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Hilda SALEH
Department of Mathematics, CT.C., Islamic Azad University, Tehran, Iran

Morteza SHAFIEE
School of Business and Law, Edith Cowan University, Perth, Australia

Kristiaan KERSTENS
IESEG School of Management, Univ. Lille, CNRS, UMR 9221 - LEM - Lille Économie Management, F-59000 Lille, France. Corresponding author.

IÉSEG School of Management Lille Catholic University 3, rue de la Digue F-59000 Lille Tel: 33(0)3 20 54 58 92
www.ieseg.fr

Modelling Nonconvex Network Production Technologies in Wheat Supply Chains: Centralized and Decentralised Approaches

Hilda Saleh[1], Morteza Shafiee[2], Kristiaan Kerstens[3]

[1]Department of Mathematics, CT.C., Islamic Azad University, Tehran, Iran

[2]School of Business and Law, Edith Cowan University, Perth, Australia

[3]Univ. Lille, CNRS, IESEG School of Management, UMR 9221 - LEM - Lille Economie Management, Lille, France. Corresponding author.

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Abstract

This study proposes a nonconvex network data envelopment analysis model for evaluating efficiency in multi-stage supply chains under centralized and decentralized decision structures. Existing network DEA models typically rely on convexity assumptions, which limit their ability to capture indivisibilities and scale discontinuities inherent in real-world supply chains. To address this gap, we develop a unified framework that accommodates nonconvex production technologies while enabling efficiency assessment at both the stage and the system levels. The model allows identification of coordination inefficiencies and structural performance differences across decision regimes. An empirical application to national wheat supply chains demonstrates significant efficiency variation across stages and decision units. It reveals that high efficiency at individual stages does not guarantee overall system efficiency. The results highlight the critical role of coordination in improving supply chain performance. The proposed framework provides a more realistic efficiency assessment tool and offers practical insights for improving resource allocation and operational coordination in complex supply chains.

Keywords: Data Envelopment Analysis, Network, Nonconvex Technology, Centralized and Decentralized Systems, Wheat Supply Chain.

1 Introduction

Supply chains encompass a series of interconnected activities, entities, and processes, ranging from raw material procurement to the production and distribution of final products to end customers. Since its introduction in the 1980s by pioneers such as Oliver (1980), supply chain management (SCM) has evolved into a cornerstone of global competitive advantage (Christopher, 2016; Mentzer et al., 2001). In contemporary markets, individual firms cannot ensure sustained success on their own. Rather, the performance of the entire supply chain—including producers, distributors, and service providers—determines value creation and competitiveness. Weaknesses or inefficiencies at any stage can propagate throughout the system, undermining overall performance and potentially leading to increased costs, reduced service quality, or lost market opportunities. These considerations are particularly critical in strategic supply chains, such as those in agriculture and food production, where inefficiencies can have significant economic and social consequences.

Considering the interdependencies and performance spillovers inherent in supply chains, effective supply chain management critically relies on rigorous performance evaluation frameworks. Such evaluations enable managers to identify bottlenecks, allocate resources more efficiently, and design informed strategies for continuous improvement across interconnected stages of the chain. To this end, a wide range of assessment approaches has been proposed, including financial and non-financial performance indicators, cost–benefit analyses, the Balanced Scorecard (BSC) (Balaji et al., 2021; Garcia-Buendia et al., 2024), simulation-based models, and multi-criteria decision-making (MCDM) methods (Tsai et al., 2021; Tsai et al., 2023). Although these tools provide valuable managerial insights, many of them are limited in their ability to account for multiple inputs and outputs simultaneously or to explicitly capture the complex interactions and dependencies among supply chain members. As a result, there is a growing need for analytical frameworks that systematically and coherently address the multidimensional, networked nature of modern supply chains.

To address these methodological limitations and to capture the multidimensional structure of supply chains, Data Envelopment Analysis (DEA) has emerged as a particularly suitable analytical framework. Originally introduced by Charnes et al. (1978), DEA is a nonparametric, linear-programming–based approach for evaluating the relative efficiency of decision-making units (DMUs) operating with multiple inputs and outputs. Unlike traditional ratio-based or indicator-based measures, DEA constructs an empirical production frontier and assesses efficiency without requiring a predefined functional form, thereby offering greater flexibility in complex operational environments. The classical Charnes et al. (1978) model assumes constant returns to scale (CRS) and measures overall technical efficiency. In contrast, the Banker et al. (1984) model extends the framework to accommodate variable returns to scale (VRS). Owing to these properties, DEA has been widely applied in diverse sectors—including healthcare, education, finance, transportation, and agriculture—where performance evaluation involves multiple, often conflicting criteria. Its ability to simultaneously handle multiple inputs and outputs while providing benchmark targets makes DEA particularly well-suited for analysing interconnected systems such as supply chains, where performance assessment must account for both resource utilization and output generation across multiple stages.

