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Abstract

This study offers a monetary growth theory that can explain the declining business dynamism observed over the past few decades in some developed countries by developing a new R&D-based growth model; the main departure from existing models is the introduction of an entry cost after innovation and an endogenous survival investment that is subject to a cash-in-advance constraint. Due to this, in our model, the entry/exit rates and firm age distribution are all endogenous. The core finding is that, theoretically, the nature of business dynamism at the macro level essentially depends on nominal factors. Specifically, lower inflation leads to declining business dynamism, characterized by lower entry and exit rates and a maturity bias in the firm age distribution, if the entry cost is sufficiently high. Empirically, we also find supportive evidence that, among a set of European countries, firm entry/exit rates are positively associated with nominal interest rates and inflation rates. Then, calibrating the model to the E.U. economy, we verify that lower inflation leads to declining business dynamism under empirically plausible values of entry, exit, and inflation rates.

JEL classification: E40, O31, O34, O41
Keywords: Cash-in-advance constraint, inflation, firm dynamics, R&D-based growth

1 Introduction

Most developed economies, including the U.S. and Europe, have experienced declining business dynamism—the process by which firms enter, exit, and survive.[1] Some scholars and policy makers are concerned that this downtrend, marked by lower entry rates and more older firms, may harm stable long-term economic growth.[2] Although recent literature has proposed explanations (e.g., Akcigit and Ates, 2021), there is no consensus and declining business dynamism remains puzzling. In this study, we explain declining business dynamism as an endogenous equilibrium phenomenon driven by nominal factors such as inflation. We focus on two key aspects of business dynamism—firm entry and exit rates—from the ten stylized facts documented by Akcigit and Ates (2021).

To this end, we propose a new monetary growth theory that captures how inflation affects firm dynamics at the macro level. Specifically, we build on Romer's (1990) research-and-development (R&D)-based growth model with expanding varieties. Following Chu and Cozzi (2014), we impose a cash-in-advance (CIA) constraint on R&D investments to incorporate money demand. Our key innovation relative to existing models is adding a CIA constraint on firms' survival investments to explain declining business dynamism. Our model has two key departures. First, to examine the entry stage, we introduce an entry cost after firms successfully innovate new products. Second, to create an endogenous firm age distribution, we introduce firm investments for market survival to delay the obsolescence of their innovated goods. Following the above-mentioned empirical evidence, we assume that survival investments also face a CIA constraint (Musso and Schiavo, 2008).[3] These two features make entry/exit rates and firm age distribution endogenous.

Our core finding is that business dynamism at the macro level depends on nominal or inflationary factors. Specifically, lower inflation (which determines the opportunity cost

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[1]See Calvino, Criscuolo, and Verlhac (2020), who show that many countries face declining business dynamism.

[2]For other characteristics, see Akcigit and Ates (2021), who document ten stylized facts relating to declining business dynamism. We focus on two crucial characteristics from the ten.

[3]Evidence shows that survival investments face financial constraints. More broadly, financing intangible assets like R&D is sensitive to cash constraints due to their firm-specific and inalienable nature. For reviews of the empirical literature, see Chen (2014) and Morikawa (2015).

of cash holdings) lowers the costs for R&D and survival investments. However, while the direct effect of lower inflation on both investments is positive, it ambiguously changes the relative profitability between the two, creating uncertain effects on innovation and survival investments. Our analysis identifies the role of entry cost in determining how inflation affects innovation and survival. When entry costs exceed a certain threshold, lower inflation discourages entry but encourages survival, reducing both entry and exit rates. This creates a maturity bias in the firm age distribution. Hence, if entries are more costly, lower inflation-or a deflationary trend-leads to declining business dynamism. Calibrating our model to E.U. data, we quantitatively confirm that nominal factors drive declining business dynamism under reasonable parameters.

Further, we prove analytically that the decline in growth rate could coincide with the decline in business dynamism. However, in our calibrated model to E.U. data, growth rate tends to be higher under declining business dynamism, resulting in dynamic welfare improvement. This is because the growth-enhancing effect of a lower exit rate dominates the growth-depressing effect of a lower entry rate in the calibrated model.

We also test our predictions concerning firm entry/exit using European cross-country data. We examine whether nominal interest and inflation rates correlate positively with firm entry/exit rates, using data from Eurostat, OECD, IMF, and the World Bank. Using generalized method of moments (GMM), we find that firm entry/exit rates are positively associated with nominal interest and inflation rates. Given the declining trends of nominal interest rates and inflation rates among the sampled countries, these results are consistent with the theoretical predictions about declining business dynamism.

As explained above, we explore how nominal factors affect declining business dynamism in R&D-based growth. This contributes to theoretical research on mechanisms behind declining business dynamism. For example, Akcigit and Ates (2021) attribute declining business dynamism to rising markups and market concentration; Akcigit and Ates (2023) point to declining knowledge diffusion; De Loecker et al. (2021) offer similar explanations in an oligopolistic model. While existing studies focus on real market mechanisms, we contribute to this line of research by emphasizing nominal factors

(lower nominal interest or inflation rates) as fundamental causes of declining business dynamism.[4]

Little research examines how nominal factors affect business dynamism. Among them, Oikawa and Ueda (2018) and Miyakawa et al. (2022) examine nominal factors in firm dynamics under R&D-based growth but differ from our study in three ways. First and foremost, they focus on optimal monetary policy, while we offer an endogenous explanation of the observed declining business dynamism. Second, we model the explicit role of money differently. Specifically, they incorporate menu costs to show how monetary policy affects resource allocation, while we impose CIA constraints on R&D investment.[5] Third, the source of firm heterogeneity also differs. Oikawa and Ueda (2018) model price heterogeneity via menu costs; Miyakawa et al. (2022) incorporate multiproduct firms with quality updates and ex-ante quality distribution à la Klette and Kortum (2004) and Lentz and Mortensen (2008). In contrast, we model firm heterogeneity in terms of age and size through endogenous survival against obsolescence.

We now review the literature on growth models with CIA constraints. Following Tobin's (1965) study of inflation and real investment, first, Stockman (1981) and Abel (1985) incorporated CIA constraints into neoclassical growth models.[6] Due to the breakthrough with the development of endogenous growth theory,[7] Chu and Lai (2013) and Chu and Cozzi (2014) applied CIA constraints to R&D (not physical) investment by incorporating the evidence that R&D faces severe financial constraints (Hall, 2008).[8] This literature has since examined how nominal interest rates or inflation affect innovation, growth, unemployment, and inequality (Chu et al. 2015, 2017, 2019a, 2019b, 2021; He and Zou 2016; He 2018; Hori 2020; Huang et al. 2021; Zheng et al. 2021). We comple-

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[4]See also Haruyama (2025), who shows declining business dynamism can explain increasing trends in income inequality in a new Schumpeterian growth model *without* nominal elements.

[5]Several studies examine inflation, innovation, and growth in different contexts; see, e.g., Arawatari et al. (2018) and Zheng et al. (2020). We complement their pioneering analysis by explicitly considering endogenous firm dynamics in the presence of survival activity and identifying lower nominal interest or slower nominal growth as a cause of declining business dynamism.

[6]Lucas (1980) pioneered this approach, following Clower (1967).

[7]Romer (1990) first developed the R&D-based growth model with an expanding variety of goods. Meanwhile, Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) developed the Schumpeterian growth model with quality improvement.

[8]For a comprehensive review, see Chu (2022a).

ment these studies by examining CIA constraints on both firm entry and survival, linking lower nominal interest/inflation rates to declining business dynamism. Unlike prior studies, we endogenize survival activity, exit rates, and age/size distributions based on CIA severity.

Next, we elaborate more on existing studies of R&D firms' survival activity. In modeling a firm's dynamic optimization for survival in the context of R&D-based growth models, we follow Dinopoulos and Syropoulos (2007) and Eicher and García-Peñalosa (2008), extended by Grieben and Sener (2009) and Davis and Sener (2012). Akiyama and Furukawa (2009), Furukawa (2013), Furukawa and Yano (2014), and Niwa (2018) also studied endogenous survival in variety expansion models. We contribute to this literature stream by linking endogenous firm survival to the inflation-growth relationship, which has long been one of the most important topics in macroeconomics.

The paper proceeds as follows. Section 2 develops our model. Section 3 examines the effects of inflation on the nature of business dynamism. Section 4 presents empirical evidence from Europe. Section 5 calibrates the model to E.U. data and provides the quantitative results. Section 6 concludes.

2   A Monetary R&D-Based Growth Model with Endogenous Business Dynamism

2.1   Consumption

We consider a variety expansion model of endogenous growth by referencing Romer (1990) and Rivera-Batiz and Romer (1991). In this model, time is continuous and extends from 0 to ∞. There is a single final good, taken as numeraire. An infinitely lived representative consumer supplies inelastically one unit of labor and consumes c_t units of final goods at each time point, t. The utility function is:

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where $\rho > 0$ is the subjective discount rate.

Following Chu and Cozzi (2014), money is introduced by assuming a CIA constraint on R&D firms' investment in invention; in addition, an extra CIA constraint is introduced for the firms' investment in market survival as our original departure. The asset accumulation constraint in real terms is thus given as:

$$\dot{a}_t + \dot{m}_t = r_t a_t + i_t b_t + w_t L + \tau_t - c_t - \pi_t m_t, \tag{2}$$

where a_t denotes the real value of financial assets, m_t the real value of cash holdings, i_t the nominal interest rate, $r_t \equiv i_t - \pi_t$ the real interest rate, $b_t \leq m_t$ the real-term amount of cash borrowed by firms, w_t the real wage rate, τ_t the real value of transfers from the government, and $\pi_t \equiv \dot{P}_t/P_t$ the inflation rate. By solving the standard dynamic optimization, we obtain the following Euler equation:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

2.2 Production

The market for a final good is perfectly competitive. The final-good firm combines labor with a constant-elasticity-of-substitution (CES) composite of intermediates:

$$Y_t = L_t^{1-\alpha} \int_0^{N_t} x_t(j)^\alpha dj, \tag{4}$$

where L_t is the amount of labor input, N_t the size or measure of intermediate inputs, $x_t(j)$ the amount of intermediate input j used, and $\alpha \in (0, 1)$ the factor share of intermediate inputs. The production technology is Cobb–Douglas in labor and the CES aggregator over intermediates. Solving profit maximization, one can show that the price elasticity of demand for any good j is $1/(1 - \alpha)$ in equilibrium.

The market for differentiated intermediates is monopolistically competitive. A monopolistic firm manufactures each input j. We assume that manufacturing one unit of good j requires one unit of final good as input; the marginal cost is equal to unity. De-

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note as $p_t(j)$ the real price of intermediate good j. Given that the price elasticity facing the monopolist is $1/(1-\alpha)$, monopolistic pricing yields $p_t(j) = 1/\alpha$. We can then obtain the equilibrium supply and profit as follows:

$$x_t(j) = \alpha^{2/(1-\alpha)} \equiv x \quad \text{and} \quad \Pi_t(j) = \alpha^{\frac{1+\alpha}{1-\alpha}} (1 - \alpha) \equiv \Pi. \tag{5}$$

where the labor market clearing condition, $L_t = 1$, is taken into account.[9]

2.3 Endogenous R&D and Entry

Following Romer (1990) and Rivera-Batiz and Romer (1991), we consider a linear R&D technology under free entry. For each potential R&D firm, inventing a new variety of intermediate goods at date t requires investing $1/\kappa$ units of final goods at date t. Parameter $\kappa > 0$ is a productivity parameter.

To introduce an explicit role of nominal factors, we follow Chu and Cozzi (2014) in assuming that R&D investments face a CIA constraint. Specifically, in paying $1/\kappa$ for R&D, a firm has to prepare ζ/κ units of cash in advance. To do so, the R&D firm has to borrow from households because it is a startup company with no cash. Parameter $\zeta \in [0, 1]$ represents the severity of the financial constraint for startup firms trying to innovate and then entering the market.[10]

Further, to consider the R&D firm's life cycle as an endogenous phenomenon in a more fully-fledged manner, we incorporate a fixed cost for entry, departing from Chu and Cozzi (2014).[11] Specifically, we consider that after a successful invention, the firm needs to pay $\varpi/\kappa > 0$ units of the final good for entry (to build a factory or, more generally, startup production).[12] Denoting as v_t the real value of an innovation, formally defined

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[9]Appendix A provides detailed derivations.

[10]Some can interpret ζ as an inverse measure of the quality of a payment system or a cash market; see Yano (2019) for a new theory of money based on the market quality economics proposed by Yano (2009).

[11]This is more in line with Chu et al. (2017), who identify the critical role of entry cost in explaining the nonmonotonic effects of inflation on innovation and growth. On the contrary, we focus on identifying a relationship between inflation and R&D firm dynamics.

