}_{>0}. \end{cases} \tag{11}$$

3 Experimental evidence

We now use different data sets to test the patterns highlighted in Section 2. We start by exploring individual choice data from experiments using a standard Ellsberg-type setting. This includes the data of Filiz-Ozbay et al. (2021) and an original data set that we collected. We then consider the data of Chew et al. (2017), whose 'partial ambiguity' setting goes beyond the standard Ellsberg framework.

3.1 Experiments using an Ellsberg-type setting

3.1.1 Designs

Filiz-Ozbay et al. (2021, hereafter FGMO) recently present the results of a study investigating preferences over the size of ambiguous Ellsberg urns in relation to ambiguity theories and the role of the ratio bias. The experiment considers risky and ambiguous Ellsberg urns with 2, 10, and 1000 balls. Following our notation, the ambiguous situations are labelled $E2$, $E10$ and $E1000$, whereas R is used to denote the risky urns containing 50% black and red balls.[10]

Subjects made binary choices between bets on different urns presented two by two. To elicit strict preferences, the same binary choice problem was presented in two versions. In each version, a correct bet on one of the urns paid \$30 whereas the one on the other urn paid \$30.25. Subjects were considered as strictly preferring one of the two urns if they chose the same bet in the two versions of the problem. The experiment entails seven binary comparisons in total, including risk vs. risk (to test for the ratio bias), risk vs. ambiguity (to test for ambiguity attitudes using urns with different sizes), and ambiguity vs. ambiguity (to test for size preferences under ambiguity). In what follows, we focus on the choices R vs. $E2$, R vs. $E10$ and R vs. $E1000$, and on the comparisons $E2$ vs. $E10$ and $E10$ vs. $E1000$ to test our different patterns.

Our experiment To complement the experiment of FGMO, we present supplementary data involving choices over Ellsberg urns of different sizes. Our design compares Ellsberg's original ambiguous urn $E100$ with two extreme cases. The

[10]Note that the probability model that the risky urns with 2, 10, and 1000 balls represent is identical (i.e., $m_{0.5}$). In consequence, we do not distinguish these urns here and denote them all by R. Note however that, in FGMO, the comparisons between risky and ambiguous urns are made for urns of equal size.

first, denoted $E1$, is the minimum-sized ambiguous urn, which contains only one ball. This urn entails only two possible compositions: the probability of drawing a red (or black) ball is either 0% or 100%, with no intermediate probabilities within these bounds. The second case is $E1000$. It presents an extremely large number of potential compositions. Risk, denoted R, is represented by a standard known urn containing 50 red and 50 black balls. Ambiguity preferences in the context of large urns is measured by comparing directly R and $E100$. Testing ambiguity attitudes with an urn like $E1$ is however non trivial.[11] For this reason, we include an additional urn, denoted $E1^*$, which presents the same number of potential models as $E1$ but is constructed with 100 balls. Specifically, the 100 balls in $E1^*$ are either *all* black or red. Figure 1 illustrates the different urns used in our experiment.

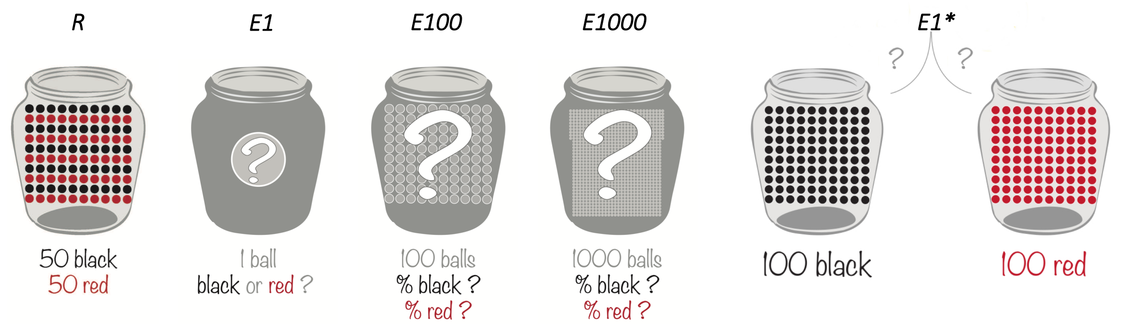

Figure 1: Urns representing the different situations in our experiment

We used binary comparisons to elicit preferences. Specifically, subjects were presented the same choice problem between bets on two urns twice. The first choice aimed at eliciting weak preferences. In that case, a correct bet on one of the urns paid €15. The same question was then repeated with a slightly increased amount (€15.10) for a correct bet on the urn that was not selected at the first stage. One urn was considered as strictly preferred over the other if it was chosen in the two versions of the problem. In the same vein, we assume indifference when preferences were reversed in favor of the bet proposing the highest prize.[12] Our experiment entails nine binary comparisons in total. Among these, we focus on two risk vs. ambiguous comparisons (R vs. $E1^*$ and R vs. $E100$) to test for ambiguity preferences and three comparisons between ambiguous urns ($E1$ vs. $E100$, $E100$ vs. $E1000$, and $E1$ vs. $E1000$) for size preferences.[13] Further details on our experimental stimuli

[11] Indeed, constructing the corresponding risky urn with only one ball is technically impossible.

[12] In principle, indifference between two subsequent bets implies that any small $\varepsilon > 0$ prize difference will lead to a preference for the bet with the highest prize. In the two experiments described, the prize differences between bets, i.e.; €0.10 in our study and \$0.25 in FGMO, is considered as sufficiently small to distinguish indifference from strict preferences.

[13] For completeness, the experiment also include the comparison R vs. $E1$ and R vs. $E1000$. However, as the number of balls in the risky and ambiguous urns are different and may lead to a potential confound of ratio bias (as demonstrated in FGMO), we do not use these comparisons in our main analysis. We

and procedure are presented in Online Appendix S1. Table 1 summarizes the main characteristics of the urns used in the two experiments.

Table 1: CHARACTERISTICS OF THE UNCERTAIN SITUATIONS IN ELLSBERG EXPERIMENTS

Uncertain situation	Experiment	Set of models (M)	Number of models ($\|M\|$)	Volatility of probabilities[a] ($\mho^2$)
R	FGMO, ABT	$\{\frac{50}{100}\}$	1	0.000
$E1$	ABT	$\{\frac{0}{1}, \frac{1}{1}\}$	2	0.250
$E2$	FGMO	$\{\frac{0}{2}, \frac{1}{2}, \frac{2}{2}\}$	3	0.167
$E10$	FGMO	$\{\frac{0}{10}, \frac{1}{10}, ..., \frac{9}{10}, \frac{10}{10}\}$	11	0.100
$E100$	ABT	$\{\frac{0}{100}, \frac{1}{100}, ..., \frac{99}{100}, \frac{100}{100}\}$	101	0.085
$E1000$	FGMO, ABT	$\{\frac{0}{1000}, \frac{1}{1000}, ..., \frac{999}{1000}, \frac{1000}{1000}\}$	1001	0.084

Notes: In an abuse of notations, we let each model $m_p \equiv x_p y$ belonging to M be fully characterized by its probability p.
[a] Assuming a uniform prior probability measure μ. FGMO refers to the experiment of Filiz-Ozbay et al. (2021), ABT refers to the new experiment we run.

3.1.2 Results

Ambiguity preferences We first test whether ambiguity preferences (i.e., preferences between risk and ambiguity) depend on the size of the Ellsberg urn considered. Table 2 presents the comparisons between the risky urn (R) and different Ellsberg ambiguous urns in the two experiments. As can be observed, the most common pattern is a preference for risk over ambiguity, although the proportions vary when using different-sized urns. Specifically, we observe that the preference for risk over ambiguity under the large-sized urns $E1000$ and $E100$ is comparable across the two studies (two-sample test of proportions, p=0.54). However, whereas FGMO found an increasing trend in the preference for risk over ambiguity when smaller urns are considered (McNemar's Chi2, p=0.02 and p=0.03 for the comparisons between $E1000$ and $E2$, $E10$, respectively), we find the opposite pattern when the extreme urn $E1^*$ is considered (McNemar's Chi2, p=0.096 for comparison between $E1^*$ and $E100$). Interestingly, the preference for risk when using $E1^*$ in our experiment is also significantly lower than that with the slightly larger $E2$ in FGMO (p<0.001), which suggests that $E1^*$ may be perceived differently than the other small-sized Ellsberg urns. We summarize these observations as follows.

Result 1: *The preference for risk over ambiguity depends on the size of the Ellsberg urn considered. Except in the case of $E1^*$, the preference for risk over ambiguity tends to decrease with the size of the ambiguous urn.*

used the comparisons $E1^*$ vs. $E1$ and $E1^*$ vs. $E100$ to test potential framing effects. The results of these comparisons are reported in Appendix A.