While the classical DEA models provide valuable efficiency measures, their standard formulations treat decision-making units as “black boxes,” ignoring the internal structure and intermediate linkages that characterize many real-world systems. To enhance discriminatory power and extend applicability to more complex settings, several methodological advancements have been proposed. Among these extensions, some studies have questioned not only the internal-structure assumption, but also the convexity assumption underlying traditional DEA frontiers. The Free Disposal Hull (FDH) model relaxes the convexity assumption of traditional DEA which implies perfect time divisibility and allows for the evaluation of efficiency in a nonconvex production possibility set (Kerstens & Van de Woestyne, 2021). When perfect time indivisibility does not hold, then there exists setup times and the ensuing setup costs. This feature is particularly relevant in environments where scale indivisibilities or technological discontinuities exist. More importantly, Network DEA (NDEA) extends the classical framework by explicitly modelling the internal structure of DMUs and the interactions among their subcomponents (see Kao (2017), for an early survey). By incorporating intermediate products and multi-stage processes, NDEA captures the flow of resources and outputs across interconnected stages, providing a more comprehensive representation of operational systems. Such an approach enables the identification of inefficiencies not only at the aggregate level, but also within specific sub-processes. Given the inherently networked and interdependent nature of supply chains, in which the outputs of one stage become the inputs to another, these structural extensions of DEA offer a conceptually consistent and analytically powerful foundation for performance evaluation beyond the limitations of traditional “black-box” models.

Building upon these structural extensions, the application of DEA to supply chain management has expanded substantially in recent years (see Agrell and Hatami-Marbini (2013), for an early survey and Hosseinzadeh Lotfi et al. (2023) for a more recent one). Recognizing that supply chains operate as interconnected multi-stage systems rather than isolated entities, researchers have increasingly adopted multi-stage and network DEA models to evaluate both overall supply chain efficiency and the performance of individual members within the network (Fathi et al., 2022; Kraude et al., 2022; Tavassoli, 2025). These approaches incorporate key performance indicators such as operational costs, inventory levels, capacity utilization, service quality, and profitability, thereby reflecting the multidimensional nature of supply chain operations. By simultaneously accounting for intermediate flows and stage-specific performance, network-based DEA models provide a richer analytical perspective than aggregate efficiency measures. They enable the decomposition of inefficiencies into those attributable to individual units and those arising from structural interactions among stages. This distinction is essential for effective resource allocation, coordination mechanisms, and process optimization within complex supply chain networks. Consequently, DEA has evolved into a robust decision-support framework for analysing supply chain performance under conditions of operational interdependence and competitive pressure.

Within this broader context, agricultural supply chains constitute a particularly critical domain for performance evaluation due to their substantial economic, social, and strategic implications. Agricultural systems are inherently characterized by seasonality, biological production processes, perishability, and significant demand and supply uncertainty, which make planning and coordination considerably more complex than in many industrial supply chains (Ahumada & Villalobos, 2009). Among agricultural commodities, wheat occupies a central position in global food systems, as it is one of the most widely produced and consumed staple crops worldwide and plays a fundamental role in food security and international trade (Fao, 2018; Shiferaw et al., 2013). Given its strategic importance, disruptions in wheat production, storage, or distribution can contribute to food price volatility and reduced accessibility to staple foods, thereby affecting household welfare and national economies (Shiferaw et al., 2013). The wheat supply chain spans multiple interdependent stages, including input procurement, cultivation, harvesting, storage, processing, transportation, and final distribution. Consequently, improving efficiency and coordination across these stages has direct implications for production costs, consumer prices, trade competitiveness, and the stability of food supply systems. A rigorous and structurally consistent performance evaluation framework is therefore essential to identify inefficiencies and support evidence-based managerial and policy decisions in this strategically significant sector.

Despite extensive research on agricultural supply chain efficiency, several methodological gaps remain, particularly for strategically important commodities such as wheat. Wheat, as one of the most globally significant cereals, plays a pivotal role in food security, national economies, and market stability. Its supply chain spans raw material procurement, production, storage, transportation, and distribution of wheat-based products to end consumers. Disruptions at any stage can result in price volatility, reduced access to essential foods, and broader economic or social instability.

While DEA-based frameworks have been widely applied to evaluate performance in networked systems (Tavakoli & Mostafaee, 2019), these approaches often face challenges when applied to complex agricultural supply chains. They may not adequately handle nonconvex production possibilities, simultaneously evaluate both centralized and decentralized network perspectives, or identify returns to scale at both the unit and the overall system levels. Building upon these gaps, the present study develops a more robust analytical framework for assessing wheat supply chain performance. Specifically, the proposed approach aims to:

i) Overcome methodological limitations of conventional DEA applications

ii) provide a unified model for both centralized and decentralized network evaluation

iii) assess efficiency in a nonconvex production space

iv) analyse the impact of returns to scale on both individual stages and the overall supply chain.

By integrating these dimensions, this study contributes to the DEA methodology while offering practical insights for managing one of the world's most vital food supply chains.

Through this approach, the study makes two contributions. First, it advances the literature on DEA applications in agricultural supply chains by addressing methodological limitations in existing frameworks and extending efficiency analysis to complex, multi-stage systems. Second, it provides practical guidance for policymakers and supply chain managers seeking to optimize resource allocation, enhance operational efficiency, and maintain stability in strategically important food supply chains. The proposed framework quantifies performance at multiple levels, reveals hidden inefficiencies, and highlights critical areas for targeted interventions, thereby supporting both strategic and operational decision-making. By combining methodological rigor with real-world applicability, the study strengthens theoretical understanding while delivering actionable insights for the management of complex supply chain systems. This framework is expected to support policymakers and managers in responding to supply chain disruptions and ensuring food security.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on DEA applications in supply chains and identifies key research gaps. Section 3 examines the Tavakoli and Mostafaee (2019) nonconvex network DEA models, outlines their methodological limitations, and introduces a modified framework tailored for nonconvex supply chain analysis from both centralized and decentralized perspectives. Section 4 presents a practical case study of the wheat supply chain, illustrating the applicability and effectiveness of the proposed model. Finally, Section 5 summarizes the main findings, discusses the managerial and policy implications, and provides recommendations to optimize performance in agricultural supply chains.