[12]Since the R&D firm can pay an entry cost to final goods firms from its cash holding (i.e., no need to borrow cash), we consider that the entry cost in our model is free from the nominal interest rate i even

as an expected present value of present and future profits,[13] the net payoff from an R&D investment at date t can be defined as:

$$v_t - \varpi/\kappa - (1 + \zeta i_t)(1/\kappa). \tag{6}$$

2.4 Endogenous Survival and Exit

To examine the role of inflation in business dynamism, in addition to the introduction of an entry cost, we further depart from Chu and Cozzi (2014) by endogenizing the exit rate for R&D firms, which is a novel approach in the literature.[14] We thus assume that each innovation (or the relevant R&D firm) faces a risk of obsolescence following a Poisson process of hazard rate δ_t. Therefore, a firm would invest resources in survival by marketing, advertisement, or protecting intellectual property rights. In describing the dynamic process of firm survival, we assume that, when the firm invests z_t/κ units of a final good, the hazard rate for obsolescence is $\delta_t \equiv \overline{\delta} - z_t^\gamma$ where $\gamma \in (0,1)$. Here, $\overline{\delta}$ gives the natural upper bound for the exit rate. One may interpret $\overline{\delta}$ as a natural exit rate because $\delta_t = \overline{\delta}$ if there is no survival activity.

There are two critical considerations for the survival of innovation. The first is that, when making a survival investment, the firm also faces a CIA constraint: to facilitate the payment of z_t/κ, it has to prepare $\xi z_t/\kappa$ amount of cash by borrowing from consumers, where $\xi \in [0,1]$; this is our core departure from the existing models and is supported empirically as discussed in the introduction.

Another critical factor is that, different from R&D investment, the existing firm already earns profit Π_t and thereby holds some cash. We assume that the firm can use a fraction $(1 - \theta)\Pi_t$ of this profit for survival investment at each date t, where $\theta \in [0,1]$ is

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if it could face a CIA constraint; see (6) below. However, we extend the baseline model in Appendix B to be more analytically symmetric by allowing the firm to borrow cash. There, we verify that the conditions for our main propositions (i.e., a sufficiently high entry cost ϖ) do not change essentially.

[13]See Appendix A for details of the within-period behavior of the R&D firm's payments along the time axis.

[14]The novelty for the growth literature on endogenous survival of R&D firms (Dinopoulos and Syropoulos, 2007, and Eicher and García-Peñalosa, 2008) is incorporating the money demand via the CIA constraint on R&D investments.

a dividend (payout) ratio.[15] If internal funds are insufficient to cover the CIA-required cash $\xi z_t / \kappa$, that is, $(1 - \theta)\Pi_t < \xi z_t / \kappa$, the firm borrows the shortfall at the nominal interest rate i_t. We label this case by $\omega = 1$. In the otherwise case with $(1 - \theta)\Pi_t \geq \xi z_t / \kappa$, labelled by $\omega = 0$, the firm need not borrow cash. To capture this distinction, we define an indicator such that $\lambda^\omega = 1$ if $\omega = 1$ (the firm borrows to meet the CIA constraint), and $\lambda^\omega = 0$ if $\omega = 0$ (the firm self-finances).[16] Combining these two cases, the total payment for the survival investment can be written as:

$$I_t \equiv \frac{z_t}{\kappa} + i_t \max \left\{ \frac{\xi z_t}{\kappa} - (1 - \theta) \Pi_t, 0 \right\}. \tag{7}$$

Now, we can express the dynamic optimization for existing firms as:[17]

$$\max_{z_t: \text{ s.t. } \delta_t \equiv \bar{\bar{\delta}} - (z_t)^\gamma} W_t dt \equiv (\Pi - I_t) \, dt + (1 - \delta_t dt) \, \dot{v}_t dt - (\delta_t dt - \eta dt) v_t, \tag{8}$$

where W_t denotes an expected net benefit for the firm at date t. We introduce $\eta > 0$ as an exogenous growth factor to ensure positive growth (and then have an endogenous evolution of firm size distribution), since our model is even more restrictive than the standard endogenous growth models, given the presence of borrowing and survival investment. In having η in (8) and (13), we follow Anderlini et al. (2013).[18] Intuitively, this reflects that firms in reality sometimes invent new products "by accident" through usual production activities. Without any intended investment or effort, firms can create new ideas by accident, or even by mistake, as a byproduct of regular activities (in our case, production or survival); see, for example, Middendorf (1981) for more details on this innovation type, i.e., "invention by accident." In our model, such an accident occurs

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[15]One can consider ratio θ to be endogenously determined through any maximization. In our model, $\theta = 0$ is optimal. Therefore, we consider θ to be exogenous because, in reality, firms do not typically invest all profit into a single plan.

[16]We implicitly assume that firms have to pay out the remainder of their available net internal reserves for survival investment $(1 - \theta) \Pi_t - \lambda^\omega \xi z_t / \kappa$ if it exists.

[17]Appendix A provides a more detailed explanation of this dynamic optimization. See Furukawa (2013) and Niwa (2018) for a discrete-time version of a similar setup.

[18]In the literature, such exogenous growth factors are often considered for a deeper understanding of the role of technological progress in various phenomena. For instance, see Lucas and Moll (2014), Benhabib et al. (2021), and Yano and Furukawa (2023). Since our model also includes endogenous growth factors of innovation and survival, it is closer to Anderlini et al. (2013), who considered both endogenous and exogenous growth factors.

with a Poisson arrival rate of $\eta > 0$. Further, the expected value for invention by accident is $(\eta dt) v_t$ during a short time interval. When this happens, the existing firm innovates another good and obtains an additional value of v_t (i.e., $2v_t$ in total).[19] Solving the standard concave programming in (8),[20] we obtain the optimal survival investment and exit rate as follows:

$$z_t^* = \left(\frac{\kappa \gamma v_t}{1 + \lambda^\omega \xi i} \right)^{\frac{1}{1-\gamma}} \text{ and } \delta_t^* = \overline{\delta} - \left(\frac{\kappa \gamma v_t}{1 + \lambda^\omega \xi i} \right)^{\frac{\gamma}{1-\gamma}}. \tag{9}$$

2.5 Monetary Authority

Following the literature (Chu and Cozzi 2014), the monetary authority exogenously sets $i_t = i$ as a stationary policy instrument. The Fisher equation is $i = \pi_t + r_t$. Denoting the aggregate *nominal* money balance as M_t, its growth rate is given by:

$$\mu_t \equiv \frac{\dot{M}_t}{M_t} = \pi_t + \frac{\dot{m}_t}{m_t} = i - r_t + \frac{\dot{m}_t}{m_t} = i - \rho - \frac{\dot{c}_t}{c_t} + \frac{\dot{m}_t}{m_t}, \tag{10}$$

where the last equality uses the Euler equation (3). Given a stationary nominal interest rate, i, real consumption c_t and aggregate real money balance m_t grow at the same rate on a balanced growth path, $\mu = i - \rho$. Then, the monetary authority returns the seigniorage revenue as a lump-sum transfer $\tau_t = \dot{m}_t + \pi_t m_t$. See Chu and Cozzi (2014) for details.

2.6 Dynamic General Equilibrium

Here, we characterize the equilibrium dynamics for the aggregate economy under four conditions under $i_t = i$ as a stationary monetary policy lever. First, we suppose free entry to the R&D market, which requires a usual zero-profit condition. From (6), it must hold

$$v_t = \frac{1 + \varpi + \zeta i}{\kappa} \tag{11}$$

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[19]For simplicity, we assume that the firm sells the ownership right for the new "invention by accident" to a randomly chosen firm from a pool of potential manufacturing firms the representative consumer owns.

[20]As a usual argument, we can ignore term $(dt)^2$.

in an equilibrium where new inventions occur in equilibrium. In our analysis, we may ignore a trivial case with no R&D investment. As shown in (11), the higher the nominal interest i, the higher the equilibrium value v_t of each innovation to cover a higher R&D cost. Naturally, this positive effect of i is magnified as the CIA constraint is severer (i.e., higher ζ).

Second, given that the benefit of owning a bond of price v_t over the small time interval of dt is $(r_t dt)v_t$, the standard Bellman equation is:

$$r_t v_t = W_t = (1 + \lambda^\omega i (1 - \theta)) \Pi - \frac{1 + \lambda^\omega \xi i}{\kappa} z_t^* + \dot{v}_t - (\delta_t^* - \eta)v_t, \tag{12}$$

from using (5), (7), (8), and (9). Note that the right-hand side of (12) can be strictly positive if, say, the profit Π is sufficiently large. The economic understanding behind (12) is as follows. If the firm with market value v_t starts producing the invented good, its net benefit equals W_t. If it sells the ownership shares and buys a bond of value v_t, it can earn an interest revenue of $r_t v_t$. Since holding the shares of the firm and holding a bond of the same value, v_t, must be indifferent in no-arbitrage equilibrium, the return from the firm shares, W_t, is equal to that from the bond, $r_t v_t$, in the equilibrium.

Third, the firm dynamics at the aggregate level can be summarized by the following differential equation:

$$\dot{N}_t = \varepsilon_t N_t - \delta_t^* N_t + \eta N_t, \tag{13}$$

in which ε_t represents an entry rate at date t, defined as a ratio between the flow of new inventions (entries) and the total number of existing good N_t.

Fourth, the final good market equilibrium condition is:

$$Y_t = c_t + N_t x_t + \left(\frac{1 + \varpi}{\kappa}\right) \varepsilon_t N_t + \frac{z_t^*}{\kappa} N_t, \tag{14}$$

where the supply of final goods Y_t is equated to the demand resulting from consumption c_t, production $N_t x_t$, invention $(1 + \varpi) \varepsilon_t N_t / \kappa$, and survival $z_t^* N_t / \kappa$.

With these four equilibrium conditions, we will derive an equilibrium dynamic system

for our model. For this goal, it is first beneficial to show that the survival investment and exit rate are time-invariant in temporary equilibrium.

Lemma 1 *For any $t \geq 0$, the equilibrium survival investment and exit rate are given by:*

$$\underbrace{z_t^* = \left(\frac{\gamma\left(1 + \varpi + \zeta i\right)}{1 + \lambda^\omega \xi i}\right)^{\frac{1}{1-\gamma}}}_{\equiv z^*} \text{ and } \delta_t^* = \overline{\delta} - \underbrace{\left(\frac{\gamma\left(1 + \varpi + \zeta i\right)}{1 + \lambda^\omega \xi i}\right)^{\frac{\gamma}{1-\gamma}}}_{\equiv \delta^*}. \tag{15}$$

Proof. Substituting (11) into (9) yields (15). ∎

From the Euler equation (3), with (11), (12), and (15), we obtain the law of motion for consumption c_t as

$$\frac{\dot{c}_t}{c_t} = \underbrace{\frac{1 + \lambda^\omega i\left(1 - \theta\right)}{1 + \varpi + \zeta i}\kappa\Pi - \frac{1 + \lambda^\omega \xi i}{1 + \varpi + \zeta i}z^* - \left(\delta^* + \rho - \eta\right)}_{\equiv g^*}. \tag{16}$$

The firm dynamics in number follow

$$\frac{\dot{N}_t}{N_t} = \varepsilon_t - \delta^* + \eta, \tag{17}$$

which straightforwardly comes from (13) and (15). Note that, in general, the entry and exit rates do not coincide. We then solve (14) for entry rate ε_t:

$$\varepsilon_t = \frac{1}{1 + \varpi}\left(\frac{1 + \alpha}{\alpha}\kappa\Pi - \kappa\frac{c_t}{N_t} - z^*\right) \equiv \varepsilon_t^* \tag{18}$$

which uses (4), (5), and (15).

We define $\left(\theta_0, \overline{\delta}_0, \kappa_0\right)$ as the threshold values of $\left(\theta, \overline{\delta}, \kappa\right)$.[21] Then, we have the following lemma on the uniqueness and global stability of a balanced growth path, which allows us to restrict our analysis to a nontrivial equilibrium with borrowing for survival.

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[21]See Appendix C for the explicit expressions of $\left(\theta_0, \overline{\delta}_0, \kappa_0\right)$, in which $\theta_0 < 1$ as long as $\xi > 0$.