Table 2: Ambiguity preferences

Part I: The experiment of FGMO ($N = 116$)

	R vs. $E2$	R vs. $E10$	R vs. $E1000$
Prefer risk	86 (74.1%)	85 (73.3%)	74 (63.8%)
Indifferent	18 (15.5%)	19 (16.4%)	28 (24.1%)
Prefer ambiguity	12 (10.3%)	12 (10.3%)	14 (12.1%)

Part II: Our experiment ($N = 84$)

	R vs. $E1^*$	R vs. $E100$
Prefer risk	40 (47.6%)	50 (59.5%)
Indifferent	34 (40.5%)	26 (31%)
Prefer ambiguity	10 (11.9%)	8 (9.5%)

Size preferences We now focus on the direct comparisons between different Ellsberg urns to test size preferences. We use a majority rule to characterize preference for large- or small-sized urns: a subject is said to exhibit a preference for small-sized (large-sized) urns if she strictly prefers a smaller (larger) urn in at least two out of the three possible pairwise comparisons. Analogously, a subject is said to be indifferent if she exhibits indifference in at least two out of the three pairwise comparisons.[14] Subjects whose preferences do not exhibit any dominant pattern remain unclassified.

Table 3 summarizes the results in both studies.[15] We observe that the majority of the subjects in FGMO exhibit a preference for larger over smaller-sized ambiguous urns. In our experiment, preference for larger urns is also the most common pattern, although a clear majority does not emerge. The proportion of subjects exhibiting this pattern is also significantly lower than that observed in FGMO (p<0.001). Finally, in contrast with FGMO, we observe that a non-negligible proportion of subjects (25%) exhibit a preference for smaller ambiguous urns (this proportion is higher than that observed in FGMO p<0.001). We summarize this second set of observations as follows.

Result 2: *Larger urns tend to be preferred to smaller ones, but this tendency is weaker when E1 is contrasted with large-sized urns.*

[14] Note that FGMO studied only two comparisons: $E2$ vs. $E10$ and $E10$ vs. $E1000$. We thus assume transitivity to infer the preference between $E2$ vs. $E1000$. Note that inferring size preferences based on our majority rule coincides with looking at weak preferences in FGMO. For example, a preference for smaller urns is exhibited if $E2 \succeq E10$ and $E10 \succeq E1000$, with at least one strict preference relation.

[15] An additional analysis with a restricted sample of subjects exhibiting transitive choices is reported in Appendix A. Our conclusions do not change in that case.

Table 3: Size Preferences under Ambiguity

Part I: The experiment of FGMO ($N = 116$)

$E1000 \succeq E10 \succeq E2$	$E1000 \sim E10 \sim E2$	$E1000 \preceq E10 \preceq E2$	other
73 (62.9%)	23 (19.8%)	5 (4.3%)	15 (12.9%)

Part II: Our experiment ($N = 84$)

$E1000 \succeq E100 \succeq E1$	$E1000 \sim E100 \sim E1$	$E1000 \preceq E100 \preceq E1$	other
38 (45.2%)	23 (27.4%)	21 (25%)	2 (2.4%)

Explaining preferences over Ellsberg urns We then explore the interaction between ambiguity and size preferences using the theoretical patterns highlighted in Section 2.[16] We use the more ambiguous and more complex relations to classify subjects as follows:

1. Preferences for large-sized Ellsberg urns over small-sized urns are related either to (1.a) *smooth ambiguity aversion* (or ambiguity aversion à la Izhakian, 2017) if the subject also exhibits a preference for risk over ambiguity, or to (1.b) *complexity seeking* if the subject also prefers ambiguity to risk.

2. Preference for small-sized Ellsberg urns over large-sized urns are related either to (2.a) *complexity aversion* if the subject also prefers risk to ambiguity, or to (2.b) *smooth ambiguity seeking* if the subject also prefers ambiguity to risk.

3. Indifference towards the size of Ellsberg urns are related either to (3.a) *SEU* if subject is also indifferent between risk and ambiguity, or to (3.2) *maxmin* preferences, irrespective of the subject's ambiguity preferences.

Table 4 reports the results of the classification. In FGMO, the majority of subjects are classified as smooth ambiguity averse, while only few subjects behave in accordance with either SEU or exhibit maxmin ambiguity or complexity attitudes. In contrast, we observe more heterogeneity in individual types in our experiment. Specifically, we find that 50% of the subjects behave in accordance with either smooth or maxmin ambiguity aversion, whereas 33% of the subjects are mainly driven by complexity attitudes (23.1% complexity aversion and 10.3% complexity seeking). This proportion of complexity-driven preferences is significantly higher than that observed in FGMO ($p<0.001$). We summarize these findings as follows.

[16]We here focus on overall ambiguity preferences, determined by the two comparisons between risk and ambiguity. To make the two studies comparable, we infer that a subject exhibits an overall preference for risk over ambiguity if we observe $R \succeq E1^*$ and $R \succeq E100$ in our study, and $R \succeq E10$ and $R \succeq E1000$ in FGMO, with at least one strict preference relation and vice-versa. Additional analyses testing the robustness of our findings with different combinations are presented in Appendix A.

Table 4: Classification of Subjects Based on Ambiguity and Size Preferences

Part I: The experiment of FGMO

Size preferences	**Ambiguity preferences**			
	Prefer risk	Indifferent	Prefer ambiguity	**Total**
Prefer larger urns	57 (62%) [Smooth AA]	4 (4.3%)	5 (5.4%) [Complexity Seeking]	66
Indifferent	10 (10.9%) [Maxmin AA]	10 (10.9%) [SEU]	1 (1.1%) [Maxmin AS]	21
Prefer smaller urns	4 (4.3%) [Complexity Averse]	0 (0%)	1 (1.1%) [Smooth AS]	5
Total	71	14	7	92

Notes: Ambiguity preferences are based on $E10$ and $E1000$. The relative share of participants is indicated in brackets.

Part II: Our experiment

Size preferences	**Ambiguity preferences**			
	Prefer risk	Indifferent	Prefer ambiguity	**Total**
Prefer larger urns	23 (29.5%) [Smooth AA]	3 (3.8%)	8 (10.3%) [Complexity Seeking]	34
Indifferent	16 (20.5%) [Maxmin AA]	5 (6.4%) [SEU]	2 (2.6%) [Maxmin AS]	23
Prefer smaller urns	18 (23.1%) [Complexity Averse]	2 (2.6%)	1 (1.3%) [Smooth AS]	21
Total	57	10	11	78

Notes: Ambiguity preferences are based on $E1^*$ and $E100$. The relative share of participants is indicated in brackets.

Result 3: *The heterogeneity of preferences measured with Ellsberg urns can be explained by subject's attitudes towards both ambiguity and complexity that the situation entails. Whereas the degree of ambiguity plays a major role in general, the role of complexity is also found non-negligible when $E1$ is contrasted with large-sized urns.*

3.1.3 Discussion

Increasing the number of balls in an unknown Ellsberg urn (say from $E1$ to $E10$, $E100$, and $E1000$) can simultaneously decrease the degree of ambiguity of the situation (e.g., under the smooth ambiguity model) and increase its degree of complexity (e.g., based on the notion of more complex [cardinality]). In what precedes, we demonstrated that these characteristics of Ellsberg urns can be considered as distinct factors driving preferences over ambiguous urns. Reconsidering the data of FGMO, we observed that the degree of ambiguity that an Ellsberg-type situation entails is the main driver of observed choices in that study. In contrast, complexity emerges as an additional important driver in our experiment. We argue that the discrepancy between these results may stem from the presence of $E1$ in our experiment. Such an urn entails, at the same time, the minimum degree of complexity and the maximum degree of ambiguity among the urns considered. Whereas subjects tend to be averse to the increasing level of ambiguity when comparing other Ellsberg urns, they seem to be more affected by the simplicity of $E1^*$ when it is contrasted with other urns. The particularity of $E1^*$ is also present in the data set of Chew et al. (2017), which we discuss next.

3.2 Beyond the Ellsberg's paradigm

The implications of our characterization of model ambiguity have so far been discussed in the context of the standard Ellsberg's paradigm only. This setup presents a specific structure of model ambiguity from which the results might be difficult to extrapolate. In what follows, we investigate the relative role of attitudes towards ambiguity and complexity in a more general ambiguous setting by analyzing the rich data of Chew et al. (2017).