2 Performance Evaluation in Multi-Stage Network Systems: From Convex to Nonconvex DEA Extension

2.1 DEA in Agricultural and Wheat Supply Chains

Efficiency assessment in agricultural supply chains has become increasingly important due to the strategic role of food systems in economic stability and food security. Wheat supply chains, in particular, have attracted significant attention because of their multi-stage structure and systemic impact. Early strategic perspectives on supply chain management emphasized integration and coordination among trading partners (Power, 2005), highlighting that performance must be evaluated beyond isolated entities.

DEA-based approaches have been widely adopted for evaluating supply chain efficiency. Liang et al. (2006) developed DEA formulations under leader–follower and cooperative structures, demonstrating that overall supply chain efficiency may mask inefficiencies at the subsystem level. Similarly, Saranga and Moser (2010) applied a two-stage value chain DEA model to Purchasing and Supply Management, illustrating the importance of decomposed efficiency measurement for strategic benchmarking.

Subsequent studies extended DEA applications to agricultural and food supply chains. Rosiana et al. (2017) applied DEA to Indonesia's coffee supply chain, while Majiwa et al. (2018) showed that standard DEA overestimates efficiency in Kenya's rice post-harvest system and that network DEA improves discriminatory power. In wheat-related contexts, studies such as Mohammadi et al. (2022), Singh et al. (2019), and Radmand et al. (2025) employed DEA and hybrid LCA–DEA frameworks to improve farm-level efficiency and sustainability.

More recently, dynamic and multi-stage models have been introduced. Mohamadi Janaki et al. (2022) applied dynamic DEA combined with SCOR-based indicators. Rajaei Qazlue et al. (2024) proposed a dynamic multi-stage network DEA model for wheat farms.

Although these studies confirm the suitability of DEA for agricultural systems, most remain confined to convex production technologies and often focus on specific subsystems (e.g., farms) rather than the entire multi-stage wheat supply chain. This limitation becomes critical when production technologies exhibit indivisibilities and discontinuities.

2.2 Network DEA Models for Multi-Stage Systems

Network DEA (NDEA) extends the classical DEA framework by explicitly accounting for DMU internal structure, multi-stage processes, and interactions among subcomponents. This approach enables a more detailed assessment of efficiency in complex, interconnected systems compared to classical "black-box" DEA models . NDEA captures the flow of resources and outputs across stages, allowing identification of inefficiencies not only at the aggregate level but also within specific subsystems.

Applications of NDEA span multiple sectors. In agriculture, Majiwa et al. (2018) applied a network DEA model to Kenya's rice industry, evaluating drying and milling nodes separately, and demonstrated that investments in post-harvest technologies improve efficiency. Rosiana et al. (2017) analysed Indonesia's robusta coffee supply chain using network DEA, showing that coordination among sellers and buyers enhances overall performance. Beyond agriculture, NDEA has been applied in banking (Zhuo et al., 2021), healthcare (Flokou et al., 2024), and manufacturing systems, demonstrating its versatility in evaluating multi-stage, interconnected operations.

Methodological extensions have further enhanced NDEA's applicability. Tavakoli and Mostafaee (2019) have extended NDEA to the nonconvex Free Disposal Hull (FDH) technologies allowing for the calculation of overall and stage efficiencies even with missing data. Ebrahimi et al. (2021) propose a dynamic NDEA model based on FDH for interval data, enabling managers to monitor efficiency across periods and divisions. Rajaei Qazlue et al. (2024) apply a dynamic multi-stage NDEA model to evaluate farm efficiency, showing that management practices significantly influence stage-specific and overall performance.

To handle uncertainty and improve robustness, fuzzy and simulation-based approaches have been integrated with NDEA. Azadi et al. (2024) propose a fuzzy network DEA model to evaluate resilience and inefficiencies under deep uncertainty, such as during disruptions caused by the COVID-19 pandemic. Tavassoli (2025) introduce a fuzzy NDEA model that addresses multiple optimal weights and fairness across stages, enabling a unique and equitable decomposition of efficiency across multi-stage networks.

Collectively, these studies demonstrate that NDEA provides a robust and flexible framework for evaluating multi-stage systems, capturing stage-specific inefficiencies, and supporting resource allocation and coordination decisions.

2.3 Nonconvex DEA

Nonconvex DEA extends the classical and network DEA frameworks by relaxing the convexity assumption of the technology. In many real-world systems, production and operational processes are inherently nonconvex due to indivisibilities, scale limitations, technological discontinuities, or discrete decision variables. By allowing efficiency evaluation in nonconvex environments, these models provide more realistic and accurate assessments of performance across multiple stages or interconnected components (Fukuyama et al., 2016; Papaioannou & Podinovski, 2025; Tavakoli & Mostafaee, 2019).