Lemma 2 *Assume that $\theta > \theta_0$, $\overline{\delta} > \overline{\delta}_0$, and $\kappa > \kappa_0$ hold. Then, the economy immediately jumps onto a unique balanced-growth path at $t = 0$ and stays there permanently. On a balanced growth path, the existing firms borrow cash ($\omega = 1$): the growth rate is*

$$g^* = \frac{1 + i\,(1 - \theta)}{1 + \varpi + \zeta i}\kappa\Pi + \gamma^{\frac{\gamma}{1-\gamma}}(1 - \gamma)\left(\frac{1 + \varpi + \zeta i}{1 + \xi i}\right)^{\frac{\gamma}{1-\gamma}} - (\overline{\delta} + \rho - \eta) > 0; \qquad (19)$$

and the firm entry rate is

$$\varepsilon^* \equiv g^* + \delta^* - \eta. \qquad (20)$$

Proof. See Appendix C. ∎

The two Lemmas, 1 and 2, completely characterize the equilibrium behavior of R&D firms and, thereby, the dynamic general equilibrium of our model. In the subsequent section, from these two theoretical results, we will draw various economic insights concerning the relationship between inflation and firm dynamics at the macro level, imposing the assumptions given in Lemma 2. Since our essential departure from the existing models is to introduce another CIA constraint on survival—in addition to the standard CIA constraint on invention à la Chu and Cozzi (2014)—we may proceed with Lemma 2 to ignore a trivial equilibrium in which incumbent firms do not borrow cash for survival, which is essentially the same as the equilibrium of the Chu and Cozzi model.

3   Inflation, Business Dynamism, and Growth

This section identifies the effects of a nominal interest rate, i, as a stationary policy instrument on business dynamism at and economic growth the macro level. As mentioned in the Introduction, we focus on the following particular aspects of business dynamism: a firm's exit rate, entry rate, and its average age.

Before proceeding, it is beneficial to verify the positive relationship between the interest rate i, as a policy instrument, and the inflation rate π. Following Chu and Cozzi

(2014) and subsequent studies, we have considered that the monetary authority exogenously sets the nominal interest rate at $i_t = i$ as a stationary policy instrument. Then, the inflation rate π_t is endogenously determined through the Fisher equation; $\pi_t = i - r_t = i - (g(i) + \rho)$, in which the second equality follows from the Euler equation, (3). Therefore, the relationship between nominal interest i and inflation π is positive as long as $dg(i)/di < 1$, which holds, for example, if the entry cost ϖ is sufficiently high.[22] Note that such a positive long-run relationship between i and π is supported by empirical studies such as those of Mishkin (1992) and Booth and Ciner (2001). Given this property, a positive (negative) effect of increasing the nominal interest rate, i, on some endogenous variable is accompanied by a positive (negative) relationship between the inflation rate, π, and the variable, which is not a causal relationship as in Chu and Cozzi (2014) and others.

3.1 Entry and Exit

We first intuitively explain the mechanism through which a higher nominal interest rate, i, affects business dynamism. A higher nominal interest, i, has two opposite effects on survival and firm exit. As a direct effect, a higher i hurts both R&D and survival investment because it increases the interest payment (total cost) of these investments due to the CIA constraint with severities ζ and ξ, respectively. This discourages both investment in innovation and survival, decreasing firm entry and increasing firm exit. However, whether the negative effect on entry is stronger or weaker than that on exit is ambiguous. On the one hand, if financial constraints are more stringent for startup firms engaging in R&D than existing firms investing in market survival ($\zeta > \xi$), a higher i tends to cause more damage to firms investing in R&D for entry. On the other hand, if the net benefit, $v - \varpi/\kappa$, of R&D is very low, R&D investment is originally very small; thus, a marginal damage by an increase in i on R&D is not so significant. Consequently, if $v_t - \varpi/\kappa$ is very low (or if the entry cost ϖ is very high), an increase in i tends to cause more damage to incumbent firms' survival investment. This implies that the effect of a

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[22]See Appendix C.

higher i on the exit rate δ^* depends on the size of an entry cost.

The following formally characterizes the effect of a higher i on firm exit δ^*.

Proposition 1 (Nominal Interest and Firm Exit) *If the entry cost is high such that* $\varpi \geq \zeta/\xi - 1$, *a higher nominal interest rate,* i, *leads to a smaller investment in survival,* z^*, *and, thereby, a higher exit rate of firms,* δ^*, *in equilibrium.*

Proof. It is straightforward from (15). See Appendix C for a proof of the positive relationship between π and δ^*. ∎

Proposition 1 shows that with a higher entry cost ϖ,[23] the relationship between the nominal interest i and the exit rate δ^* is positive. As mentioned, a higher entry cost, ϖ, implies a lower net benefit, $v - \varpi/\kappa$, of innovation. Thus, when ϖ is higher, R&D investment is originally very small; an increase in the higher nominal interest rate, i, causes insignificant damage on R&D. The damage to survival is more vital, which leads to a shift of final goods as a production resource from survival to R&D. This explains why the high entry cost is a condition under a higher nominal interest, discouraging survival investment, leads to a higher exit rate.

Next, we examine the effects on firm entry. Intuitively, as a direct effect, a higher interest rate, i, discourages R&D investment for firm entry, which, in turn, has a negative effect on the entry rate, ε^*. However, as mentioned above, this entry-discouraging effect of higher i on R&D can be insignificant and the damage on survival investment is vital if the net benefit, $v_t - \varpi/\kappa$, of R&D is very small. This has an indirect positive effect on R&D because it causes a resource shift from survival to R&D. Therefore, the effect of higher i on entry can be positive or negative, depending on some parameters including the size of an entry cost, ϖ. The following formally proves this.

Proposition 2 (Nominal Interest and Firm Entry) *If the entry cost is high such that* $\varpi \geq \zeta/\min\{\xi, 1-\theta\} - 1$, *a higher nominal interest,* i, *leads to a higher firm entry rate,* ε^*, *in equilibrium.*

---

[23]We implicitly assume that start-up firms face a more stringent CIA constraint, i.e., $\zeta > \xi$, which alligns with empirical evidence (Hall 2008). Otherwise, if $\xi \geq \zeta$, the condition $\varpi \geq \zeta/\xi - 1$ always holds, i.e., the relationship between i and δ^* is always positive. Therefore, whether $\xi > \zeta$ or not, Proposition 1 holds; however, we should restrict our analysis to a more realistic case of $\zeta > \xi$.

Proof. See Appendix C. ∎

Propositions 1 and 2 reveal that if the entry cost ϖ is relatively high, a lower nominal interest rate, i, can be a cause of declining business dynamism (in the sense of lowering entry/exit rates). Before proceeding, we should explore the relationship between inflation and business dynamism. As explained above, the inflation rate is endogenously determined by the Fisher equation and the Euler equation: $\pi(i) = i - (g(i) + \rho)$. The relationship between i and π is positive if $dg(i)/di < 1$. Since this positive relationship holds under the condition required for the unique existence of well-defined equilibrium shown in Lemma 2,[24] we can state that: *If the entry cost is sufficiently high, inflation and business dynamism also exhibits a positive relationship, implying low inflation is accompanied by declining business dynamism.* Notably, this is consistent with data; see Section 4 for an empirical analysis with the evidence supporting our theoretical findings.

3.2 Firm Age Distribution

To see another aspect of business dynamism, we next formally verify that the higher entry and exit rates caused by a higher interest rate i (Propositions 1 and 2) lead to a smaller share of old firms, noting that a larger share of old firms is the third factor of business dynamism. In doing this, we first derive the density function of firm age. To do so, we denote a firm's birth date as $b \geq 0$. Then, firms with b have age $t - b$. Additionally, we denote the number of firms of age $t - b$ as $n_t(t - b)$. Keeping b constant, the law of motion for the evolution of $n_t(t - b)$ is:

$$\dot{n}_t(t - b) = -\delta^* n_t(t - b) \tag{21}$$

with initial condition $n_b(0) = \varepsilon^* N_b + \eta N_b = (g^* + \delta^*) N_b = (g^* + \delta^*) (N_0 e^{g^* b})$. Deriving a particular solution to this differential equation from (21), we have $n_t(t - b) = (g^* + \delta^*) N_b e^{-\delta^*(t-b)}$. Dividing both sides by $N_t = N_0 e^{g^* t}$ yields a density function for

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[24] See Appendix C for a formal proof.

firms of age $t - b \equiv a > 0$:

$$f_t(a) \equiv \frac{n_t(a)}{N_t} = (\varepsilon^* + \eta) \, e^{-(g^* + \delta^*)a}, \tag{22}$$

which is free from t, since firm age, a, is fixed. Consistent with empirical evidence,[25] the firm age distribution in our economy obeys an exponential function. We can therefore show that the average firm age is $1/(g^* + \delta^*) = 1/(\varepsilon^* + \eta)$.

The inspection of (22) reveals how a higher nominal interest rate, i affects the firm age distribution. In the case with a higher entry cost ϖ, a higher i leads to an increase both entries and exits (Propositions 1 and 2). This implies that more young firms are more likely to enter the market while older firms are more likely to exit. Therefore, a higher i tends to decrease the average firm age. Given the positive relationship between i and π, a lower inflation rate π, or a deflationary trend, implies a maturity bias in the firm age distribution, which is one of the characteristics of declining business dynamism.

3.3 Growth and Welfare

As we considered in the introduction, there is a growing concern that declining business dynamism can be associated with slower growth. Therefore, we then explore the macroeconomic consequences to economic growth and welfare. In the following analysis, we retain the assumptions in Propositions 1 and 2, i.e., $\varpi \geq \zeta / \min\{\xi, 1 - \theta\} - 1$.

Seeing (19), an increase in the nominal interest i can positively or negatively affect the equilibrium growth rate g^*. When a lower i leads to lower entry and exit rates, ε^* and δ^*, as in Propositions 1 and 2, it has two opposite effects on g^*. On the one hand, a lower entry rate, ε^*, slows the growth of firm number N_t, which implies a lower equilibrium growth rate, g^*. On the other hand, a lower exit rate, δ^*, mitigates the slowing growth of firm number N_t due lowering ε^*, which implies a higher equilibrium growth rate, g^*. This is why the growth effect of the nominal interest rate i is ambiguous.

The following proposition formally characterizes the effect of i on g^*.

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[25]See, for example, Coad (2010).

Proposition 3 *Under a larger entry cost such that $\varpi \geq \zeta / \min\{\xi, 1 - \theta\} - 1$, a lower nominal interest rate, i, leads to a lower economic growth rate, g^*, in equilibrium if*

$$\kappa > \frac{1}{\Pi} \frac{\xi (1 + \varpi) - \zeta}{(1 - \theta)(1 + \varpi) - \zeta} \left(\frac{\gamma (1 + \varpi + \zeta i)}{1 + \xi i} \right)^{\frac{1}{1 - \gamma}}. \tag{23}$$

Proof. From (19), we can show that $dg^*/di > 0$ holds if and only if (23), implying the effect of higher i on g^* is positive as κ is sufficiently high. This proves the relationship between lower i and lower g^*. ∎

Proposition 3 suggests that in economies with more productive R&D firms, a lower nominal interest rate, i, may lead to slower economic growth. Under the assumption of a large entry cost, the more efficient the R&D technology, the more likely it is that declining business dynamism and slower growth simultaneously take place caused by a lower nominal interest rate. Applying a similar argument that follows Proposition 2 concerning the effect of i on the equilibrium inflation rate, inflation and economic growth would also exhibit a positive relationship. This implies lower inflation accompanied by slower growth, which is in line with our consideration in the introduction.

From (1), the equilibrium social welfare is:

$$U^* = \frac{1}{\rho} \left(\ln c_0 + \frac{g^*}{\rho} \right), \tag{24}$$

where $c_0 \equiv \frac{1+\alpha}{\alpha} \Pi - \frac{1}{\kappa} \left(z^* + (g^* + \delta^* - \eta)(1 + \varpi) \right)$. In some existing models including Chu and Cozzi (2014), the effects of higher i on U^* can be are always negative, in which the Friedman rule holds that $i = 0$ is optimal. However, since an increase in the nominal interest rate i may lead to a higher growth rate g^* in our model, the effect on the equilibrium welfare U^* is potentially ambiguous. Since it is difficult to identify the effect on initial consumption level c_0 analytically, we will explore the welfare effects quantitatively in Section 5.

In this section, we have revealed the crucial role of an entry cost in determining the effects of a higher interest rate on business dynamism and growth using the model we develop in Section 2. In summary, if the entry cost is larger than the cutoff level, a

lower interest rate causes declining business dynamism (i.e., lower entry/exit rates and a higher average firm age). If the R&D productivity is also larger than the cutoff, it further discourages economic growth (i.e., a lower economic growth rate).