3.2.1 Chew et al.'s (2017) design

Chew et al. (2017, henceforth CMZ) proposed an elegant design in a framework going beyond the standard Ellsberg paradigm. Specifically, they consider three different (symmetric) forms of *partial ambiguity* using urns containing 100 red or black balls, but with constraints on their possible compositions. Specifically, by letting $n \in \{0, 1, ..., 50\}$, partial ambiguity may take the form of:

- *Interval ambiguity,* when the proportion of red (or black) balls is in the set $I_n = [\frac{50-n}{100}; \frac{50+n}{100}]$;
- *Disjoint ambiguity,* when the proportion of red (or black) balls is in the set $D_n = [\frac{0}{100}; \frac{n}{100}] \cup [\frac{100-n}{100}; \frac{100}{100}]$; and
- *Two-point ambiguity,* when the proportion of red (or black) balls is in the set $T_n = \left\{\frac{50-n}{100}, \frac{50+n}{100}\right\}$.

These situations span the space of possible urn compositions between the risky urn R, and the standard Ellsberg ambiguous urn $E100$. For the purpose of our study, we concentrate on the data from the first supplementary experiment of CMZ, which considers the cases $n \in \{0, 10, 20, 30, 40, 50\}$. This experiment offers a rich domain of model ambiguity situations, with varying degrees of ambiguity and complexity. A summary of the situations considered and their main characteristics is presented in Table 5. As can be observed, the degrees of complexity and ambiguity (as summarized by the number of potential models $|M|$ and the volatility index $\mho^2$, respectively) are, in turn, positively correlated under I_n ($r = 0.93$, $p = 0.006$), negatively correlated under D_n ($r = -0.99$, $p < 0.001$), and uncorrelated under T_n ($r = 0.46$, $p = 0.36$).

Table 5: Characteristics of the uncertain situations in Chew et al. (2017)

Uncertain situation	Set of models (M)	Number of models ($\|M\|$)	Volatility of probabilities[a] ($\mho^2$)
I_0	$\left\{\frac{50}{100}\right\}$	1	0.000
I_{10}	$\left\{\frac{40}{100}, \frac{41}{100}, \ldots, \frac{59}{100}, \frac{60}{100}\right\}$	21	0.004
I_{20}	$\left\{\frac{30}{100}, \frac{31}{100}, \ldots, \frac{69}{100}, \frac{70}{100}\right\}$	41	0.014
I_{30}	$\left\{\frac{20}{100}, \frac{21}{100}, \ldots, \frac{79}{100}, \frac{80}{100}\right\}$	61	0.031
I_{40}	$\left\{\frac{10}{100}, \frac{11}{100}, \ldots, \frac{89}{100}, \frac{90}{100}\right\}$	81	0.055
I_{50}	$\left\{\frac{0}{100}, \frac{1}{100}, \ldots, \frac{99}{100}, \frac{100}{100}\right\}$	101	0.085
D_0	$\left\{\frac{0}{100}, \frac{100}{100}\right\}$	2	0.250
D_{10}	$\left\{\frac{0}{100}, \ldots, \frac{10}{100}\right\} \cup \left\{\frac{90}{100}, \ldots, \frac{100}{100}\right\}$	22	0.204
D_{20}	$\left\{\frac{0}{100}, \ldots, \frac{20}{100}\right\} \cup \left\{\frac{80}{100}, \ldots, \frac{100}{100}\right\}$	42	0.164
D_{30}	$\left\{\frac{0}{100}, \ldots, \frac{30}{100}\right\} \cup \left\{\frac{70}{100}, \ldots, \frac{100}{100}\right\}$	62	0.131
D_{40}	$\left\{\frac{0}{100}, \ldots, \frac{40}{100}\right\} \cup \left\{\frac{60}{100}, \ldots, \frac{100}{100}\right\}$	82	0.104
D_{50}	$\left\{\frac{0}{100}, \frac{1}{100}, \ldots, \frac{99}{100}, \frac{100}{100}\right\}$	101	0.085
T_0	$\left\{\frac{50}{100}\right\}$	1	0.000
T_{10}	$\left\{\frac{40}{100}, \frac{60}{100}\right\}$	2	0.010
T_{20}	$\left\{\frac{30}{100}, \frac{70}{100}\right\}$	2	0.040
T_{30}	$\left\{\frac{20}{100}, \frac{80}{100}\right\}$	2	0.090
T_{40}	$\left\{\frac{10}{100}, \frac{90}{100}\right\}$	2	0.160
T_{50}	$\left\{\frac{0}{100}, \frac{100}{100}\right\}$	2	0.250

Notes: In an abuse of notations, we let each model $m_p \equiv x_p y$ belonging to M be fully characterized by its probability p. [a] Assuming a uniform prior probability measure μ.

In total, the experiment entails 15 bets on different urns, each of which pays 40 Singapore Dollars (correspond to about USD30). The certainty equivalent (CE) for each bet is elicited using a choice list containing 10 decision problems. In line with the analysis in CMZ, we use the switching point of each list as a proxy for the CE (see Appendices D and E in CMZ for further details).

3.2.2 Original observations of CMZ

The original analysis provided by CMZ (see their Appendix D) highlights three interesting behavioural patterns in relation to our conjectures. (1) The first pattern is a decreasing trend in the CEs of T_n when n increases, except a reversal at T_{50}. This pattern corroborates some of the observations previously made in the context of standard Ellsberg urns. In particular, because the situations T_n all present only two potential models, the decreasing trend in the CEs suggests an aversion to higher degrees of ambiguity in those situations. In addition, the reversal at T_{50} demonstrates further the special status of this situation, which presents the same properties as the situation $E1^*$ our experiment.[17] (2) The second pattern observed is a decreasing trend in the CEs of D_n when n increases. As an increase in n is associated with both a higher degree of complexity ($|M|$) and a lower degree of ambiguity ($\mho^2$), this pattern suggests a dominating effect of complexity aversion over ambiguity aversion. (3) Finally, the third pattern is a decreasing trend in the CEs of I_n as n increases, which can be due to an aversion to both more complex and/or more ambiguous situations.

3.2.3 Further analysis and results

Building on the original observations of Chew et al. (2017), we further test the effects of the degrees of ambiguity and complexity that different partial ambiguity situations entail. Our analysis focuses on explaining the heterogeneity in preferences that may depend on the two effects. Out of 106 subjects, 16 (15.1%) exhibit the same CEs in all situations in accordance with SEU. These subjects are neither affected by the ambiguity nor the complexity of the situation. For the rest of the subjects, we run a finite mixture regression with two latent class models as follows:

Latent class model 1 (Complexity):

$$CE_i = \beta_0 + \beta_1 \ln\left(|M_i|\right) + \varepsilon_i, \tag{12}$$

where $|M_i|$ is the cardinality of the set of models in situation i.

Latent class model 2 (Ambiguity):

$$CE_i = \alpha_0 + \alpha_1 \ln\left(\mho_i^2\right) + \varepsilon_i, \tag{13}$$

where $\mho_i^2$ is the volatility index in situation i.

We use natural logarithm of the indexes $|M_i|$ and $\mho_i^2$ to account for potential nonlinearities in the impact of those indexes.[18] Negative (positive) coefficients of those indexes indicate decreasing (increasing) CEs with respect to the degree of complexity and ambiguity, and thus aversion (seeking) towards ambiguity and/or complexity.

Table 6 presents the results of the regression. We find that the majority of non-SEU preferences (69%) is captured by the first latent class model, indicating

[17] Remark that T_{50} and $E1^*$ present exactly the same sets of potential models. These situations may be seen as presenting a minimal degree of complexity because their set M consists of degenerate distributions only. Note that, in general, situations T_n are ranked as being equally complex when using our definition of more complex [cardinality], but cannot be ordered by our definition of more complex [coarser].

[18] Note that this specification outperforms the specification without log-transformation based on both AIC and BIC scores.

Table 6: Finite mixture regression of switching points in Chew et al. 2017

	Class 1 (Complexity)	Class 2 (Ambiguity)
Degree of complexity ($\ln(\mho^2)$)	-0.245***	
Degree of ambiguity ($\ln(\|M\|)$)		-0.155*
Constant	5.831***	1.012***
Class prevalence	69%	31%

Notes: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

a prevailing role of complexity aversion. Turning to the second latent class model, we also find an aversion to increasing the degree of ambiguity. Overall, the main driver of preferences in the experiment of Chew et al. (2017) is thus an aversion towards the degree of complexity, whose affect is found to be more frequent and more pronounced than aversion towards the degree of ambiguity.

4 Concluding remarks

This paper proposes a characterization of ambiguous situations in terms of their degrees of ambiguity and complexity. To do so, we combine the notions of "more ambiguous" proposed, respectively, by Jewitt and Mukerji (2017) and Izhakian (2020) with two new definitions of "more complex". We argue that such a characterization can usefully uncover the observed heterogeneity in preferences under ambiguity. We derived testable predictions, which we confronted to different experimental data sets. We showed that the experiment of Filiz-Ozbay et al. (2021) using Ellsberg urns of different sizes suggests a predominant role of the degree of ambiguity. On the contrary, we found more evidence for the effect of complexity in our new experiment and a dominating role played by complexity in the experiment of Chew et al. (2017).