Several nonconvex DEA approaches have been proposed to address these challenges. The Free Disposal Hull (FDH) model allows for efficiency evaluation without assuming convex combinations of inputs and outputs, making it suitable for systems with discrete or indivisible resources (Fukuyama et al., 2016). Extensions such as the Multicomponent FDH (MFDH) further enhance discriminatory power by allowing each DMU to consist of independent components with shared and specific inputs/outputs, enabling the construction of hypothetical units for benchmarking purposes (Papaioannou & Podinovski, 2025). Kerstens and Van de Woestyne (2021) have illustrated that convexity not only affects the technology and returns to scale, but also the cost function and economies of scale.

Applications of nonconvex DEA are broad and span multiple sectors. For example, studies have analysed banking efficiency (De Borger et al., 1998), healthcare operations (Flokou et al., 2024), education (Papaioannou & Podinovski, 2025), and manufacturing systems (Fukuyama et al., 2016). In these contexts, nonconvex models provide finer distinctions among units, allow for stage-specific efficiency evaluation, and accommodate irregular production technologies that classical convex DEA models cannot capture.

Further extensions incorporate dynamic, interval-based, and fuzzy modelling to address uncertainty and variability in real systems. For instance, Ebrahimi et al. (2021) develop a dynamic network DEA based on FDH with interval data, while Azadi et al. (2024) and Tavassoli (2025) propose fuzzy network DEA frameworks to account for deep uncertainty and fairness issues in multi-stage processes. These approaches demonstrate the flexibility and robustness of nonconvex DEA in evaluating complex, multi-stage, and uncertain systems.

Overall, nonconvex DEA provides a generalized and adaptable framework for efficiency analysis across diverse operational environments. This discussion lays the methodological foundation for the subsequent application of nonconvex Network DEA to agricultural supply chains, specifically wheat, where discrete decisions, resource limitations, and environmental variability create an inherently nonconvex production technology, as will be illustrated in the case study developed in Section 4. Table 1 summarizes key studies on nonconvex DEA across different sectors, highlighting model type, application area, and main findings.

Table 1. Key Studies in Nonconvex DEA

Authors (Year)	Methodological	Technology	DEA Type	Application Context
Flokou et al. (2024)	DEA and FDH comparison	Nonconvex	FDH	Greek healthcare units' efficiency
Papaioannou and Podinovski (2025)	FDH Extension (Multicomponent)	Nonconvex	FDH	Enhanced discrimination without convexity
Ebrahimi et al. (2021)	Dynamic FDH-based NDEA	Nonconvex	Dynamic NDEA	Supply Chain Sustainability with Interval Data
Tavakoli and Mostafaee (2019)	NDEA under FDH	Nonconvex	NFDH	Supply Chain Performance in Iran
Fukuyama et al. (2016)	Nonconvex FDH Measures	Nonconvex	FDH	Efficiency analysis with strong monotonicity

2.4 Research Gap and Motivation

The preceding review highlights the growing use of DEA and its network extensions for evaluating multi-stage and interconnected systems across various sectors, including agriculture, banking, healthcare, and education (Ebrahimi et al., 2021; Majiwa et al., 2018; Papaioannou & Podinovski, 2025; Tavakoli & Mostafaee, 2019). NDEA models have proven effective in capturing internal structures, intermediate flows, and stage-specific efficiencies. At the same time, nonconvex DEA approaches, such as FDH and MFDH, allow for more realistic assessment in environments with discrete decisions, scale limitations, and technological discontinuities (Fukuyama et al., 2016; Papaioannou & Podinovski, 2025).

Despite these methodological advancements, several gaps remain in the literature. First, most existing studies focus either on network structures or on nonconvex environments, but rarely integrate both aspects simultaneously. For example, Tavakoli and Mostafaee (2019) extend NDEA to nonconvex FDH technologies, yet applications are largely limited to banking contexts, leaving broader multi-stage operational systems underexplored. Similarly, fuzzy, interval, and dynamic extensions have been proposed (Azadi et al., 2024; Ebrahimi et al., 2021; Tavassoli, 2025), but their combined potential for analysing complex, multi-stage networks in applied contexts—such as supply chains—remains underutilized.

Second, while NDEA enables stage-specific efficiency assessment, few studies explicitly address integrating centralized and decentralized perspectives to capture both overall system efficiency and individual-stage performance. This is particularly relevant for multi-member systems where local bottlenecks or inefficiencies may be masked by aggregate efficiency scores (Rajaei Qazlue et al., 2024). Third, the application of DEA to strategically important agricultural supply chains, such as wheat, remains limited. Although classical DEA and NDEA have been used to evaluate efficiency in agricultural systems (Majiwa et al., 2018; Mohamadi Janaki et al., 2022; Singh et al., 2019), the inherent nonconvexities arising from discrete planting decisions, resource constraints, and environmental variability are rarely addressed in a NDEA framework.

Building on these gaps, the present study develops a nonconvex network DEA framework tailored for multi-stage systems, providing:

1) Integration of centralized and decentralized perspectives, allowing efficiency assessment at both system and stage levels.
2) Accommodation of nonconvex technologies, reflecting real-world operational constraints.
3) Applicability to strategic supply chains, with the wheat supply chain serving as an illustrative case study.