In ending this section, we should note a critical caveat on our assumption that firm size grows exogenously with "invention by accident." Further, firms are homogeneous in their productivity. Nevertheless, in reality, most firms voluntarily invest to control their size and productivity. In addition, if inferior firms in size and productivity tended to survive and remain in the market, the downward trend of entry and exit could depress aggregate growth more than before. Therefore, one would develop a richer model by considering endogenous firm size and productivity. Although this direction to deepen the analysis is interesting, we will leave a full-fledged analysis of endogenous firm size and productivity for future research.[26]

4   Cross-country Evidence

In Section 3, we show that if the entry cost is sufficiently high, both nominal interest rate and inflation rate exhibit positive relationships with firm entry/exit rates. This section presents some cross-country evidence to support these theoretical predictions. Note that while cross-country nominal interest rate and inflation data are available from, for instance, the International Monetary Fund and the World Bank respectively, corresponding data for firm entry/exit rates are available only from Eurostat for some European countries. Due to data limitation, our empirical analysis below uses a relatively selective sample of European countries.

Importantly, we would like to emphasize that the main purpose of the regression analysis is about showing evidence consistent with the model predictions rather than claiming causality. Before we proceed, we briefly discuss the related empirical literature. To the best of our knowledge, there is no prior empirical research documenting cross-

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[26]See Appendix B for an off-hand extension with in-house innovation and endogenous η. Such a minor extension does not alter the effects of higher i on entry/exit rates; however, the growth effect is more likely to be negative as in standard models.

country relationships (correlational or causal) between firm entry/exit rates and nominal inflation rate or interest rate. One reason is that cross-country data on firm entry/exit rates are quite limited. Some studies in the literature focus on the U.S. where quarterly data on firm entry and exit over a long time-horizon are available. For instance, Colciago and Silverstrini (2022), Hamano and Zanetti (2022), and Hartwig and Lieberknecht (2025) study the impact of monetary policy on firm dynamics in the U.S. under the vector autoregression (VAR) framework. Colciago and Silverstrini (2022) find that after an unexpected monetary easing, firm entry increases initially but undershoots in the long run, while Hamano and Zanetti (2022) find that a contractionary policy could prevent firm entry. Hartwig and Lieberknecht (2025) find that lower nominal interest rate is associated with higher firm entry and lower firm exit. Other cross-country studies examine the impact of monetary policy or inflation on various outcomes other than firm entry/exit. For instance, Cevik et al. (2024) provide cross-country firm-level evidence about the impact of inflation shocks on firm productivity. Durante et al. (2022) examine the impact of monetary policy shocks in terms of EONIA (Euro Overnight Index Average) swaps on the investments of a sample of firms in Germany, France, Italy, and Spain. In a different context, Garcia-Macia (2023) studies the impact of inflation on public finances in a broad set of countries. In his regression analysis, he uses the growth of the commodity import price index interacted with a dummy indicating whether the country's exchange rate is fully pegged as the instruments for inflation rate.

4.1 Data

Our main data for firm entry and exit come from Eurostat's Structural Business Statistics.[27] This data set provides standardized data about business dynamism for a number of European countries. Specifically, we consider the following two key variables:

1. Firm entry rate (*entry*), defined as the number of enterprise births divided by the number of active enterprises.

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[27]See https://ec.europa.eu/eurostat/web/structural-business-statistics.

2. Firm exit rate (*exit*), defined as the number of enterprise deaths divided by the number of active enterprises.

To measure nominal interest rate (*i*), we use the short-term interest rates obtained from the OECD and the IMF.[28] Data for inflation rates (π) (and GDP, which is to control for country size) are extracted from the World Bank.

The raw sample contains an unbalanced panel of 409 observations with non-missing birth and death rates for 34 countries over 2004-2018. These countries include Austria, Belgium, Bulgaria, Croatia, Cyprus, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, North Macedonia, Norway, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, and Turkey, and United Kingdom.[29] Table 1 reports the summary statistics of the variables. In a typical year, the average nominal interest rate and inflation rate are both about 2.1%, and the average firm entry and exit rates are about 10.5% and 8.9% respectively.[30]

Table 1: Summary statistics

	N	Mean	S.D.	1st Quartile	Median	3rd Quartile
Firm entry rate (*entry*, in percentage)	409	10.505	3.692	7.980	10.010	12.220
Firm exit rate (*exit*, in percentage)	409	8.928	3.551	6.790	8.340	10.580
Nominal interest rate (*i*, in percentage)	384	2.083	2.541	0.210	1.238	3.904
Inflation rate (π, in percentage)	409	2.123	2.221	0.697	1.823	2.895
GDP (in constant 2010 billion US$)	409	612.786	895.330	51.921	243.604	537.421

Note: Each observation is a country by year over 2004-2018. Data come from Eurostat's Structural Business Statistics (for firm entry and exit rates), the OECD and the IMF (for nominal interest rate), and the World Bank (for inflation rate).

For the regression analysis below, we collapse the time-series data into 3-year means.[31] This results in another unbalanced panel of 150 country-period observations.[32] Figure 1

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[28]See https://www.oecd.org/en/data/indicators/short-term-interest-rates.html and https://data.imf.org/. Specifically, according to the OECD, these short-term interest rates are based on "three-month money market rates where available. Typical standardised names are 'money market rate' and 'treasury bill rate'." If data are not available for the countries in the sample, we replace the missing values by "Financial, Interest Rates, Government Securities, Government Bonds" from the IMF.

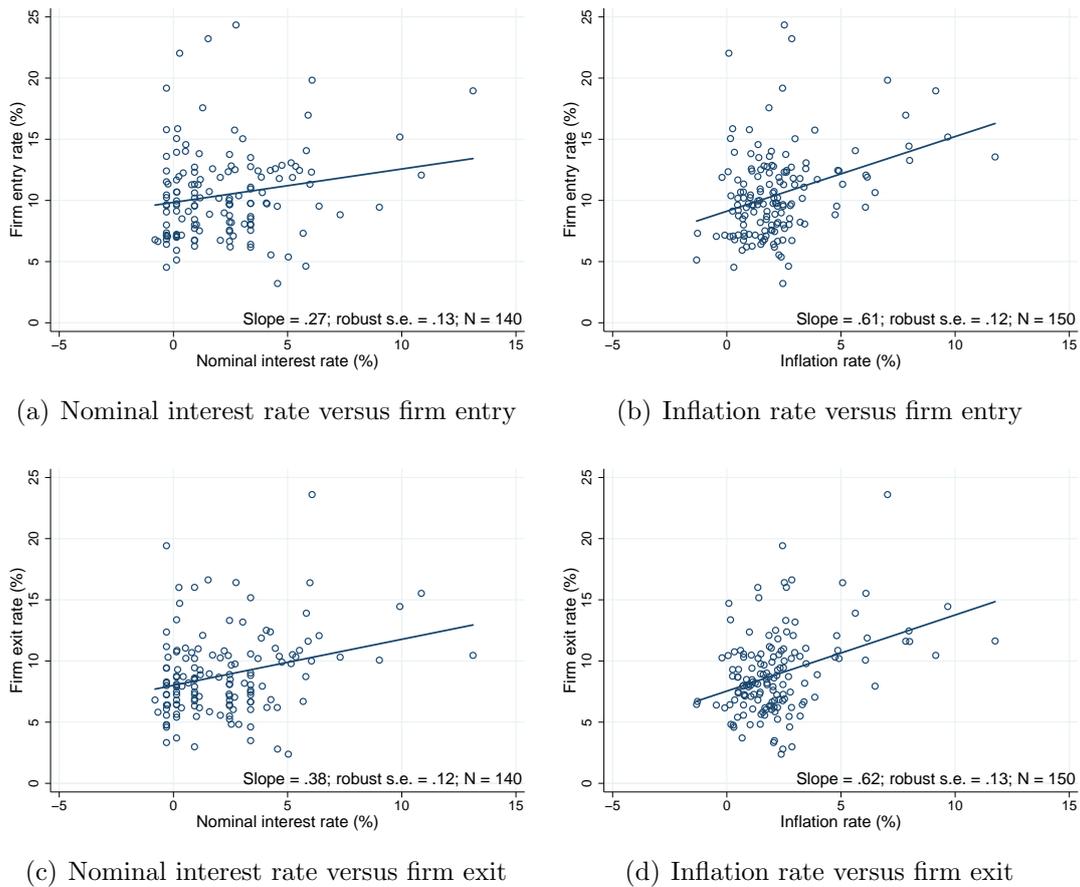[29]The sample size becomes 384 when we exclude missing values of nominal interest rate.

[30]Note that in the raw data, the nominal interest rates are identical among Eurozone countries.

[31]We do not collapse these data into 5-year means (as many other empirical macro studies do) mainly because we only have a relatively short sample period.

[32]The sample size becomes 140 when we exclude missing values of nominal interest rate.

shows the scatter plots between nominal interest rate, inflation rate and firm entry/exit rates. We can see that, unconditionally, both nominal interest rate and inflation rate are positively associated with firm entry/exit rates at the country level. On the other hand, in unreported analyses, we find that both nominal interest rate and inflation rate display declining trends over the sample period.

Figure 1: Scatter plots

(a) Nominal interest rate versus firm entry

(b) Inflation rate versus firm entry

(c) Nominal interest rate versus firm exit

(d) Inflation rate versus firm exit

Note: Each observation is a country by 3-year period over 2004-2018. Data come from Eurostat's Structural Business Statistics (for firm entry and exit rates), the OECD and the IMF (for nominal interest rate), and the World Bank (for inflation rate and GDP).

4.2 Regression Analysis

We first estimate the following regression by simple ordinary least squares (OLS):

$$y_{ct} = \alpha_c + \alpha_t + \beta X_{ct-1} + \gamma \log \text{GDP}_{ct-1} + \varepsilon_{ct}, \tag{25}$$

22

where c is a country, t is a 3-year period, y_{ct} is the dependent variable, which is either $entry_{ct}$ (firm entry rate) or $exit_{ct}$ (firm exit rate), X_{ct-1} is either the nominal interest rate (i) or the inflation rate (π) lagged by one period, GDP is measured in constant 2010 US\$, α_c and α_t are country and period fixed-effects, and ε_{ct} is the error term.

Panel A of Table 2 reports the results of the relationship between firm entry/exit rates and nominal interest rate while Panel A of Table 3 reports the relationship between firm entry/exit rates and inflation rate. In these regressions, we cluster the standard errors by country. In both cases, after controlling for country and period fixed-effects and lagged log GDP, there is a positive association between firm entry rate and nominal interest rate and inflation rate (in Column (3)) but there is no significant association between firm exit rate and nominal interest rate and inflation rate (in Column (6)).

Note that β from (25) can only inform us of the *correlation* between y and X. While we do not intend to document *causal* relationships, standard arguments about potential endogeneity apply here. For instance, there may be time-varying country-specific policies that affect both nominal interest rate and inflation rate and firm entry/exit, leading to omitted-variable bias. Ideally, we would like to find an instrument that is correlated with nominal interest rate and inflation rate but not the error term so that we can estimate (25) by two stage least squares (2SLS). In practice, an "external" instrument can be difficult to identify. Instead, we attempt to address the potential endogeneity issue by using the generalized method of moments (GMM) approach (see Arellano and Bond, 1991; Blundell and Bond, 1998). This approach employs lagged values of the right hand side variables as the "internal" instruments and has been used in the recent empirical growth literature (such as Murtin and Wacziarg, 2014).

We follow Roodman (2009)'s suggestions to estimate (25) by two-step system GMM. Specifically, when we choose the set of instruments, we pay attention to the following "rules of thumb:" First, the number of instruments should be strictly less than the number of countries in the sample. Second, the p-value for Arellano-Bond test for AR(2) in differences should be greater than 0.1, so that we cannot reject the null hypothesis that "the error terms are not serially correlated." Third, the p-value for Hansen test of

Table 2: Nominal interest rate and firm dynamics

Panel A: OLS regression results

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:		entry			exit	
i_{t-1}	0.263***	0.270***	0.288***	0.208***	0.004	0.023
	(0.062)	(0.067)	(0.075)	(0.054)	(0.027)	(0.032)
log GDP$_{t-1}$			2.600			2.741
			(3.494)			(1.975)
Country fixed-effects	No	Yes	Yes	No	Yes	Yes
Period fixed-effects	No	Yes	Yes	No	Yes	Yes
Observations	140	140	140	140	140	140
R^2	0.065	0.870	0.872	0.047	0.841	0.843

Panel B: GMM regression results

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:		entry			exit	
i_{t-1}	0.124	0.120	0.112	0.472*	0.476*	0.672**
	(0.198)	(0.198)	(0.352)	(0.272)	(0.264)	(0.260)
log GDP$_{t-1}$	-0.968	-0.933	-0.723	-0.814	-0.678	-0.455
	(0.628)	(0.627)	(0.599)	(0.960)	(0.801)	(0.640)
Observations	140	140	140	140	140	140
No. of countries	30	30	30	30	30	30
No. of instruments	23	21	15	23	21	15
Nature of instruments	Z_{t-2} & Z_{t-3}	Z_{t-2} to Z_{t-4}	Z_{t-3} & Z_{t-4}	Z_{t-2} & Z_{t-3}	Z_{t-2} to Z_{t-4}	Z_{t-3} & Z_{t-4}
AB2 p-value	0.347	0.360	0.335	0.131	0.119	0.125
Hansen p-value	0.329	0.246	0.450	0.206	0.244	0.324

Note: Each observation is a country by 3-year period over 2004-2018. In Panel B, all specifications include country and period fixed-effects. $Z = [\pi, \log \text{GDP}]$ is the vector of instruments. "AB2 p-value" is the p-value for Arellano-Bond test for AR(2) in differences. "Hansen p-value" corresponds to the p-value for Hansen test of joint validity of instruments. Standard errors, clustered at the country level, are reported in parentheses. *: significance at 10% level; **: significance at 5% level; ***: significance at 1% level.

joint validity of instruments should also be greater than 0.1 but should not be too large, so that we cannot reject the null hypothesis that "all instruments are jointly exogenous."