The results presented in this paper may have important implications for both descriptive and prescriptive research on ambiguity. First, although the mainstream descriptive research on ambiguity has clarified that ambiguity attitudes are essentially source dependent (Abdellaoui et al., 2011, see also Wakker, 2010 for a review), the contextual factors that determine source-dependent preferences have so far been understudied. Our characterization of ambiguous situations could thus enable tractable analyses of source preferences, which can ultimately help to better understand and predict them in different contexts. Second, understanding the underlying factors that determine ambiguity preferences is also crucial for the normative validity of ambiguity theories, which has been a debate in the literature (Gilboa et al., 2008, 2009, 2012). Whereas the families of ambiguity preferences that we consider in this paper (i.e., the smooth and maxim families) have mainly normative underpinnings, preferences due to complexity may potentially relate to cognitive limitations of decision-makers, and therefore leave little room for normative interpretations. Future studies on complexity and ambiguity with subject pools possessing different levels of cognitive abilities can clarify to what extend ambiguity aversion prevails as an inability to deal with complexity or is more in line with a specific treatment of probabilities that are not objectively given.

Appendix

A Further results

A.1 Ambiguity and size preferences in the new experiment

Ambiguity preferences Table A.1 presents the results of the comparisons R vs. $E1$ and R vs. $E1000$ in our experiment. As can be observed the preference for risk when measured using $E1$ is slightly lower than what was found using $E1^*$ (p=0.07), thus reinforcing Result 1. Interestingly, remark that the preference for risk measured with $E1000$ is less pronounced in our experiment than in FGMO (p=0.051). This difference can be due to a ratio bias, which may have increased the attractiveness of $E1000$ compared to R (which was constructed with 100 balls in our experiment).

Table A.1: Ambiguity preferences using $E1$ and $E1000$ in the new experiment ($N = 84$)

	R vs. $E1$	R vs. $E1000$
Prefer risk	32 (38.1%)	42 (50%)
Indifferent	42 (50%)	26 (31%)
Prefer ambiguity	10 (11.9%)	16 (19%)

Size preferences Table A.2 presents the three pairwise comparisons between $E1$, $E100$, and $E1000$ in our experiment. We observe that, for each of the comparisons, the majority of subjects is *not* indifferent to the size of the urns considered. Focusing on strict preferences, a preference for larger urns is more common than for smaller urns in the comparisons $E1$ vs. $E1000$ and $E100$ vs. $E1000$ (one-sample test of proportion, p=0.02 and p=0.04 respectively). The preference for the small urn in the comparison $E1$ vs. $E100$, although more common than the preference for the large urn, is not significant (one-sample test of proportion, p=0.385).

Table A.2: Size preferences in the new experiment ($N = 84$)

	$E1$ vs. $E100$	$E100$ vs. $E1000$	$E1$ vs. $E1000$
Prefer larger urn	29 (34.5%)	30 (35.7%)	39 (46.4%)
Indifferent	19 (22.6%)	38 (45.2%)	26 (31%)
Prefer smaller urn	36 (42.9%)	16 (19%)	19 (22.6%)

Table A.3 presents the results of the comparisons of $E1^*$ with $E1$ and $E100$, respectively. Recall that $E1^*$ has the same set of potential models as $E1$, but contains a different number of balls (i.e., 100 balls in $E1^*$ and 1 ball in $E1$). On the contrary, $E1^*$ has the same number of balls as $E100$, but is characterized by a different set of potential models. We observe that a large majority of subjects (70%) is indifferent between $E1^*$ and $E1$. In contrast, a majority of subjects exhibits a strict preference in the comparison $E1^*$ vs. $E100$. Nevertheless, as in the comparison $E1$ vs. $E100$, there is no significant preference for the large-sized urn (p=0.90).

Table A.3: Preferences for $E1^*$ in the new experiment ($N = 84$)

	$E1^*$ vs. $E1$	$E1^*$ vs. $E100$
Prefer $E1^*$	10 (11.9%)	34 (40.5%)
Indifferent	59 (70.2%)	17 (20.2%)
Prefer standard Ellsberg urn	15 (17.9%)	33 (39.3%)

A.2 Robustness results

Table A.4 replicates the classification analysis provided in the body of the paper using different measures of ambiguity preferences. We use the urns $E2$ and $E10$ in FGMO and $E1$ and $E100$ in our experiment. The results are similar to the ones presented in our main analysis. Specifically, smooth ambiguity aversion is the most common pattern in both experiments although its proportion is significantly lower in our experiment (p<0.001). The proportion of complexity-driven preferences is also more prevalent in our experiment than in FGMO (p<0.001).

Table A.4: Alternative classification of Subjects Based on Ambiguity and Size Preferences

Part I: The experiment of FGMO, based on $E2$ and $E10$

Size preferences	Ambiguity preferences			
	Prefer risk	Indifferent	Prefer ambiguity	**Total**
Prefer larger urns	63 (66.3%) [Smooth AA]	1 (1.1%)	6 (6.3%) [Complexity Seeking]	70
Indifferent	9 (9.5%) [Maxmin AA]	10 (10.5%) [SEU]	2 (2.1%) [Maxmin AS]	21
Prefer smaller urns	3 (3.2%) [Complexity Averse]	0 (0%)	1 (1.1%) [Smooth AS]	4
Total	75	11	9	95

Notes: Ambiguity preferences are based on $E2$ and $E10$. Classification categories reported in square brackets.

Part II: New experiment, based on $E1$ and $E100$

Size preferences	Ambiguity preferences			
	Prefer risk	Indifferent	Prefer ambiguity	**Total**
Prefer larger urns	21 (28.4%) [Smooth AA]	5 (6.8%)	7 (9.5%) [Complexity Seeking]	33
Indifferent	17 (23%) [Maxmin AA]	5 (6.8%) [SEU]	0 (0%) [Maxmin AS]	22
Prefer smaller urns	17 (23%) [Complexity Averse]	2 (2.7%)	0 (0%) [Smooth AS]	19
Total	55	12	7	74

Notes: Ambiguity preferences are based on $E1$ and $E100$. Classification categories reported in square brackets.

A.3 Analysis with transitive subjects

We here focus on transitive preferences among the three Ellsberg urns $E1$, $E100$, $E1000$ in our experiment. Table A.5 reports the results on size preferences for this restricted sample of subjects. A preference for larger urns remains the most common pattern. The proportion of preferences for small-sized urns is now 24%.

Table A.5: Size Preferences, New Experiment (restricted sample of transitive preferences, $N = 62$)

$E1000 \succeq E100 \succeq E1$	$E1000 \sim E100 \sim E1$	$E1000 \preceq E100 \preceq E1$	other
28 (45.2%)	8 (12.9%)	15 (24.2%)	11 (17.7%)

Table A.6 reports the results of the interaction between ambiguity and size preferences with the restricted sample of transitive subjects. We find that 40% of the subjects are classified as smooth ambiguity averse (which is closer but still less than what is observed in FGMO, $p<0.01$). Complexity-driven preferences amounts to a substantial 36%.

Table A.6: Classification of Subjects Based on Ambiguity and Size Preferences (restricted sample of transitive preferences)

Size preferences	**Ambiguity preferences**			
	Prefer risk	Indifferent	Prefer ambiguity	**Total**
Prefer larger urns	20 (40%) [Smooth AA]	2 (4%)	5 (10%) [Complexity Seeking]	27
Indifferent	4 (8%) [Maxmin AA]	3 (6%) [SEU]	1 (2%) [Maxmin AS]	8
Prefer smaller urns	13 (26%) [Complexity Averse]	2 (4%)	0 (0%) [Smooth AS]	15
Total	37	7	6	50

Notes: Ambiguity preferences are based on $E1^*$ and $E100$. Classification categories reported in square brackets.

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Online Appendix

S1 Experimental Design

We use a within-subject design to study individual choices under risk and Ellsberg ambiguity. The experiment entails betting on the color of a ball drawn from an urn in different situations. All situations entail a standard two-color Ellsberg (1961) setting. The experiment uses real monetary incentives.

S1.1 The choice situations

Subjects in our experiment are confronted with five different uncertain situations. These situations are represented by urns containing balls that can be either red or black. They are characterised as follows:

1. Risk (denoted R): the urn contains 50 red and 50 black balls;
2. Ellsberg's ambiguity with 1 ball (denoted $E1$): the urn contains 1 ball, which can be either red or black;
3. Ellsberg's ambiguity with 100 balls (denoted $E100$): the urn contains 100 balls, each of which can either be red or black;
4. Ellsberg's ambiguity with 1000 balls (denoted $E1000$): the urn contains 1000 balls, each of which can be either red or black.
5. Degenerate ambiguity (denoted $E1^*$): the urn contains 100 balls, but they are either *all* red or *all* black.