By addressing these methodological and practical gaps, the proposed framework contributes to both DEA methodology and supply chain management, offering a more accurate, flexible, and actionable tool for assessing performance in complex, multi-stage, and networked systems.

3 Proposed Method

3.1 Problem Statement

One of the most important issues in any supply chain is the type of leadership within it. Chen and Yan (2011) propose two leadership structures. First, in some supply chains, each part of the chain has an independent manager. In other words, each manager selfishly focuses only on the performance of just the part under his leadership. This supply chain is called a decentralized supply chain (DSC) (Chen & Yan, 2011). For performance supply chains, such as those shown in Figure 1, Chen and Yan (2011) propose model (1).

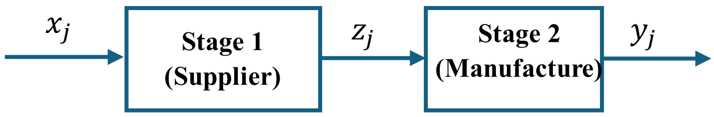

Figure 1: A two-member supply chain

Such that $x_j^1 = (x_{1j}^1, \ldots, x_{mj}^1)$ are inputs of stage 1 and $y_j^2 = (y_{1j}^2, \ldots, y_{sj}^2)$ are the final outputs produced by 2. Also, $z_j = (z_{1j}, \ldots, z_{Dj})$ are intermediate values.

$min\ \theta_{decentral}$

$s.t$ (1)

$$\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq \theta_{decentral} x_{io}^1,\ i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{dj},\ d = 1, \dots, D,$$

$$\sum_{j=1}^{n} \lambda_j^2 z_{dj} \leq z_{do}, d = 1, \dots, D,$$

$$\sum_{j=1}^{n} \lambda_j^2 y_{rj}^2 \geq y_{ro}^2,\ r = 1, \dots, s$$

$$\lambda_j^1, \lambda_j^2 \geq 0, j = 1, \dots, n$$

Second, in a centralized supply chain (CSC), the entire chain is under the leadership of an integrated management, and all members, in addition to considering their own performance, must strive to improve the global chain's performance. Therefore, model (2) is introduced to calculate the efficiency of two-member centralized supply chains under a convex technology.

$$\min \theta_{Central}$$

$$s.t \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq \theta_{Central} x_{io}^1,\ i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{dj},\ d = 1, \dots, D,$$

$$\sum_{j=1}^{n} \lambda_j^2 y_{rj}^2 \geq y_{ro}^2,\ r = 1, \dots, s,$$

$$\lambda_j^1, \lambda_j^2 \geq 0, j = 1, \dots, n$$

Now consider a two-member supply chain (supplier-manufacture) shown in Figure 2. As can be seen, $x_j^k = (x_{1j}^k, \dots, x_{mj}^k), k = 1,2$ are the independent inputs of stages 1 and 2, and $y_j^k = (y_{1j}^k, \dots, y_{sj}^k), k = 1,2$ are the independent outputs produced by stages 1 and 2. Also, $z_j = (z_{1j}, \dots, z_{Dj})$ are intermediate values (intermediate products) that are produced by stage 1 and are considered as inputs in stage 2.

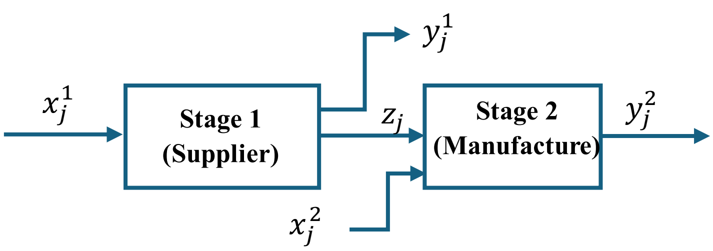

Figure 2: A general two-member supply chains

Considering the two-stage structure and the internal connections between the stages of a supply chain, as stated in the introduction, NDEA models are powerful tools for evaluating multi-stage supply chains. However, as shown in the literature review, although nonconvexity is present in many supply chain indicators, it has been largely ignored in the evaluation of supply chains using NDEA models. Therefore, we focus on this topic in this section.

Tavakoli and Mostafaee (2019) introduce model (1) to calculate the efficiency of two-stage processes under the nonconvex technology. so that:

$$\Gamma \in \{CRS, VRS, NIRS, NDRS\}.$$

They then extend the proposed model to *K*-stage networks.

In model (3), first, the overall network efficiency is calculated, and then the efficiency of each stage is determined based on it. Therefore, in a network with *k* stages, *K+1* separate models must be solved to calculate the overall efficiency and the efficiency of each of the stages 1, 2, …, *K*.

$$\begin{aligned}
&\theta_o^{\Gamma} = \min \theta \\
&s.t \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3) \\
&\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \leq \theta x_{io}^k, i = 1, \dots, m, k = 1,2, \\
&\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \theta z_{do}, d = 1, \dots, D, \\
&\sum_{j=1}^{n} \lambda_j^2 z_{dj} \leq \theta z_{do}, d = 1, \dots, D, \\
&\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \geq y_{ro}^k, r = 1, \dots, S, k = 1,2, \\
&\lambda_j^k = \delta_j^k w_j^k, j = 1, \dots, n, \\
&\sum_{j=1}^{n} w_j^k = 1, k = 1,2, \\
&w_j^k \in \{0,1\}, \quad \delta_j^k \in \Gamma, \quad j = 1, \dots, n, k = 1,2.
\end{aligned}$$

such that:

$\Gamma^{CRS} = \{\delta^k \mid \delta^k \geq 0\}$, $\Gamma^{VRS} = \{\delta^k \mid \delta^k = 1\}$, $\Gamma^{NIRS} = \{\delta^k \mid 0 \leq \delta^k \leq 1\}$ and $\Gamma^{NDRS} = \{\delta^k \mid \delta^k \geq 1\}$.