Panel B of Tables 2 and 3 respectively report the GMM regression results. For each outcome variable, we try different combinations of instruments. Let $Z = [X, \log \text{GDP}]$ where X is the nominal interest rate for Table 2 and is the inflation rate for Table 3. In Columns (1) and (4), the instruments for X_{t-1} are Z_{t-2} and Z_{t-3}. In Columns (2) and (5), the instruments for X_{t-1} are Z_{t-2} to Z_{t-4} In Columns (3) and (6), the instruments for X_{t-1} are Z_{t-3} and Z_{t-4}.

In Panel B of Table 2, we can see that firm entry rate is positively but not significantly related to nominal interest rate whereas firm exit rate is positively and significantly related

Table 3: Inflation rate and firm dynamics

Panel A: OLS regression results

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:		entry			exit	
π_{t-1}	0.446***	0.252**	0.252**	0.334***	-0.030	-0.027
	(0.069)	(0.096)	(0.097)	(0.088)	(0.062)	(0.062)
log GDP$_{t-1}$			0.154			2.480
			(3.132)			(1.793)
Country fixed-effects	No	Yes	Yes	No	Yes	Yes
Period fixed-effects	No	Yes	Yes	No	Yes	Yes
Observations	150	150	150	150	150	150
R^2	0.117	0.867	0.867	0.076	0.840	0.842

Panel B: GMM regression results

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:		entry			exit	
π_{t-1}	0.428**	0.528***	0.559***	0.489**	0.538**	0.548***
	(0.186)	(0.189)	(0.164)	(0.240)	(0.215)	(0.184)
log GDP$_{t-1}$	-0.844*	-0.803*	-1.002*	-0.500	-0.486	-0.316
	(0.494)	(0.438)	(0.541)	(0.456)	(0.470)	(0.560)
Observations	150	150	150	150	150	150
No. of countries	34	34	34	34	34	34
No. of instruments	23	21	15	23	21	15
Nature of instruments	Z_{t-2} & Z_{t-3}	Z_{t-2} to Z_{t-4}	Z_{t-3} & Z_{t-4}	Z_{t-2} & Z_{t-3}	Z_{t-2} to Z_{t-4}	Z_{t-3} & Z_{t-4}
AB2 p-value	0.540	0.417	0.394	0.088	0.090	0.078
Hansen p-value	0.496	0.504	0.797	0.662	0.588	0.569

Note: Each observation is a country by 3-year period over 2004-2018. In Panel B, all specifications include country and period fixed-effects. $Z = [\pi, \log \text{GDP}]$ is the vector of instruments. "AB2 p-value" is the p-value for Arellano-Bond test for AR(2) in differences. "Hansen p-value" corresponds is the p-value for Hansen test of joint validity of instruments. Standard errors, clustered at the country level, are reported in parentheses. *: significance at 10% level; **: significance at 5% level; ***: significance at 1% level.

to nominal interest. On the other hand, in Panel B of Table 3, we can see that both firm entry rate and firm exit rate are positively and statistically related to inflation rate. If we strictly follow Roodman (2009)'s suggestions, the results in Columns (4)-(6) of Panel B of Table 3 should be interpreted with caution; it is because the p-values for Arellano-Bond test for AR(2) in differences are less than 0.1, so that serial correlation of the error terms could be a concern.

To summarize, we in general find consistent evidence at the country level to support Proposition 1 and Proposition 2.

5 Quantitative Exercises

In this section, we calibrate our model to the E.U. data and quantitatively evaluate how our model explains the observed positive effects of inflation on firm entry and exit rates, which we empirically characterized in Section 4.[33]

Before proceeding, we should note that the parameter $\alpha \in (0,1)$ soley relate to two different concepts, the labor share $1 - \alpha$ in the production function (4) and the monopolistic markup $1/\alpha$. Therefore, to allow for a more realistic quantitative analysis in our calibration exercise,[34] we introduce an additional markup parameter, $\phi > 1$, following existing studies (e.g., Chu et al. 2023), which may capture patent breadth as in Gilbert and Shapiro (1990).[35] Otherwise, it may also capture price regulations as in Evans et al. (2003). Since the monopolistic pricing becomes $p_t(j) = \phi$ in this extended model, the demand and profit functions, (5), slightly change to $x_t(j) = (\alpha/\phi)^{1/(1-\alpha)}$ and $\Pi_t(j) = (\phi - 1)(\alpha/\phi)^{1/(1-\alpha)}$.[36]

The extended model features the structural parameters $\{\alpha, \gamma, \kappa, \rho, \zeta, \xi, \eta, \theta, \varpi, \overline{\delta}, \phi\}$ and a policy instrument i. For a benchmark model, we normalize $\gamma = 0.5$, which implies a square-root production function for innovation. For the discount factor ρ, we set it to 0.03, which falls into the conventional range in the literature. We then set the labor intensity to an empirically reasonable value, $1 - \alpha = 0.56$, as in Chu et al. (2019a); for related empirics, see Karabarbounis and Neiman (2014). While $\alpha = 0.44$ implies the markup ratio to be $1/\alpha \approx 2.3$ in the original model, we seek a more empirically plausible value in the quantitative exercise. Specifically, we set $\phi = 1.5$, which is slightly above the empirically plausible range of markup, 1.05–1.4, suggested by Jones and Williams (2000) but consistent with most of the estimates of the more recent study by Autor et al. (2020, Figure 10). As for the share $(1 - \theta)$ of internal cash available for survival investment in the total profit, we set it to 0.294 (i.e., $\theta = 0.706$) taken from the data on the fraction of

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[33]We can show a similar quantitative result using U.S. data; see Furukawa and Niwa (2021).

[34]Without introducing ϕ, either the labor income share or the price markup would deviate from realistic values.

[35]See Chu (2022b) for an extensive review of growth-theoretic studies on patent breadths.

[36]With this extension, the expressions of equations in Sections 2 and 3 do not change except for a minor change in (20); thus, the main results still hold.

Table 4: Calibrated parameters

α	γ	ρ	ζ	ξ	$\overline{\delta}$	ϖ	θ	η	κ	i	ϕ
0.44	0.5	0.03	0.65	0.627	0.873	0.606	0.706	0.00093	15.422	0.08	1.5

cash payouts, i.e., paying dividends and repurchase shares in 2005 for the E.U. (von Eije and Megginson, 2008). We follow Chu et al. (2019a) to consider the case with $\zeta = 0.65$.

We calibrate the six remaining structural parameters, $\{\kappa, \xi, \eta, \varpi, \overline{\delta}, i\}$, by matching theoretical moments to the data. As for ϖ and κ, we target the data of entry and exit rates in the E.U. in 2005, i.e., 10.826% and 8.419%,[37] respectively. To match the entry and exit rates, ε^* and δ^*, calculated from the model to these values from data, we calibrate the entry cost ϖ and the R&D productivity κ as $\varpi = 0.6056$ and $\kappa = 15.4224$. As for the natural exit rate, $\overline{\delta}$, we target the data of R&D intensity (R&D expenditure as a percentage of GDP) in the E.U. in 2005, i.e., 1.84%.[38] As for the exogenous growth rate η, we calibrate it to match the long-run growth rate to 2.5%, which falls within the standard range.[39] We, in turn, calibrate the CIA parameter ξ to match the average money-output, m_t/Y_t, calculated from the model to its empirical value, which is 7.83% in the E.U. (in 2005, base money (M0) per GDP).[40]. To complete the calibration, we use a 2005 inflation rate of 2.547% (for the countries covered in Section 4) to calibrate the monetary policy parameter; $i = 0.080$. The calibrated parameters are summarized in Table 4.

In the calibrated model, the relationship of the inflation rate, π, (or the nominal interest rate, i) to the firm entry and exit rates, ε^* and δ^*, are positive. Thus, our model is quantitatively equipped to explain the empirical fact shown in Section 4—lower inflation rates accompany lower entry and exit rates—under realistic values of the entry, exit, and inflation rates. Figure 2 illustrates these relationships.[41]

We also see the growth and welfare effects of inflation through changes in business

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[37]Based on our own calculations from the data we use for our empirical analysis; see Section 4 for details on the data.

[38]The data are taken from the Eurostat.

[39]This value is also consistent with a three-year average of the growth rate from 2014-2016 in the E.U.

[40]The data are from the Statistical Data Warehouse of the European Central Bank.

[41]We can illustrate similar figures by taking the nominal interest rate i on the horizontal axis because there is a one-to-one correspondence between i and π in our calibrated model.

dynamism. As mentioned in the introduction, there is a general concern that declining business dynamism may hurt long-term growth. Yet, although it can be either positive or negative theoretically (Proposition 3), our calibrated model exhibits a negative relationship between the inflation rate (or the nominal interest rate) and the growth rate, as shown in Figure 2. This quantitative result implies that the long-run growth rate can be even higher under declining business dynamism, thereby increasing the dynamic social welfare. This is because the growth-enhancing effect of a lower exit rate dominates the growth-depressing effect of a lower entry rate in our calibrated economy. Importantly, we should carefully interpret this counterintuitive result because it can result from a simplistic feature of our model, which states that a lower exit rate is always beneficial for growth. However, in reality, less productive companies staying in the market as "zombies" can stifle potentially more productive companies from entering the market; unlike in our model, lowering the exit rate can negatively affect growth. It should be meaningful to fix our model's limitation by introducing firm heterogeneity in productivity. Further, in our model, firm exits occur following an endogenous hazard rate. Although the firm voluntarily invests against market exit, there is no focus on the decision-making aspect of exiting the market voluntarily. It should also be interesting to extend the analysis in this direction. Nevertheless, since the present study aims to explain declining business dynamism as an equilibrium phenomenon, we would leave this interesting extension for future research.

6   Concluding Remarks

We offer a new growth theory that explains what causes declining business dynamism, by developing a new a monetary growth model. Specifically, based on the seminal monetary R&D-based growth model developed by Chu and Cozzi (2014), we propose two new considerations: (a) firms successfully innovating new goods need to further pay an entry cost, (b) after entry, incumbent firms invest in market survival, facing a CIA constraint. These two new features make entry/exit rates and the firm distribution in age endogenous.

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Figure 2: Simulations
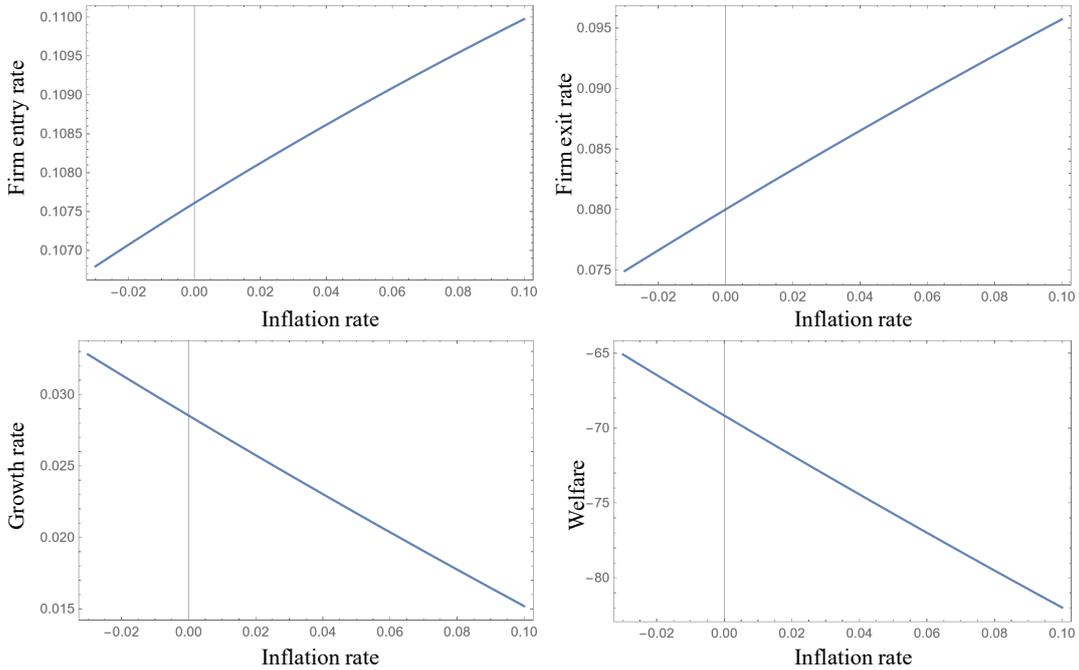
Using this model, we theoretically find that if the entry cost is sufficiently large, a decrease in the inflation rate, or a deflationary trend, can be accompanied by declining business dynamism, which is featured by lower entry and exit rates and a higher average firm age (or a larger share of older firms). Empirically, we also find evidence that firm entry/exit rates are positively associated with nominal interest rates and inflation rates among a set of European countries, supporting our theoretical prediction. Calibrating the model to the E.U. data, we show that a decrease in inflation decreases both entry and exit rates under empirically plausible values of the entry, exit, and inflation rates.