Urns R and $E100$ are the same as the ones described by Ellsberg (1961) in his original two-color problem. Urns $E1$ and $E1000$ are similar, in spirit, to $E100$ but contain, respectively, less and more balls in them. Finally, $E1^*$ is technically identical to $E1$ (i.e., they share the same set of potential probability models) but contain a different total number of balls.[19]

Subjects were asked to choose between bets on the color of a ball drawn from these urns. For each of the five uncertain situations, subjects were given the choice of the color on which to bet and were offered €15 for a correct bet and €0 otherwise. We elicited direct (strict) preferences between betting on one urn or the other using a random lottery pairs (RLP) design (see Harrison and Rutström, 2008).[20]

[19] In $E1$ and $E1^*$, the layer of risk is degenerate as the probabilities in each model are either 0 or 1. In spirit, $E1^*$ also corresponds to one of the situations used in Chew et al. (2017) to study the notion of *partial ambiguity*, which implicitly relies on the partial information available to pin down the potential probability models describing the phenomenon of interest (see also Berger, 2022). Specifically, in $E1^*$, the information available is that all the balls have the same color.

[20] In the context of another study, we also elicited the certainty equivalents (CEs) of each bet using

S1.2 Procedure

The experiment was run on computers at the Bocconi Experimental Laboratory for Social Sciences. In total, 84 university students participated to the experiment. Four sessions were organized with 19 to 24 subjects per session. Subjects were paid in cash at the end of the experiment. Average earnings were approximately €14.5, including a €5 participation fee. Each session lasted approximately one hour, including instructions and payment. The experiment started with the experimental instructions, examples of the stimuli, and related comprehension questions. Complete instructions are available in online Appendix S2.

Stimuli During the experiment, subjects faced five different uncertain situations, represented by the urns described in Section S1.1. All the urns were constructed before each session by an assistant, who was not present in the lab during the experiment. Thus, no one in the room (including the experimenters) had more information about the content of the urns than that described in the experimental instructions. The subjects were told that they would have the opportunity to look at the urns at the end of the experiment to verify the truthfulness of the instructions.

We presented the different uncertain situations two-by-two in a randomized sequence and asked subjects on which urn they prefer to bet . After the selection of one of the two urns, the same question was asked once more, but this time with a slightly increased amount (€15.10) for a correct bet on the urn that was not selected at the first stage. Subjects were then considered as strictly preferring one of the two urns if they chose it in both stages and as indifferent if they reversed their choice in the second stage.[21] In total, nine of the ten possible binary choices over the five urns were presented to the subjects, resulting in eighteen choice questions.[22] At the end of the experiment (and before the payment stage), subjects answered a short survey with a few socio-economic questions.

Payment and incentives Each subject received a €5 flat payment for taking part to the experiment. In addition, they were paid depending on one of the decisions they made in the experiment. A prior random incentive system was implemented to determine the choice question that was actually used for determining the subjects'

a price-list design. The order of the RLP and CE elicitations was randomized and no order effect was detected. For details, see Online Appendix S4.

[21]Note that if a subject was indeed indifferent between the two bets in the first place, any $\varepsilon > 0$ prize increase would lead to a reversal in the second stage. Although we cannot rule out the possibility of a strict, but low, preference for the urn initially chosen, which is then reversed by the additional prize in the second stage, we believe that the additional €0.10 prize is sufficiently small to distinguish strict preference from indifference.

[22]The binary choice between $E1^*$ and $E1000$ was not presented.

payment.[23] After all subjects answered all the questions, a ball was randomly drawn from the urn corresponding to the relevant choice question and each subject's decision in that question was implemented. Each subject was then paid the amount corresponding to her decision. See online Appendix S2 for more details.

S2 Experimental Instructions

Welcome page and examples

Welcome to this experiment!

You are participating in a study on decision making under uncertainty. The experiment will take approximately 60 minutes.

You will be confronted with different uncertain scenarios. Each uncertain scenario involves urns from which a ball is drawn randomly. In each scenario, you are asked to make choices between two options.

In each uncertain scenario, you will be the one who chooses the color of the ball on which to bet to potentially win a gain in €.

>>

There are 14 *uncertain scenarios* in total. In every uncertain scenario, the urns contain only red and black balls. There is **no** other color.

The urns were constructed before the experiment by one of the collaborators, who is not present in the room now.

The urns associated with different uncertain scenarios are (visibly) placed on the table in front of the room. The experimenters also have no information about their compositions except the descriptions provided to you in the instructions. You will be invited to check them at the end of the experiment.

Next, you will see some examples of uncertain scenarios.

>>

[23]Under this random incentive system, the randomization is performed before subjects begin answering questions (Johnson et al., 2020). Such a *prior* incentive system aims to enhance isolation to minimize potential biases, thereby preventing subjects from hedging over the randomization between problems (see Baillon et al., 2022; for a demonstration of its incentive compatibility in Ellsberg-type experiments, and Epstein and Halevy, 2019; for a recent application).

Example 1

There are **100** balls in an urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.

A ball will be randomly drawn from the urn and its color will be observed.

Suppose you have chosen black as the color associated with winning €15.

The following list contains 16 choice questions. On each line, you are asked to make a decision between **Option 1** (on the left) and **Option 2** (on the right). **Option 1** gives you €15 if the ball drawn is black and €0 otherwise. **Option 2** gives you a sure amount of money ranging between €15 and €0.

Notice that **Option 2** gives you sure amounts that are decreasing as you move down the table whereas **Option 1** is the same throughout. For example, **Option 2** gives you a sure €15 on line 1, and €0 for sure on line 16.

You indicate your decision on each line by marking the circle next to the option.

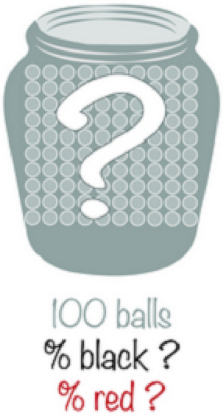
Option 1			Option 2
Have €15 if the ball drawn is black	○	○	Have €15 for sure
Have €15 if the ball drawn is black	○	○	Have €14 for sure
Have €15 if the ball drawn is black	○	○	Have €13 for sure
Have €15 if the ball drawn is black	○	○	Have €12 for sure
Have €15 if the ball drawn is black	○	○	Have €11 for sure
Have €15 if the ball drawn is black	○	○	Have €10 for sure
Have €15 if the ball drawn is black	○	○	Have €9 for sure
Have €15 if the ball drawn is black	○	○	Have €8 for sure
Have €15 if the ball drawn is black	○	○	Have €7 for sure
Have €15 if the ball drawn is black	○	○	Have €6 for sure
Have €15 if the ball drawn is black	○	○	Have €5 for sure
Have €15 if the ball drawn is black	○	○	Have €4 for sure
Have €15 if the ball drawn is black	○	○	Have €3 for sure
Have €15 if the ball drawn is black	○	○	Have €2 for sure
Have €15 if the ball drawn is black	○	○	Have €1 for sure
Have €15 if the ball drawn is black	○	○	Have €0 for sure

>>

Below, you are asked some comprehension questions testing your understanding of Example 1. Please indicate your answers.

Example 1

Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is unknown.



Q. Please indicate whether the following sentence is true or false for Example 1.

The number of **red balls** in the urn may be any number between 0 and 100.

○ True ○ False

It is possible that the urn is composed of only **red balls**.

○ True ○ False

It is possible that the urn is composed of only **black balls**.

○ True ○ False

It is possible that the urn is composed of an equal proportion of **red and black balls**.

○ True ○ False

The urn might contain a ball of a color other than red and black.

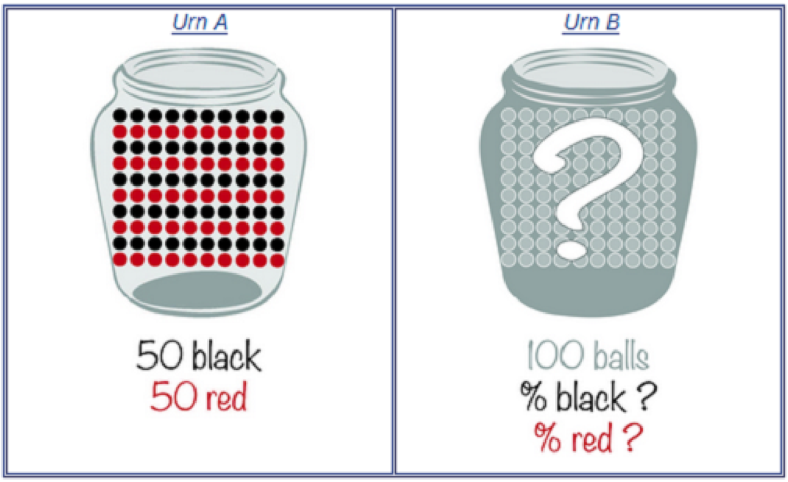
○ True ○ False

>>

Example 2

There are two urns with balls that can either be red or black.