As can be seen in model (3), the constraints of $\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq \theta x_{io}^1$, $\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \theta z_{do}$, and $\sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 \geq y_{ro}^1$ respectively correspond to the independent input, intermediate products (which are considered as the output of stage 1), and the independent outputs of stage 1 of the two-stage process, which is shown in Figure (2).

As is rather well-known, to calculate the efficiency of the overall network and of each stage separately, an efficient DMU produces the highest output with the lowest input at each stage and the highest final output for the entire system.

However, this principle is violated in model (3) with respect to the intermediate products. Since the intermediate products are considered as the outputs of Stage 1, an efficient DMU — as stated above — should produce the highest possible level of these intermediate values. Yet, the constraint $\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \theta z_{do}$, combined with the objective function $min\ \theta$, works against this principle. Since θ is minimised, the right-hand side θz_{do} decreases, which continuously lowers the reference benchmark for the intermediate outputs of Stage 1. Consequently, the model always projects the intermediate products to values lower than their observed levels, rather than higher. This directly contradicts the fundamental definition of efficiency stated above, in which an efficient DMU is expected to maximise its outputs — including intermediate ones — for a given level of input. Therefore, this behaviour of model (3) is not logically consistent with the standard principles of efficiency measurement.

Also, as explained earlier, the intermediate products are considered inputs for stage 2 and outputs for stage 1. Now consider the constraints $\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \theta z_{do}$ and $\sum_{j=1}^{n} \lambda_j^2 z_{dj} \leq \theta z_{do}$ in the model (3). Considering that in the model (10), θ is reduced, therefore intermediate products are always required to decrease (without considering any input or output role). This issue is also evident in the example given by Tavakoli and Mostafaee (2019). If one sees the values presented in the fifth column of Table 3 of Tavakoli and Mostafaee (2019), it is clear that in the projection point that is presented for inefficient units, the intermediate productions have always decreased compared to their initial value.

As explained earlier, the model (3) is presented based on the decomposition approach. In other words, the efficiency of the stages is calculated based on the total efficiency. An alternative approach, recently presented by Despotis, Koronakos and Sotiros (2016a), and then by Despotis, Koronakos and Sotiros (2016b), Despotis, Sotiros and Koronakos (2016) and expanded in Sahoo et al. (2021), is the composition approach. In this composition approach, unlike the decomposition approach, the total efficiency is calculated from the stage efficiencies. Therefore, in this section, we introduce a new model to calculate the efficiency of the two-stage process under a nonconvex technology using a composition approach.

3.2 Research Methodology

3.2.1 Performance evaluation of decentralized supply chains under nonconvexity

One of the important issues in any supply chain is the type of leadership within it. In some supply chains, each part of the chain has an independent manager. In other words, each manager selfishly focuses only on the performance of the part under his leadership. This supply chain is called a decentralized supply chain (DSC) by (Chen & Yan, 2011).

To evaluate a DSC, suppose we have n DSCs, each of which is denoted by DMU_j from now on. Suppose $DMU_j, j = 1, \ldots, n$ is under evaluation.

Based on Kerstens and Van de Woestyne (2021), a unified algebraic representation of the technology for a decentralized supply chain under nonconvex technologies and different RTS is presented using (4):

$$T_{decentral}^{NC} = \{(x^1, x^2, y^1, y^2) | \sum_{j=1}^{n} \delta_j^k w_j^k x_j^k \le x^k, \sum_{j=1}^{n} \delta_j^k w_j^k y_j^k \ge y^k, \sum_{j=1}^{n} \delta_j^1 w_j^1 z_{dj} \ge z, \sum_{j=1}^{n} \delta_j^2 w_j^2 z_j \le z, k = 1,2, \delta \in \Gamma, w \in \Omega\} \quad (4)$$

such that:

$\Gamma = \Gamma^{CRS} = \{\delta = (\delta_1^1, \ldots, \delta_n^1, \delta_1^2, \ldots, \delta_n^2) | \delta_j^k \ge 0\}$,

$\Gamma = \Gamma^{VRS} = \{\delta = (\delta_1^1, \ldots, \delta_n^1, \delta_1^2, \ldots, \delta_n^2) | \delta_j^k = 1\}$,

$\Gamma = \Gamma^{NIRS} = \{\delta = (\delta_1^1, \ldots, \delta_n^1, \delta_1^2, \ldots, \delta_n^2) | 0 \le \delta_j^k \le 1\}$

or

$\Gamma = \Gamma^{NDRS} = \{\delta = (\delta_1^1, \ldots, \delta_n^1, \delta_1^2, \ldots, \delta_n^2) | \delta_j^k \ge 1\}$.