Our model abstracts several important aspects of reality. First, we do not focus on the distribution of firm size, which is also an important issue in macroeconomics; for example, see Axtell (2001) for empirics and Luttmer (2007) for theory. Our model could be extended by introducing the empirically plausible type of firm size distribution, i.e., a Pareto distribution. Second, consumers are homogeneous in the model. Therefore, there is no direct contribution to the vast literature on growth and income/wealth inequality (e.g., Acemoglu and Cao, 2015, and Jones and Kim, 2018).[42] Our model could be ex-

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[42]See, for example, Haruyama (2025) for a recent contribution to this literature, who develops a

tended by assuming that different consumers own shares of different firms that vary in size and profit. Under such an extension, consumers are heterogeneous in their interest incomes. Third, in our model, firm growth is driven by an exogenous factor. This could be extended by introducing in-house R&D (e.g., Peretto 1996). Forth, our results should depend on the setting where when firms get cash, they have no choice but to borrow from consumers. Introducing an alternative way to raise fund from, for example, a venture capital market would be interesting. We leave these possible extensions for future research.

Finally, we discuss the theoretical insights and policy implications of our study. Theoretically, we offer an explanation for the mechanism through which a decrease in the inflation—or a deflationary trend—can be a cause of declining business dynamism via the CIA channel, which is a novel contribution to the literature. As in existing models, a lower inflation rate yields a lower cost of money holding and, thereby, money borrowing because of the lower nominal interest, which encourages R&D investment for entry. However, in our model, firms are also financially constrained in the survival and exit stage in the equilibrium where incumbent firms borrow cash. As a result, a lower inflation rate directly encourages survival investment and decreases firm exit, due to lower interest payment. Notably, it further has an indirect, general-equilibrium effect that emerges from a shift of labor resources between innovation and survival investment. The direction of this indirect effect is potentially ambiguous, and depends on the size of the entry cost. The reason is as follows. If the entry cost after successful R&D investment is very high; the net payoff of R&D investment is very small. In this case, R&D activity is originally very weak; the positive effect of lower inflation on entry is insignificant. Therefore, a decrease in the inflation rate, i.e., a deflationary trend, causes a shift of resources from entry to survival, discouraging R&D for firm entry and encouraging survival to delay exit. This explains why a lower inflation rate leads to a lower entry and exit rate and a higher average firm age.

These findings deliver the important policy implication that monetary policy control-

Schumpeterian growth model in which a double Pareto distribution of income emerges in equilibrium as a result of entrant (drastic) and incumbent innovations.

ling the interest rate, or targeting the inflation rate, can essentially affect the nature of business dynamics. For example, a low-interest-rate policy can encourage both innovation and survival investment, thereby, enhancing long-run growth as a standard effect, while we qualitatively prove that the general-equilibrium reallocation between entry and survival can reverse the growth effect ambiguous. However, as an additional effect, it can also cause declining business dynamism, depending on the size of an entry cost. These findings suggest that the monetary authorities should consider not only standard macroeconomic variables, such as the growth rate, but also variables for firm demographics, in creating a desirable monetary policy.

References

[1] Abel, A. B., 1985. Dynamic behavior of capital accumulation in a cash-in-advance model. *Journal of Monetary Economics* 16, 55–71.

[2] Acemoglu, D., and Cao, D., 2015. Innovation by entrants and incumbents. *Journal of Economic Theory* 157, 255–294.

[3] Akcigit, U., and Ates, S., 2021. Ten facts on declining business dynamism and lessons from endogenous growth theory. *American Economic Journal: Macroeconomics* 13, 257–98.

[4] Akcigit, U., and Ates, S., 2023. What happened to U.S. business dynamism? *Journal of Political Economy* 131, 2059—2124.

[5] Aghion, P., and Howitt, P., 1992. Research and development in the growth process, *Journal of Economic Growth* 1, 49–73.

[6] Akiyama, T., and Furukawa, Y., 2009. Intellectual property rights and appropriability of innovation. *Economics Letters* 103, 138–141.

[7] Anderlini, L., Felli, L., Immordino, G., and Riboni, A., 2013. Legal institutions, innovation and growth. *International Economic Review* 54, 937–956.

[8] Autor, D., Dorn, D., Katz, L.F., Patterson, C., and Van Reenen, J., 2020. The fall of the labor share and the rise of superstar firms. *Quarterly Journal of Economics* 135, 645–709.

[9] Arawatari, R., Hori, T., and Mino, K., 2018. On the nonlinear relationship between inflation and growth: A theoretical exposition. *Journal of Monetary Economics* 94, 79–93.

[10] Arellano, M., Bond, S., 1991. Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *Review of Economic Studies* 58, 277–297.

[11] Axtell, R., 2001. Zipf distribution of U.S. firm sizes. *Science* 293 (5536), 1818–20.

[12] Benhabib, J., Perla, J., and Tonetti, C. 2021. Reconciling models of diffusion and innovation: A theory of the productivity distribution and technology frontier. *Econometrica* 89, 2261–2301.

[13] Booth, G., and Ciner, C., 2001. The relationship between nominal interest rates and inflation: International evidence. *Journal of Multinational Financial Management* 11, 269–280.

[14] Blundell, R., Bond, S., 1998. Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics* 87(1), 115–143.

[15] Calvino, F., Criscuolo, C., and Verlhac, R., 2020. Declining business dynamism: Structural and policy determinants. *OECD Science, Technology and Innovation Policy Papers* No. 94.

[16] Cevik, S., Fan, A., and Naik, S. (2024). Is inflation good for business? The firm-level impact of inflation shocks in the Baltics, 1997–2020. *Economic Change and Restructuring*, 57(5), 1-19.

[17] Chen, S., 2014. Financial constraints, intangible assets, and firm dynamics : Theory and evidence. *IMF Working Papers* No. 14/88.

[18] Chu, A., 2022a. Inflation, innovation and growth: A survey. *Bulletin of Economic Research* 74, 863–878.

[19] Chu, A., 2022b. Patent policy and economic growth: A survey. *Manchester School* 90, 237–254.

[20] Chu, A., and Cozzi, G., 2014. R&D and economic growth in a cash-in-advance economy. *International Economic Review* 55, 507–524.

[21] Chu, A. C., Cozzi, G., Fan, H., and Furukawa, Y. 2021. Inflation, unemployment, and economic growth in a Schumpeterian economy. *Scandinavian Journal of Economics* 123, 874–909.

[22] Chu, A., Cozzi, G., Fan, H., Furukawa, Y., and Liao, C., 2019a. Innovation and inequality in a monetary Schumpeterian model with heterogeneous households and firms. *Review of Economic Dynamics* 34, 141–164.

[23] Chu, A., Cozzi, G., Furukawa, Y., and Liao, C., 2017. Inflation and economic growth in a Schumpeterian model with endogenous entry of heterogeneous firms. *European Economic Review* 98, 392–409.

[24] Chu, A., Cozzi, G., Furukawa, Y., and Liao, C., 2019b. Inflation and innovation in a Schumpeterian economy with North-South technology transfer. *Journal of Money, Credit and Banking* 51, 683–719.

[25] Chu, A., Cozzi, G., Furukawa, Y., and Liao, C., 2023. Should the government subsidize innovation or automation? *Macroeconomic Dynamics* 27, 1059–1088.

[26] Chu, A., Cozzi, G., Lai, C., and Liao, C., 2015. Inflation, R&D and growth in an open economy. *Journal of International Economics* 96, 360–374.

[27] Chu, A., and Lai, C.-C., 2013. Money and the welfare cost of inflation in an R&D growth model. *Journal of Money, Credit and Banking* 45, 233–249.

[28] Clower, R.W., 1967. A reconsideration of the microfoundations of money. *Western Economic Journal* 6, 1–8.

[29] Coad, A., 2010. The exponential age distribution and the Pareto firm size distribution. *Journal of Industry, Competition and Trade* 0, 389–395.

[30] Colciago, A., and Silvestrini, R. (2022). Monetary policy, productivity, and market concentration. *European Economic Review*, 142, 103999.

[31] Davis, L., and Şener, F., 2012. Private patent protection in the theory of Schumpeterian growth. *European Economic Review* 56, 1446–1660.

[32] De Loecker, J., Eeckhout, J., and Mongey, S., 2021. Quantifying market power and business dynamism in the macroeconomy. *NBER Working Paper* 28761.

[33] Dinopoulos, E., and Syropoulos, C., 2007. Rent protection as a barrier to innovation and growth. *Economic Theory* 32, 309–332.

[34] Durante, E., Ferrando, A., and Vermeulen, P. (2022). Monetary policy, investment and firm heterogeneity. *European Economic Review*, 148, 104251.

[35] Eicher, T., and García-Peñalosa, C., 2008. Endogenous strength of intellectual property rights: Implications for economic development and growth. *European Economic Review* 52, 237–258.

[36] Evans, L., Quigley, N., and Zhang, J., 2003. Optimal price regulation in a growth model with monopolistic suppliers of intermediate goods. *Canadian Journal of Economics* 36, 463–474.

[37] Furukawa, Y., 2013. The Struggle to Survive in the R&D Sector: Implications for Innovation and Growth. *Economics Letters* 121, 26–29.

[38] Furukawa, Y., and Niwa, S., 2021. Deflation and declining business dynamism in a cash-in-advance economy. *RIETI Discussion Paper Series* 21-E-058.

[39] Furukawa, Y., and Yano, M., 2014. Market quality and market infrastructure in the South and technology diffusion. *International Journal of Economic Theory* 10, 139–146.

[40] Garcia-Macia, D. (2023). The Effects of Inflation on Public Finances, IMF Working Paper (No. 2023/093).

[41] Grieben, W. H., and Şener, F., 2009. Globalization, rent protection institutions, and going alone in freeing trade. *European Economic Review* 53, 1042–1065.

[42] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies* 58, 43–61.

[43] Hall, B. H., 2008. The financing of innovation. Handbook of Technology and Innovation Management. Edited by S. Shane. Wiley-Blackwell.

[44] Hamano, M., and Zanetti, F. (2022). Monetary policy, firm heterogeneity, and product variety. *European Economic Review*, 144, 104089.

[45] Hartwig, B., and Lieberknecht, P. 2025. Monetary Policy, Firms' Extensive Margin and Productivity. *International Journal of Central Banking* 21, 1–82.

[46] Haruyama, T. 2025. A Schumpeterian exploration of Gini and top/bottom income shares. *Macroeconomic Dynamics* 29, e49. doi:10.1017/S1365100524000403

[47] He, Q., 2018. Inflation and fertility in a Schumpeterian growth model: Theory and evidence. *International Review of Economics & Finance* 58, 113–126.

[48] He, Q., and Zou, H., 2016. Does inflation cause growth in the reform-era China? Theory and evidence. *International Review of Economics & Finance* 45, 470–484.

[49] Hori, T., 2020. Monetary policy, financial frictions, and heterogeneous R&D firms in an endogenous growth model. *Scandinavian Journal of Economics*, forthcoming.

[50] Huang, C., Chang, J., and Ji, L., 2021. Inflation, market structure, and innovation-driven growth with various cash constraints. *Oxford Economic Papers* 73, 1270–1303.

[51] Jones, C. I. and Kim, J., 2018. A Schumpeterian model of top income inequality, *Journal of Political Economy* 126, 1785–1826.

[52] Jones, C., and Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth* 5, 65-85.

[53] Karabarbounis, L., and Neiman, B., 2014. The global decline of the labor share. *Quarterly Journal of Economics* 129, 61–103.

[54] Klette, T.J., and Kortum, S., 2004. Innovating firms and aggregate innovation. *Journal of Political Economy* 112, 986–1018.