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. The proportion of red and black balls is **unknown**.



A ball will be randomly drawn from each of the two urns and its color will be observed.

You are asked to place a bet on the color of the ball drawn from one of the two urns. If your bet is correct, you will receive an amount of money in €. If it is not correct, nothing happens.

Suppose you have chosen black as the color associated with winning a gain in €.

The following line contains 1 choice question. You are asked to make a decision between 2 options.
Option 1 (on the left) gives you €15 if the ball drawn from **Urn A** is black and €0 otherwise.
Option 2 (on the right) gives you €15 if the ball drawn from **Urn B** is black and €0 otherwise.

You indicate your decision by marking the circle below the option.

On which urn do you want to bet?

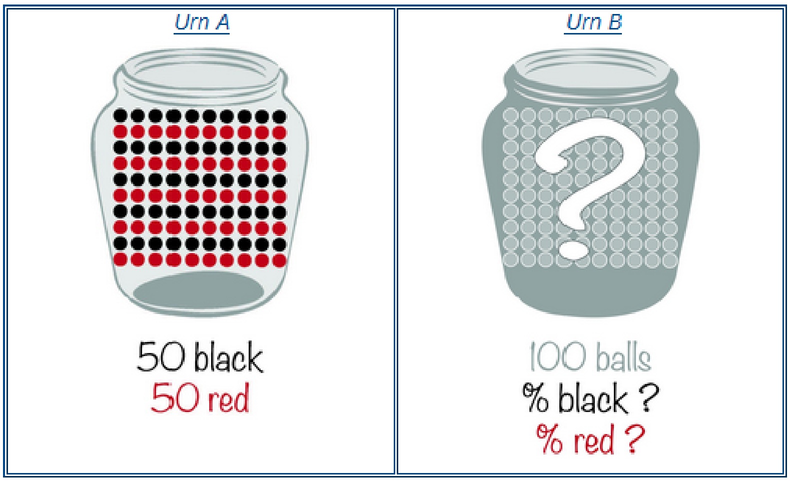
Option 1	Option 2
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Below, you are asked some comprehension questions testing your understanding of Example 2. Please indicate your answers.

Example 2

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



Q. Please indicate whether the following sentence is true or false for Example 2.

It is possible that Urn A and Urn B have exactly the same composition with **50% black** and **50% red balls.**

○ True ○ False

It is possible that there are more **red balls** in Urn B than in Urn A.

○ True ○ False

It is possible that there are more **black balls** in Urn B than in Urn A.

○ True ○ False

It is possible that there are more **black balls** and more **red balls** in Urn B than in Urn A.

○ True ○ False

Urn B might contain a ball of a color other than red and black.

○ True ○ False

>>

Before starting the incentivized part of the experiment, we clarify how subjects were going to be paid.

Your payment

Each of you will receive €5 for your participation.
In addition, you will also be paid based on one of your decisions in the experiment.

Which decision will matter for your payment?

During this experiment, you will make decisions in 14 different uncertain scenarios (of type I and II).
Type I sceanrios: in 5 scenarios, you will face a list of 16 choice questions (as in the tables we showed you in Example 1). Thus, there are 80 choice questions in these scenarios in total.
Type II sceanrios: in 9 scenarios, you will face 2 choice questions (as the one we showed you in Example 2). Thus, there are 18 choice questions in these scenarios in total.
In total, there are thus 98 choice questions.

One of the 98 choice questions will be picked in the following manner: We have 98 sealed envelopes each containing one of the 98 choices you will face in the 14 uncertain scenarios. We will randomly pick one envelope. The uncertain scenario and the choice question there contained, as well as your corresponding recorded decision in that choice question, will determine your payment.

The content of the envelope picked will be revealed to you only at the end of the experiment. Hence, each of the 98 choice questions has an equal chance of being picked. Therefore, in every choice question it is in your best interest to make your decisions whilst keeping in mind that it could be the one determining your payment at the end.

How will the payments be undertaken?

At the end of the experiment:

1. The picked envelope containing the uncertain scenario and the choice questions is opened.
2. The urn(s) in the relevant uncertain scenario is/are selected. A ball is drawn from the urn(s).
3. Your recorded decision in the relevant choice question is observed. Your payment is given based on the option you chose.

>>

Payment example 1

For example, suppose the envelope contains the following scenario and choice question:

Uncertain Scenario X, Choice Question 1

There are **100** balls in an urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.

A ball will be randomly drawn from the urn and its color will be observed.

Suppose you have chosen black as the color associated with winning €15.

Option 1			Option 2
Have €15 if the ball drawn is black	◉	○	Have €15 for sure

Furthermore, suppose you chose Option 1. Then, you are paid €15 if the color of the ball drawn from the urn is black and 0€ otherwise.

In the case you chose Option 2, you would have received €15 for sure.

>>

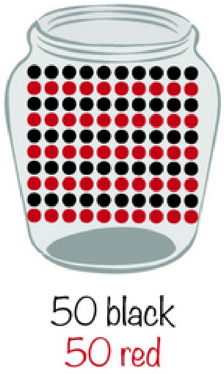
Payment example 2

For example, suppose the envelope contains the following scenario and choice question:

Uncertain Scenario Y, Choice Question 1

There are two urns with balls that can either be red or black.

- Urn A is composed of **50 Red** and **50 Black** balls.
- Urn B contains **100** balls. The proportion of red and black balls is **unknown**.

Urn A	*Urn B*
	

A ball will be randomly drawn from one of the two urns and its color will be observed.
Suppose you have chosen black as the color associated with winning a gain in €.

Option 1	Option 2
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	⊙

Furthermore, suppose you chose Option 2. Then, you are paid €15 if the color of the ball drawn from Urn B is black and 0€ otherwise.

In the case you chose Option 1, you would have received €15 if the color of the ball drawn from Urn A were black and 0€ otherwise.

>>

Every subject made decisions in 14 different scenarios in total (9 scenarios taking the form of random lottery pairs and 5 scenarios taking the form of certainty equivalents). The order in which the type of scenarios appears is randomized.

Random lottery pairs

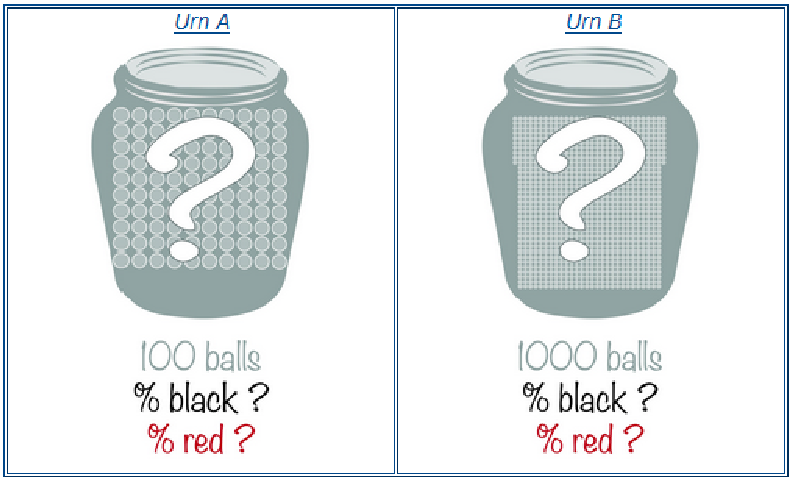
Each of the 9 scenarios presents two choice questions, presented in a sequence. The order in which the scenario appear is randomized.

Question 1

Uncertain scenario θ

There are two urns with balls that can either be red or black.

- *Urn A contains **100** balls. The proportion of red and black balls is **unknown**.*
- *Urn B contains **1000** balls. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ○ I would like to receive money if the ball drawn is black
- ○ I would like to receive money if the ball drawn is red

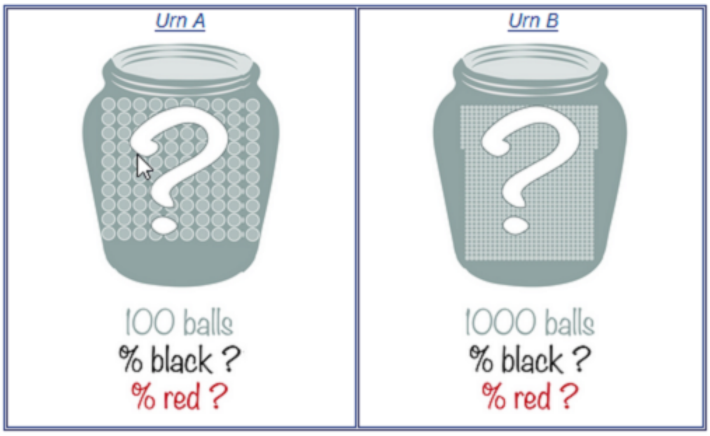
You will receive €15 if your bet is correct.
On which urn do you want to bet?