Also: $\Omega = \Omega^{NC} = \{w = (w_1^1, \ldots, w_n^1, w_1^2, \ldots, w_n^2) | \sum_{j=1}^{n} w_j^k = 1, w_j^k \in \{0,1\}, k = 1,2\}$.

It is necessary to explain that, if $\Omega = \Omega^{C} = \{w = (w_1^1, \ldots, w_n^1, w_1^2, \ldots, w_n^2) | \sum_{j=1}^{n} w_j^k = 1, w_j^k \ge 0, k = 1,2\}$, then $T_{decentral}^{NC}$ converts to $T_{decentral}^{C}$ as proposed by Chen and Yan (2011).

To calculate the efficiency of DMU_o, we first obtain the efficiency of stages 1 and 2 in its input-oriented form, i.e., θ^1 and θ^2, and then calculate the total efficiency based on these efficiencies. Therefore, we introduce model (5) to calculate the efficiency of two-stage units under some nonconvex technology (4):

$$\begin{aligned}
&\min \theta^1 \\
&\min \theta^2 \\
&S.t \qquad (5) \\
&\sum_{j=1}^{n} \lambda_j^k x_{ij}^k \le \theta^k x_{io}^k, i = 1, \ldots, m, k = 1,2, \\
&\sum_{j=1}^{n} \lambda_j^1 z_{dj} \ge z_{do}, d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j^2 z_{dj} \le z_{do}, d = 1, \ldots, D, \\
&\sum_{j=1}^{n} \lambda_j^k y_{rj}^k \ge y_{ro}^k, r = 1, \ldots, S, k = 1,2, \\
&\lambda_j^k = \delta_j^k w_j^k, j = 1, \ldots, n, \\
&\sum_{j=1}^{n} w_j^k = 1, k = 1,2, \\
&w_j^k \in \{0,1\}, \quad \delta_j^k \in \Gamma, \quad j = 1, \ldots, n, k = 1,2
\end{aligned}$$

Although the constraints of stages 1 and 2 in the selfish perspective seek to increase their output and decrease their input, respectively, constraints: $\sum_{j=1}^{n}\lambda_j^1 z_{dj} \geq z_{do}$and $\sum_{j=1}^{n}\lambda_j^2 z_{dj} \leq z_{do}$ lead to $\sum_{j=1}^{n}\lambda_j^1 z_{dj} \geq \sum_{j=1}^{n}\lambda_j^2 z_{dj},$ and this issue is logical. Because suppliers must provide the materials the manufacturer needs, otherwise, this cooperation is not be possible. Also, if in $\sum_{j=1}^{n}\lambda_j^1 z_{dj} \geq \sum_{j=1}^{n}\lambda_j^2 z_{dj}$, the inequality is strict, then there will be waste in raw materials (Chen & Yan, 2011).

Let $(\theta^{1*}, \theta^{2*}, \lambda_j^{k*}, \delta_j^{k*}, w_j^{k*})$ be the optimal solution of model (5), then the efficiency of stages 1 and 2 is equal to θ^{1*} and θ^{2*}, respectively. If $\theta^{k*} = 1$, then stage k is FDH-efficient in the input-orientation. But, if $\theta^{k*} < 1$, then stage k is called FDH-inefficient. Finally, the overall efficiency of a DSC is computed using (6).

$$\theta^{T} = \frac{\theta^{1*} + \theta^{2*}}{2} \quad (6)$$

So, a DSC is FDH-efficient if $\theta^T = 1$ and FDH-inefficient if $\theta^T < 1$. Also, a DSC is FDH-efficient if and only if stages 1 and 2 are both efficient.

Consider model (5) again. Model (5) is a bi-objective mixed-integer programming model. Suppose $(\theta^{1*}, \theta^{2*}, \lambda_j^{k*}, \delta_j^{k*}, w_j^{k*})$ is a feasible solution for model (5). This solution is a Pareto optimal solution for model (5) if and only if no feasible solution such as $(\theta^1, \theta^2, \lambda_j^k, \delta_j^k, w_j^k)$ can be found such that: $(\theta^1, \theta^2) \leq (\theta^{1*}, \theta^{2*})$. Therefore, an ideal solution to this problem is one that simultaneously minimizes both objective functions without worsening the other.

Also, model (5) contains some binary variables; therefore, solving it is computationally intensive. To provide a simpler method for calculating the optimal solution of model (5), a new Theorem 1 is presented.

Theorem 1. Suppose that DMU_o is under evaluation, and we define:

$$\mu_{po}^1 = max\left\{\frac{x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1, \dots, m\right\}$$

$$\eta_{op}^1 = max\left\{\frac{y_{ro}^1}{y_{rp}^1} \middle| y_{rp}^1 + y_{ro}^1 > 0, r = 1, \dots, s\right\}$$

$$\psi_{op}^1 = max\left\{\frac{z_{do}}{z_{dp}} \middle| d = 1, \dots, D\right\}$$

$$\psi_{oq}^2 = min\left\{\frac{z_{do}}{z_{dq}} \middle| d = 1, \dots, D\right\}$$

$$\mu_{qo}^2 = max\left\{\frac{x_{iq}^2}{x_{io}^2} \middle| x_{iq}^2 + x_{io}^2 > 0, i = 1, \dots, m\right\}$$

$$\eta_{oq}^2 = max\left\{\frac{y_{ro}^2}{y_{rq}^2} \middle| y_{rq}^2 + y_{ro}^2 > 0, r = 1, \dots, s\right\}$$

and

$$J_q = \left\{ j \left| \frac{y_{rj}^2}{y_{rq}^2} \leq \frac{z_{dj}}{z_{dq}}, j = 1, \ldots, n \right. \right\}$$

For the case: $\Gamma = CRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p \in J} max\{\eta_{op}^1, \psi_{op}^1\} \mu_{po}^1, \min_{q \in J} \eta_{oq}^2 \mu_{qo}^2)$ is a Pareto optimal solution of model (5) such that: $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$.