[55] Lentz, R., and Mortensen, D.T., 2008. An empirical model of growth through product innovation. *Econometrica* 76, 1317–1373.

[56] Lucas, R.E., 1980. Equilibrium in a pure currency economy. *Economic Inquiry* 18, 203–220.

[57] Lucas, R. E., and Moll, B., 2014. Knowledge growth and the allocation of time. *Journal of Political Economy* 122, 1–51.

[58] Luttmer E., 2007. Selection, growth, and the size distribution of firms. *Quarterly Journal of Economics* 122, 1103–1144.

[59] Middendorf, W. H., 1981. What every engineer should know about inventing. New York: Marcel Dekker. (Reprinted in 2019 by CRC Press.)

[60] Mishkin, F., 1992. Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates. *Journal of Monetary Economics* 30, 195–215.

[61] Miyakawa, D., Oikawa, K., and Ueda, K., 2022. Reallocation effects of monetary policy. *International Economic Review* 63, 947–975.

[62] Morikawa, M. 2015. Financial constraints on intangible investments: Evidence from Japanese firms. In Bounfour, A., and Miyagawa, T. (ed.) *Intangibles, Market Failure and Innovation Performance*, Springer, 139–155.

[63] Murtin, F., Wacziarg, R., 2014. The democratic transition. *Journal of Economic Growth* 19, 141–181.

[64] Musso, P., and Schiavo, S., 2008. The impact of financial constraints on firm survival and growth. *Journal of Evolutionary Economics* 18, 135–149.

[65] Niwa, S., 2018. Effects of a blocking patent on R&D with endogenous survival activities. *Journal of Economics* 125, 263–277.

[66] Oikawa, K., and Ueda, K., 2018. The optimal inflation rate under Schumpeterian growth. *Journal of Monetary Economics* 100, 114–125.

[67] Peretto, P., 1996. Sunk costs, market structure, and growth. *International Economic Review* 37, 895–923.

[68] Rivera-Batiz, L., and Romer, P. M., 1991. Economic integration and endogenous growth. *Quarterly Journal of Economics* 106, 531–555.

[69] Romer, P., 1990. Endogenous technological progress. *Journal of Political Economy* 98, S71–S102.

[70] Roodman, D., 2009. A note on the theme of too many instruments. *Oxford Bulletin of Economics and Statistics* 71(1), 135–158.

[71] Segerstrom, P., Anant, T. C. A., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review* 80, 1077–91.

[72] Small and Medium Enterprise Agency of Japan, 2020. *2020 White Paper on Small and Medium Enterprises in Japan.*

[73] Stockman, A. C., 1981. Anticipated inflation and the capital stock in a cash in-advance economy. *Journal of Monetary Economics* 8, 387–393.

[74] Tobin, J., 1965. Money and economic growth. *Econometrica* 33, 671–684.

[75] von Eije, H., and Megginson, W. L.,2008. Dividends and share repurchases in the European Union. *Journal of Financial Economics* 89, 347–374.

[76] Yano, M., 2009. The foundation of market quality economics. *Japanese Economic Review* 60, 1–32.

[77] Yano, M., 2019. A new theory of money: From ancient Japanese copper Coins to virtual Currencies. *RIETI Discussion Paper Series* No. 19-E-062.

[78] Yano, M., and Furukawa, Y., 2023. Two-dimensional constrained chaos and industrial revolution cycles. *Proceedings of the National Academy of Sciences* 120, e2117497120.

[79] Zheng, Z., Huang, C., and Yang, Y., 2021. Inflation and growth: A non-monotonic relationship in an innovation-driven economy. *Macroeconomic Dynamics* 25(5), 1199–1226.

[80] Zheng, Z., Mishra, T., and Yang, Y., 2020. Inflation and income inequality in a variety-expansion growth model with menu costs. *Economics Letters* 194, 109–373.

Online appendix for

Explaining Declining Business Dynamism: A Monetary Growth-Theoretic Approach

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Appendix A: Detailed Derivations and Discussions on the Model

Profit Maximization for Final-Good Firms (Section 2.2)

To maximize profit $Y_t - w_t L_t - \int_0^{N_t} p_t(j) x_t(j) dj$, the first-order condition gives rise to

$$\frac{\partial \Pi}{\partial x_t(j)} = \alpha L_t^{1-\alpha} x_t(j)^{\alpha-1} - p_t(j) = 0 \tag{A1}$$

Arranging (A1) yields

$$x_t(j) = \alpha^{1/(1-\alpha)} L_t p_t(j)^{-1/(1-\alpha)}. \tag{A2}$$

The price elasticity of demand for good j is defined as:

$$\epsilon \equiv -\frac{dx_t(j)}{dp_t(j)} \frac{p_t(j)}{x_t(j)}, \tag{A3}$$

which is equal to $1/(1-\alpha)$. Substituting the equilibrium monopolistic price $p_t(j) = 1/\alpha$ into (A2) yields the equilibrium quantity and profit given by (5).

The Behavior of the R&D Firm's Payments along the Time Axis (Section 2.3)

The within-period behavior of the R&D firm's payment proceeds over time is as follows. In the first step, to invest in $1/\kappa$ units of the final good, the firm borrows ζ/κ units of cash from the representative consumer and prepays ζ/κ to final good firms to purchase $1/\kappa$ units of the final good. After inventing a new good, the firm sells ownership shares or patents in the market to earn v_t. Then, the firm—from its cash holdings of v_t (in real terms)—(a) pays a balance of $(1-\zeta)/\kappa$ to the final good firm, (b) repays ζ/κ units of cash along with the interest $i\zeta/\kappa$ (i.e., a total of $(1+\zeta i)/\kappa$), and (c) spends ϖ/κ to purchase the final good for entry. After that, the firm will begin to produce its invented good and earn profits, which are distributed to the agent who hold the ownership shares.

Financing of Survival Investment and the Total Payment for Survival I_t (Section 2.4)

The following provides additional details on how incumbent firms finance their survival investment z_t/κ under the CIA constraint.

As described in the main text, an incumbent firm must prepare $\xi z_t/\kappa$ units of cash in advance in order to carry out a survival investment of z_t/κ units of the final good, where $\xi \in [0,1]$ measures the severity of the liquidity requirement. Thus, the firm must verify whether its internal funds are sufficient to meet the CIA requirement. At each date t, as described in the main text, the incumbent firm earns profit Π_t; the firm can devote a fraction $(1-\theta)\Pi_t$ of this profit to survival investment.

Depending on whether internal funds can cover the CIA-required cash $\xi z_t/\kappa$, two cases arise. If $(1-\theta)\Pi_t \geq \frac{\xi z_t}{\kappa}$, there are sufficient internal funds; then, the firm does not need to borrow additional cash. All required liquidity for survival investment is covered by internal funds. If $(1-\theta)\Pi_t < \frac{\xi z_t}{\kappa}$, there are insufficient internal funds; then, the firm borrows the shortfall $\frac{\xi z_t}{\kappa} - (1-\theta)\Pi_t$ from households at the nominal interest rate i_t.

For expositional convenience, define the indicator λ^ω such that $\lambda^0 = 0$ when there is no borrowing and $\lambda^1 = 1$ when there is borrowing, Here, $\omega \in \{0, 1\}$ indexes the two cases above. This allows the two cases to be written in a unified expression.

Combining the two cases, the total payment associated with survival investment can be expressed as

$$I_t = \frac{z_t}{\kappa} + i_t \max \left\{ \frac{\xi z_t}{\kappa} - (1 - \theta)\Pi_t,\, 0 \right\} = \frac{z_t}{\kappa} + \lambda^\omega i_t \left(\frac{\xi z_t}{\kappa} - (1 - \theta)\Pi_t \right), \qquad (A4)$$

which is the compact expression for (7). The second line follows from substituting the definition of λ^ω.

Finally, if internal funds exceed the CIA requirement, any remaining balance $(1 - \theta)\Pi_t - \xi z_t/\kappa$ is implicitly assumed to be paid out to households, consistent with the representative-agent setup.

Dynamic Optimization of Existing Firms for Survival (Section 2.4) Since firm exit occurs at the Poisson hazard rate of δ_t, during the short time interval dt, the firm obtains $(\dot{v}_t + \eta v_t)\, dt$ with probability $(1 - \delta_t dt)$ and loses the current value v_t with probability $\delta_t dt$. Further, in nominal terms, (8) is written as

$$W_t^n + \pi v_t^n = (\Pi_t^n - I_t^n)\, dt + (1 - \delta_t dt)\, \dot{v}_t^n dt - (\delta_t dt - \eta dt) v_t^n, \qquad (A5)$$

where variables with superscript n denotes nominal values. Given that $\dot{v}_t^n = \dot{P}_t v_t + P_t \dot{v}_t$. This implies that W_t in (8) is defined as the real value of innovation *minus* a nominal benefit of inflation.

Appendix B: Extensions

This appendix investigates several extensions mentioned in the text.

Extension with an extra CIA constraint for entry. We introduce an extra CIA constraint that requires the firm after successful R&D to prepare $\chi \varpi / \kappa$ amount of cash in investing ϖ / κ units of final goods in entry, where $\chi \in [0, 1]$. We will show that this modification does not essentially change our main result, while making the proof more complex. With this modification, the free entry condition (11) changes to have a CIA parameter, such that

$$v_t = \frac{1 + \varpi + (\zeta + \chi \varpi) i}{\kappa}. \tag{B1}$$

Accordingly, (15) becomes

$$z^{**} = \left(\frac{\gamma \left(1 + \varpi + (\zeta + \chi \varpi) i \right)}{1 + \xi i} \right)^{\frac{1}{1-\gamma}} \text{ and } \delta^{**} = \overline{\delta} - \left(\frac{\gamma \left(1 + \varpi + (\zeta + \chi \varpi) i \right)}{1 + \xi i} \right)^{\frac{\gamma}{1-\gamma}}. \tag{B2}$$

With these changes, (16) changes to

$$\frac{\dot{c}_t}{c_t} = \frac{(1 + i (1 - \theta))}{1 + \varpi + (\zeta + \chi \varpi) i} \kappa \Pi - \frac{1 + \xi i}{1 + \varpi + (\zeta + \chi \varpi) i} z^* - (\delta^* + \rho - \eta) \equiv g^*. \tag{B3}$$

From (17), (B2), and (B3), the entry rate is:

$$\varepsilon^* = \frac{1 + i (1 - \theta)}{1 + \varpi + (\zeta + \chi \varpi) i} \kappa \Pi - \frac{1 + \xi i}{1 + \varpi + (\zeta + \chi \varpi) i} z^* - \rho. \tag{B4}$$

By (B4), we can show that δ^{**} is increasing in i if $\varpi > (\zeta - \xi) / (\xi - \chi)$; ε^{**} is increasing in i if $\varpi > (\zeta - (1 - \theta)) / ((1 - \theta) - \chi)$. With a higher entry cost, the effect of increasing i on business dynamism is negative, in line with Propositions 1 and 2 in the main text. ∎

Extension with in-house innovation and endogenous η. Suppose the arrival rate η_t at date t is a function of an investment of z_t^n units of final goods for in-house innovation, which gives the firm the invention of an extra differentiated product. Consider a strictly concave success function, such as $\eta_t = \tilde{\eta}(z_t^n)^\beta$ with $\beta \in (0, 1)$. Equations (7) and (8)

change to

$$\tilde{I}_t \equiv \frac{z_t + z_t^n}{\kappa} + \lambda^\omega i_t \left(\frac{\xi \left(z_t + z_t^n \right)}{\kappa} - (1 - \theta) \, \Pi_t \right) \tag{B5}$$

and

$$\max_{z_t: \text{ s.t. } \delta_t \equiv \bar{\delta} - (z_t)^\gamma, \ \eta_t \equiv \tilde{\eta}(z_t^n)^\beta} W_t \equiv (\Pi_t - I_t) + \dot{v}_t - (\delta_t - \eta_t) v_t. \tag{B6}$$

Solving this concave programming, with (11) and $i_t = i$, we can obtain

$$z_t^n = \left(\frac{\beta \tilde{\eta} \left(1 + \varpi + \zeta i \right)}{1 + \xi i} \right)^{\frac{1}{1-\beta}} \text{ and } \eta_t = \tilde{\eta}^{\frac{1}{1-\beta}} \left(\frac{\beta \left(1 + \varpi + \zeta i \right)}{1 + \xi i} \right)^{\frac{\beta}{1-\beta}} \equiv \eta^* \tag{B7}$$

for any t. (B7) shows that under a higher entry cost, ϖ, an increase in i leads to a lower rate, η^*, of in-house innovation.