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Question 2 If Urn A is chosen first:

- Urn A contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.
- Urn B contains **1000** balls. Each of the 1000 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



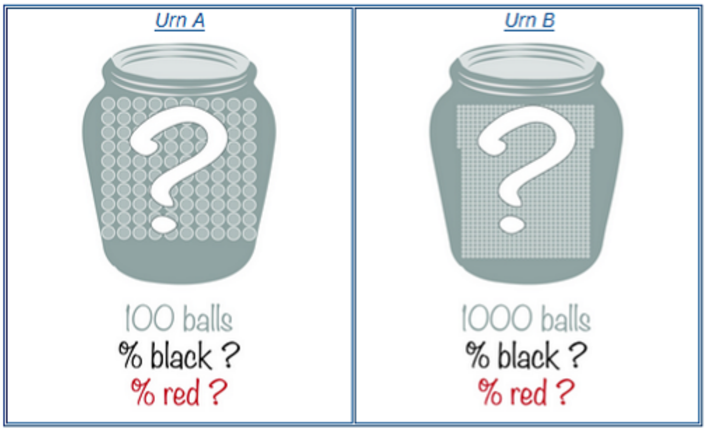
You chose to bet on Urn A.
Now, we ask you to reconsider your choice if a correct bet on Urn B gives a slightly higher gain.

On which urn do you want to bet:

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15.10 if my bet is correct
○	○

Question 2 If Urn B is chosen first:

- Urn A contains **100** balls. Each of the 100 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.
- Urn B contains **1000** balls. Each of the 1000 balls is either **black** or **red**. The proportion of red and black balls is **unknown**.



You chose to bet on Urn B.
Now, we ask you to reconsider your choice if a correct bet on Urn A gives a slightly higher gain.

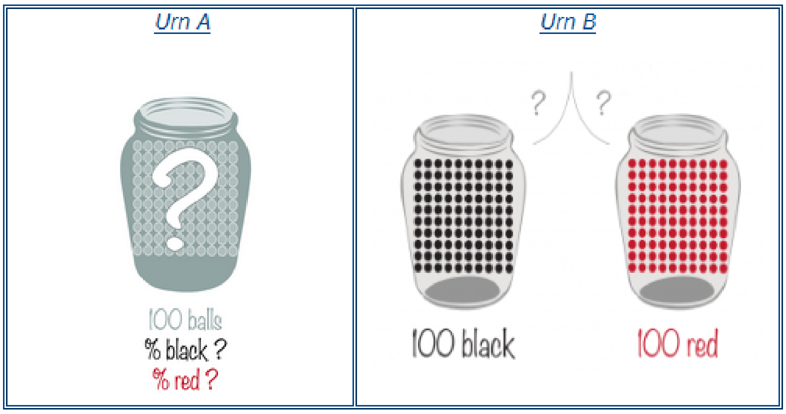
On which urn do you want to bet:

I would like to bet on Urn A, and receive €15.10 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

Uncertain scenario λ

There are two urns with balls that can either be red or black.

- *Urn A contains **100** balls. The proportion of red and black balls is **unknown**.*
- *Urn B contains **100** balls. The balls are either **all black** or **all red.** The exact composition is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ○ I would like to receive money if the ball drawn is black
- ○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

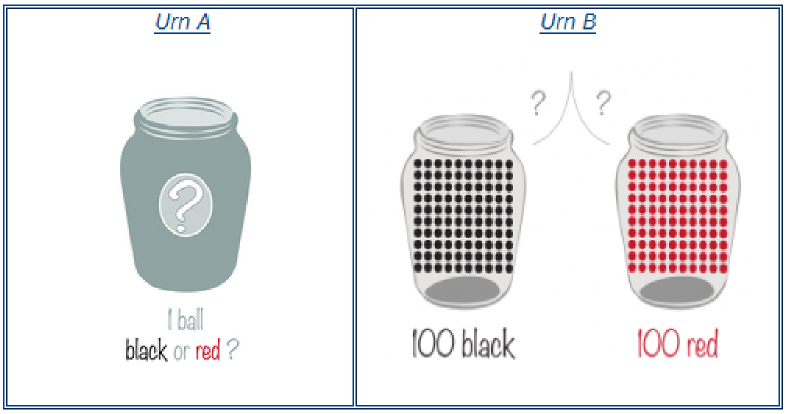
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario ζ

There are two urns with balls that can either be red or black.

- *Urn A contains **1** ball. The color of the ball is **unknown**.*
- *Urn B contains **100** balls. The balls are either **all black** or **all red.** The exact composition is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

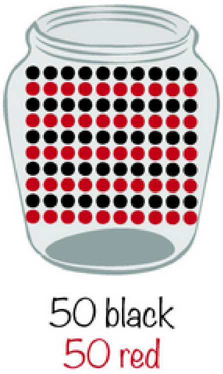
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario α

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Red** and **50 Black** balls.*
- *Urn B contains **1** ball. The color of the ball is **unknown**.*

Urn A	*Urn B*
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

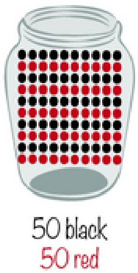
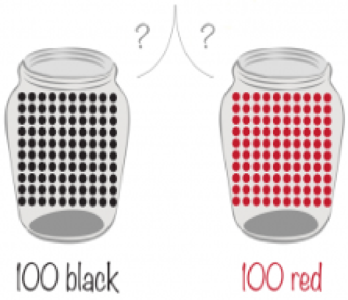
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario 5

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Black** and **50 Red** balls.*
- *Urn B contains **100** balls. The balls are either **all Black** or **all Red.** The exact composition is **unknown**.*

Urn A	*Urn B*
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

- ◯ I would like to receive money if the ball drawn is black
- ◯ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

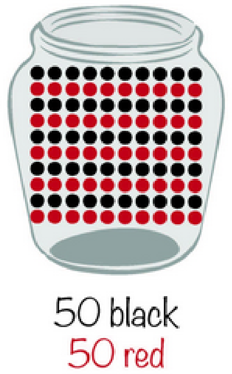
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
◯	◯

>>

Uncertain scenario β

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Red** and **50 Black** balls.*
- *Urn B contains **100** balls. The proportion of red and black balls is **unknown**.*

Urn A	*Urn B*
	

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

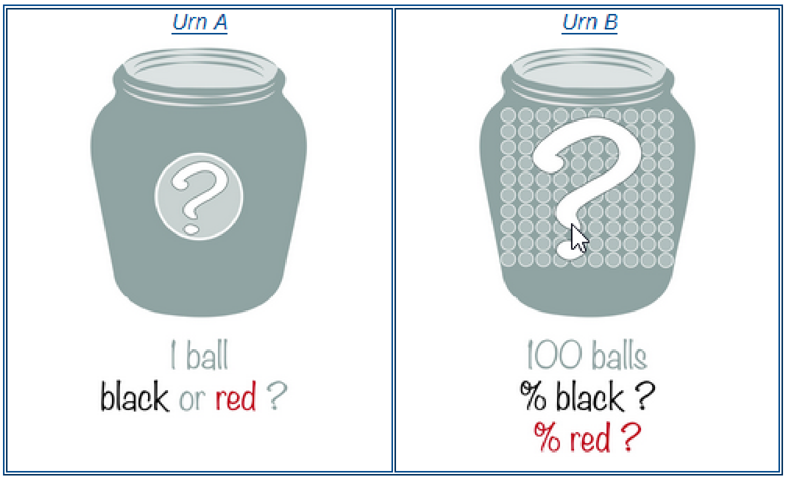
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario η

There are two urns with balls that can either be red or black.

- Urn A contains **1** ball. The color of the ball is **unknown**.
- Urn B contains **100** balls. The proportion of red and black balls is **unknown**.



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

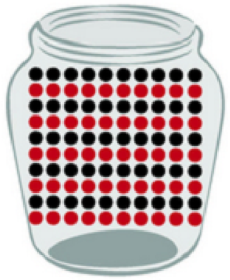
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario γ

There are two urns with balls that can either be red or black.

- *Urn A is composed of **50 Black** and **50 Red** balls.*
- *Urn B contains **1000** balls. The proportion of red and black balls is **unknown**.*

Urn A	*Urn B*
	
50 black 50 red	1000 balls % black ? % red ?