Since the equilibrium investment in survival, z^*, remains the same as (15). By (16) and (20), the entry rate is free from η, such that

$$\varepsilon^* \equiv g^* + \delta^* - \eta = \frac{(1 + i \, (1 - \theta))}{1 + \varpi + \zeta i} \Pi - \frac{1 + \xi i}{1 + \varpi + \zeta i} z^* - \rho. \tag{B8}$$

Therefore, this simplistic extension does not alter the effects of higher i on the exit rate δ^* and the entry rate ε^* (identified in Propositions 1 and 2). However, the relationship between i and the growth rate g^* can differ. From a revised version of (19) with η^* instead of η, we can find that an increase in i has an extra effect on g^* through (A19), which is negative under a higher entry cost, ϖ. Therefore, if the existing firm could endogenously grow, a lower nominal interest rate, i, would be more likely to enhance economic growth. ∎

Appendix C: Proofs

This appendix shows formal proofs that complement the results in the text.

Proof of Lemma 2. Define $e_t \equiv c_t/N_t$. Using Lemma 1, we combine (16)–(18) to obtain

$$\frac{\dot{e}_t}{e_t} = g^* - \varepsilon(e_t) - (\eta - \delta^*) \equiv \epsilon(e_t) \tag{C1}$$

where $\varepsilon(e_t) \equiv \varepsilon_t^*$ from (18) with $\varepsilon' < 0$. Because g^* and δ^* are independent of t, (C1) is an autonomous dynamic system for e_t. Applying the standard argument using the transversality condition, (C1) has a globally saddle-point stable steady state, e^*, as long as $\epsilon(0) < 0$. Under $\epsilon(0) < 0$, e_t jumps to $e^* \equiv \epsilon^{-1}(0)$ at time 0. Along this unique balanced growth path, from (C1) with $\dot{e}_t = 0$, $\varepsilon_t = \varepsilon^* \equiv g^* + \delta^* - \eta$ holds for any $t \geq 0$, which is the same as (20). Note that the equilibrium growth rate for c_t and N_t is already given in (16), which is the same as (19).

In the rest of this proof, we will derive conditions under which (a) the existing firms borrow cash in equilibrium, (b) the stability condition $\epsilon(0) < 0$ holds, and (c) g^*, ε^*, and δ^* are all positive.

(a) We first derive a condition under which the existing firms borrow cash ($\omega = 1$): They do so if and only if $\frac{\xi z^*}{\kappa}\big|_{\omega=1} > (1-\theta)\Pi$ and $\frac{\xi z^*}{\kappa}\big|_{\omega=0} > (1-\theta)\Pi$, which requires

$$\xi \left(\gamma \frac{1 + \varpi + \zeta i}{1 + \xi i} \right)^{\frac{1}{1-\gamma}} > (1-\theta)\kappa\Pi. \tag{C2}$$

The left-hand side has a positive lower bound as $i \to \infty$ ($i = 0$) for $\varpi > (\zeta - \xi)/\xi$ ($\varpi < (\zeta - \xi)/\xi$). Therefore, there exists some value of θ, say θ_0, such that (C2) holds for $\theta \in (\theta_0, 1]$; specifically, if $\varpi > (\zeta - \xi)/\xi$ holds as in our main propositions, it is explicitly defined by $\theta_0 \equiv 1 - \xi^{\gamma/(1-\gamma)} (\gamma\zeta)^{1/(1-\gamma)} / (\kappa\Pi)$. With $\theta \in (\theta_0, 1]$, we hereafter focus on the equilibrium where the existing firms borrow cash ($\omega = 1$).

(b) Using (16)–(18), we express $\epsilon(0) < 0$ by

$$\underbrace{\left(\frac{1+i\,(1-\theta)}{1+\varpi+\zeta i} - \frac{1}{1+\varpi}\frac{1+\alpha}{\alpha}\right)}_{\epsilon_1}\kappa\Pi < \underbrace{\left(\frac{1+\xi i}{1+\varpi+\zeta i} - \frac{1}{1+\varpi}\right)}_{\epsilon_2}z^* + \rho \qquad \text{(C3)}$$

using (15). Note that (C3) always holds if $\epsilon_1 < 0$ and $\epsilon_2 > 0$, which hold if

$$1 - \theta < \frac{1}{\alpha}\left(\frac{\zeta\,(1+\alpha)}{1+\varpi} + \frac{1}{i}\right) \equiv A \;\; \text{and} \;\; \frac{\zeta - \xi}{\xi} < \varpi.$$

From $\theta > \theta_0$, $1 - \theta < \xi^{\gamma/(1-\gamma)}\,(\gamma\zeta)^{1/(1-\gamma)}\,/(\kappa\Pi)$; thus, the first inequality always holds if $\kappa > \kappa_1 \equiv \xi^{\gamma/(1-\gamma)}\,(\gamma\zeta)^{1/(1-\gamma)}\,/(A\Pi)$. Therefore, the system stability is ensured with the second inequality and $\kappa > \kappa_1$.

(c) To think of the positivity of $(g^*, \varepsilon^*, \delta^*)$, it is helpful to recall

$$g^* = \frac{1+i\,(1-\theta)}{1+\varpi+\zeta i}\kappa\Pi + \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)\left(\frac{1+\varpi+\zeta i}{1+\xi i}\right)^{\frac{\gamma}{1-\gamma}} - (\overline{\delta} + \rho - \eta) \equiv h_g(\underset{+}{\kappa}), \qquad \text{(C4)}$$

$$\varepsilon^* = g^* + \delta^* - \eta = \frac{1+i\,(1-\theta)}{1+\varpi+\zeta i}\kappa\Pi - \gamma^{\frac{1}{1-\gamma}}\left(\frac{1+\varpi+\zeta i}{1+\xi i}\right)^{\frac{\gamma}{1-\gamma}} - \rho \equiv h_\varepsilon(\underset{+}{\kappa}), \qquad \text{(C5)}$$

and

$$\delta^* = \overline{\delta} - \left(\gamma\frac{1+\varpi+\zeta i}{1+\xi i}\right)^{\frac{\gamma}{1-\gamma}}, \qquad \text{(C6)}$$

There exists some value of κ, say κ_2, such that (C4) and (C5) hold for any $\kappa > \kappa_2$ because $h_g(\kappa)$ and $h_\varepsilon(\kappa)$ are increasing in κ. Further, we can take some sufficiently high $\overline{\delta}$ that can ensure $\delta^* > 0$; we label the threshold as $\overline{\delta}_0$. While writing κ_2 and $\overline{\delta}_0$ explicitly is possible, we omit to do so for descriptive simplicity. In summary, the proof has shown that in the equilibrium with $\theta > \theta_0$ where existing firms borrow cash, the equilibrium dynamic system is globally stable with $g^* > 0$, $\varepsilon^* > 0$, and $\delta^* > 0$ under $\kappa > \kappa_0 \equiv \max\{\kappa_1, \kappa_2\}$ and $\overline{\delta} > \overline{\delta}_0$. ∎

Proof for the relationship between i and π. By (19), the equilibrium real interest

is (with $\omega = 1$):

$$r = \frac{1 + i(1 - \theta)}{1 + \varpi + \zeta i} \kappa \Pi + \gamma^{\frac{\gamma}{1-\gamma}} (1 - \gamma) \left(\frac{1 + \varpi + \zeta i}{1 + \xi i} \right)^{\frac{\gamma}{1-\gamma}} - (\bar{\delta} - \eta) \equiv r(i). \qquad \text{(C7)}$$

From the Fisher equation, $\pi = i - r(i)$. To verify the effect of i on π, we have

$$\frac{d\pi}{di} = 1 - \left[\frac{(1 - \theta)(1 + \varpi) - \zeta}{(1 + \varpi + \zeta i)^2} \kappa \Pi + \gamma^{\frac{1}{1-\gamma}} \left(\frac{1 + \varpi + \zeta i}{1 + \xi i} \right)^{\frac{2\gamma - 1}{1-\gamma}} \frac{\zeta - \xi(1 + \varpi)}{(1 + \xi i)^2} \right]. \qquad \text{(C8)}$$

For a sufficiently large ϖ, $d\pi/di > 0$ can hold because the term $\zeta - \xi(1 + \varpi)$ is negative. Specifically, under the conditions given in Propositions 1 and 2, $d\pi/di > 0$ if and only if

$$1 > \frac{\xi(1 + \varpi) - \zeta}{(1 + \varpi + \zeta i)^2} \left[\frac{(1 - \theta)(1 + \varpi) - \zeta}{\xi(1 + \varpi) - \zeta} \kappa \Pi - \left(\frac{\gamma(1 + \varpi + \zeta i)}{1 + \xi i} \right)^{\frac{1}{1-\gamma}} \right] \qquad \text{(C9)}$$

from (C8). Supposing $\xi > 1 - \theta$, the right-hand side is bounded from above by $(1 - \theta) \kappa \Pi / \xi$ since it is increasing in ϖ. Thus, if $\xi > 1 - \theta$, (C9) always holds if (C2) holds (which requires a sufficiently high $\theta < 1$). ∎

Proof of Proposition 2. Substituting (15) and (19) into (20) yields the entry rate as

$$\varepsilon^* = \frac{1 + (1 - \theta) i}{1 + \varpi + \zeta i} \kappa \Pi - \gamma^{\frac{1}{1-\gamma}} \left(\frac{1 + \varpi + \zeta i}{1 + \xi i} \right)^{\frac{\gamma}{1-\gamma}} - \rho. \qquad \text{(C10)}$$

By differentiating ε^* with respect to i, $d\varepsilon^*/di > 0$ if and only if

$$\underbrace{((1 - \theta)(1 + \varpi) - \zeta) \kappa \Pi}_{\equiv \vartheta_L} > \underbrace{\frac{\gamma^{(2-\gamma)/(1-\gamma)}}{1 - \gamma} \left(\frac{1 + \varpi + \zeta i}{1 + \xi i} \right)^{1/(1-\gamma)} (\zeta - \xi(1 + \varpi))}_{\equiv \vartheta_R(i)}. \qquad \text{(C11)}$$

Suppose $(1 - \theta)(1 + \varpi) - \zeta > 0$ (i.e., $\varpi > \zeta/(1 - \theta) - 1$). Then, (C11) always holds if $\zeta - \xi(1 + \varpi) \leq 0$ (i.e., $\varpi > \zeta/\xi - 1$), since the left-hand side, ϑ_L, is positive and the right-hand side, $\vartheta_R(i)$, is zero or negative.

Otherwise, if $\zeta - \xi(1 + \varpi) > 0$, ϑ_L and $\vartheta_R(i)$ are both positive, and $\vartheta_L(i)$ is increasing in i. There are three possible cases: (i) $\vartheta_L > \vartheta_R(0)$ and $\vartheta_L \geq \vartheta_R(\infty)$, (ii) $\vartheta_L > \vartheta_R(0)$ and

$\vartheta_L < \vartheta_R(\infty)$, and (iii) $\vartheta_L \le \vartheta_R(0)$ and $\vartheta_L < \vartheta_R(\infty)$. Define

$$\tilde{\gamma} \equiv \left(\frac{(1-\theta)(1+\varpi) - \zeta}{\zeta - \xi(1+\varpi)} \kappa \Pi \frac{1-\gamma}{\gamma^{(2-\gamma)/(1-\gamma)}} \right)^{1-\gamma}. \tag{C12}$$

In case (i), where $\tilde{\gamma} \ge \zeta/\xi$, (C11) always holds. In case (ii), where $1 + \varpi < \tilde{\gamma} < \zeta/\xi$, (C11) holds if and only if $i \ge \tilde{i}$ in which $\tilde{i} \equiv (\tilde{\gamma} - (1+\varpi))/(\zeta - \tilde{\gamma}\xi)$. In case (iii), where $\tilde{\gamma} \le 1 + \varpi$, (C11) is always violated. All these prove that the effect of a higher i on ε^* is monotonically positive if $\varpi > \zeta/\min\{\xi, (1-\theta)\} - 1$; otherwise, it is positive, negative, or inverted U-shaped. ∎

Proof for the effect on the firm age distribution. Differentiating (22) yields:

$$\frac{d}{di} f_t(a) < 0 \iff (\varepsilon^*)' (1 - a(\varepsilon^* + \eta)) e^{-(\varepsilon^* + \eta)a} < 0. \tag{C13}$$

As shown in Proposition 2, under $\varpi \ge \zeta/\min\{\xi, 1 - \theta\} - 1$, $(\varepsilon^*)' > 0$ holds. In this case, there necessarily exists some cutoff value of a, $\tilde{a} \equiv 1/(\varepsilon^* + \eta)$, such that $(\varepsilon^*)' (1 - a(\varepsilon^* + \eta)) < 0$ for any $a > \tilde{a}$. Therefore, a higher i decreases the density of older firms. ∎