A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

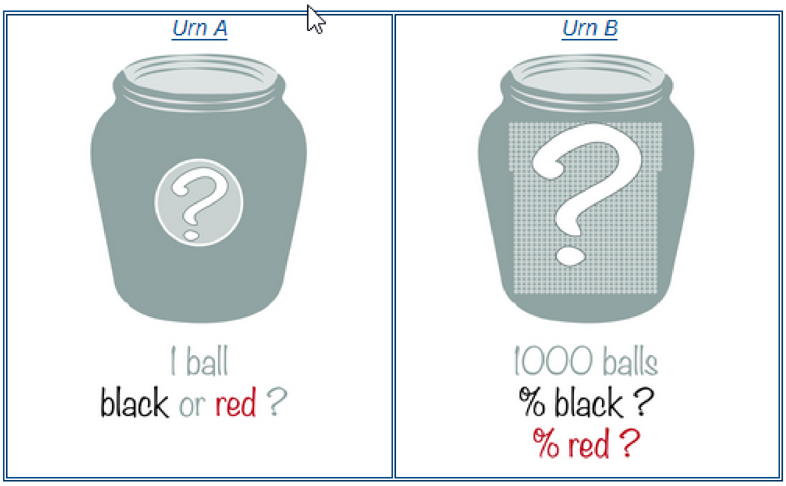
I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Uncertain scenario κ

There are two urns with balls that can either be red or black.

- *Urn A contains **1** ball. The color of the ball is **unknown**.*
- *Urn B contains **1000** balls. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from each of the two urns and its color will be observed.
You are asked to choose an urn and place a bet on the color of this ball. If your bet is correct, you will receive money. If it is not correct, nothing happens.

Please select your winning color:

○ I would like to receive money if the ball drawn is black

○ I would like to receive money if the ball drawn is red

You will receive €15 if your bet is correct.
On which urn do you want to bet?

I would like to bet on Urn A, and receive €15 if my bet is correct	I would like to bet on Urn B, and receive €15 if my bet is correct
○	○

>>

Certainty equivalents

Each of the 5 certainty equivalent scenarios is presented on two pages. Subjects first choose their winning color and then indicate their decisions in the choice list. The order in which the scenarios appear is randomized.

Page 1

Uncertain scenario u

*There is **1** ball in the urn. This ball is either black or red. The color of the ball is **unknown**.*

The ball will be drawn from the urn and its color will be observed.

Please select your winning color:

- ○ I would like to have €15 if the ball drawn is black
- ○ I would like to have €15 if the ball drawn is red

>>

Page 2

The ball in the urn is either black or red. The color of the ball is unknown.

Please indicate your decision on each line by marking the circle next to the option.

Option 1			Option 2
Have €15 if the ball drawn is black	○	○	Have €15 for sure
Have €15 if the ball drawn is black	○	○	Have €14 for sure
Have €15 if the ball drawn is black	○	○	Have €13 for sure
Have €15 if the ball drawn is black	○	○	Have €12 for sure
Have €15 if the ball drawn is black	○	○	Have €11 for sure
Have €15 if the ball drawn is black	○	○	Have €10 for sure
Have €15 if the ball drawn is black	○	○	Have €9 for sure
Have €15 if the ball drawn is black	○	○	Have €8 for sure
Have €15 if the ball drawn is black	○	○	Have €7 for sure
Have €15 if the ball drawn is black	○	○	Have €6 for sure
Have €15 if the ball drawn is black	○	○	Have €5 for sure
Have €15 if the ball drawn is black	○	○	Have €4 for sure
Have €15 if the ball drawn is black	○	○	Have €3 for sure
Have €15 if the ball drawn is black	○	○	Have €2 for sure
Have €15 if the ball drawn is black	○	○	Have €1 for sure
Have €15 if the ball drawn is black	○	○	Have €0 for sure

>>

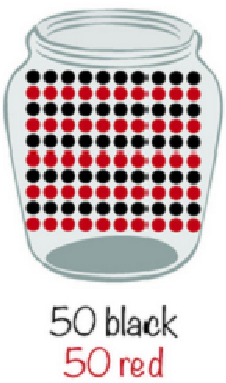
Uncertain scenario $\mathcal{E}$

*There are **100** balls in the urn. Each of the 100 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.*

A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario ρ

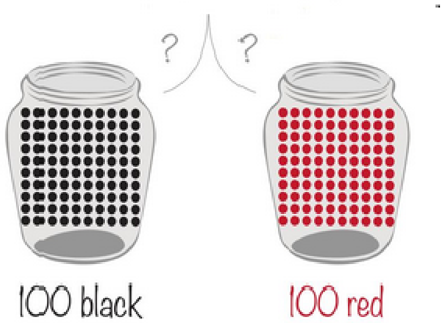
*The urn is composed of **50 Red** and **50 Black** balls.*



A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario σ

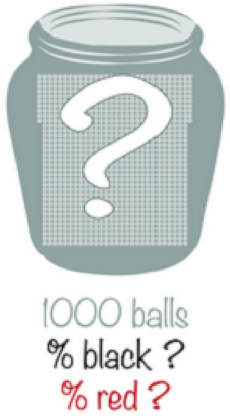
*There are **100** balls in the urn. The 100 balls in the urn are either **all black** or **all red.** The exact composition is **unknown**.*



A ball will be randomly drawn from the urn and its color will be observed.

Uncertain scenario μ

*There are **1000** balls in the urn. Each of the 1000 balls in the urn is either black or red. The proportion of red and black balls is **unknown**.*



A ball will be randomly drawn from the urn and its color will be observed.

S3 Order Effects

In Table S3.1, we report the results of the Fisher exact tests comparing the choices made when the CE elicitation part appeared first and when the binary choices appeared first. None of the tests is significant at 5%, suggesting that choices made are not associated with the order of the task appearance.

Table S3.1: ATTITUDES MEASURED WITH CERTAINTY EQUIVALENTS

Pairwise comparisons	R vs. $E1$	R vs. $E100$	R vs. $E1000$	R vs. $E1^*$	$E1$ vs. $E100$	$E1$ vs. $E1000$	$E100$ vs. $E1000$	$E1$ vs. $E1^*$	$E100$ vs. $E1^*$
p-value	0.413	0.152	0.461	0.296	0.443	0.070	0.109	0.942	0.276

S4 Certainty Equivalents data

A second set of questions enables us to elicit the certainty equivalents (CEs) of the five uncertain situations. We used the following price-list design: for each uncertain situation, subjects made 16 binary choices between the prospect of receiving €15 if their bet was correct and receiving a sure amount of money ranging from €15 to €0 (with a decrease of €1 between each choice). Subjects were expected to choose the sure amount when it was higher than the CE of the uncertain situation and to switch to preferring the bet as the sure amount decreased to the point that is lower than the CE for the situation. We then use the switch point to compute the CE: it corresponds to the midpoint of the indifference interval implied by the switch point in that situation. Switching in the middle of the list implies a CE equal to the expected payoff (€7.5). The order of the uncertain situations R, $D100$, $E100$, $E1$, and $E1000$ was randomized.

Table **??** presents ambiguity attitudes measured with the CEs. Comparing the results with those obtained in Table **??** , we note that the share of ambiguity neutral in Part I (two sample proportion tests, ambiguity attitudes: measured with $E1$, p=0.391; measured with $E100$, p=0.015; measured with $E1000$, p=0.044) and indifferent in Part II is higher (two sample proportion tests, size preferences: between $E1$ and $E100$, p<0.001; between $E1$ and $E1000$, p=0.030; between $E100$ and $E1000$, p=0.015). It can be explained by the lack of precision of CE (that goes in step of €1) and does not allow to detect weak preferences.

Table S4.1: Ambiguity Attitudes measured with Certainty Equivalents

	Size of the ambiguous urn		
	$E1$ ($N = 83$)	$E100$ ($N = 83$)	$E1000$ ($N = 83$)
Ambiguity Aversion	26 (31.3%)	35 (42.2%)	33 (41.25%)
Ambiguity Neutrality	47 (56.6%)	41 (49.4%)	37 (46.25%)
Ambiguity Seeking	10 (12.1%)	7 (8.4%)	10 (12.5%)

Table S4.2: Size Preferences with Certainty Equivalents

	Size of the urns		
	$E1$ vs. $E100$ ($N = 83$)	$E1$ vs. $E1000$ ($N = 80$)	$E100$ vs. $E1000$ ($N = 81$)
Prefer Larger Urn	19 (22.9%)	21 (26.5%)	18 (22.2%)
Indifferent	41 (49.4%)	38 (47.5%)	52 (64.2%)
Prefer Smaller Urn	23 (27.7%)	21 (26.5%)	11 (13.6%)