For the case: $\Gamma = VRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p \in J} \mu_{po}^1, \min_{q \in J} \mu_{qo}^2)$ is a Pareto optimal solution of model (5) such that: $\eta_{op}^1 \leq 1$, $\eta_{qo}^2 \leq 1$, $\psi_{op}^1 \geq 1$, and $\psi_{oq}^2 \leq 1$.

For the case: $\Gamma = NIRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p \in J} max\{\eta_{op}^1, \psi_{op}^1\} \mu_{po}^1, \min_{q \in J} \eta_{oq}^2 \mu_{qo}^2)$ is a Pareto optimal solution of model (5), such that: $\eta_{op}^1 \leq 1$, $\eta_{qo}^2 \leq 1$, $\psi_{op}^1 \geq 1$, $\psi_{oq}^2 \leq 1$, $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$.

For the case: $\Gamma = NDRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p \in J} max\{\eta_{op}^1, \psi_{op}^1, 1\} \mu_{po}^1, \min_{q \in J} max\{1, \eta_{oq}^2\} \mu_{qo}^2)$ is a Pareto optimal solution of model (5) such that: $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$.

Proof: The details are provided in Appendix A.

3.2.2 Performance Evaluation of Centralized Supply Chains under Nonconvexity

Unlike DSC, in a centralized supply chains (CSC), the entire chain is under the leadership of an integrated management, and all members, in addition to considering their own performance, must strive to improve the chain's overall performance.

Similar to a decentralized supply chain, $T_{central}^{NC}$ is defined as follows:

$$T_{Central}^{NC} = \{(x^1, x^2, y^1, y^2) | \sum_{j=1}^n \delta_j^k w_j^k x_j^k \leq x^k, \sum_{j=1}^n \delta_j^k w_j^k y_j^k \geq y^k, \sum_{j=1}^n \delta_j^1 w_j^1 z_{dj} \geq \sum_{j=1}^n \delta_j^2 w_j^2 z_j, k = 1,2, \delta \in \Gamma, w \in \{0,1\}\} \quad (8)$$

If $\Omega = \Omega^c$, then $T_{Central}^{NC}$ converts to $T_{Central}^{C}$, which is proposed by Chen and Yan (2011). Therefore, model (9) is introduced to calculate the efficiency of two-member centralized supply chains under some nonconvex technology.

$$\begin{aligned}
& \min \theta^1 \\
& \min \theta^2 \\
& S.t \qquad (9) \\
& \sum_{j=1}^n \lambda_j^k x_{ij}^k \leq \theta^k x_{io}^k, i = 1, \ldots, m, k = 1,2, \\
& \sum_{j=1}^n \lambda_j^1 z_{dj} \geq \sum_{j=1}^n \lambda_j^2 z_{dj}, d = 1, \ldots, D, \\
& \sum_{j=1}^n \lambda_j^k y_{rj}^k \geq y_{ro}^k, r = 1, \ldots, S, k = 1,2, \\
& \lambda_j^k = \delta_j^k w_j^k, j = 1, \ldots, n,
\end{aligned}$$

$$\sum_{j=1}^{n} w_j^k = 1, k = 1,2,$$

$$w_j^k \in \{0,1\}, \quad \delta_j^k \in \Gamma, \quad j = 1,\ldots,n, k = 1,2$$

In a centralized view, under the constraint $\sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{dj}$, the supplier's priority is to meet the producer's demand.

Let $(\theta^{1*}, \theta^{2*}, \lambda_j^{k*}, \delta_j^{k*}, w_j^{k*})$ be the optimal solution of model (9), then the efficiency of stages 1 and 2 is equal to θ^{1*} and θ^{2*}, respectively. If $\theta^{k*} = 1$, then stage k is FDH-efficient in the input-orientation. But if $\theta^{k*} < 1$, then stage k is called FDH-inefficient.

Despotis, Koronakos and Sotiros (2016b) and Sahoo et al. (2021) use the weak link concept in supply chains to define the overall efficiency of a two-stage series as equal to the efficiency of the weakest stage. In CSC, since all members are trying to improve the chain's performance, the weak performance of a member directly impacts the performance of other members and the entire chain. As a result, the total efficiency of DMU_o (θ_o^T) is obtained using (10):

$$\theta_o^T = min\{\theta^{1*}, \theta^{2*}\} \tag{10}$$

As can be seen, model (9) is a bi-objective mixed-integer programming. As in section 1.1, to provide a simpler method for calculating the optimal solution of model (9), a new Theorem 2 is presented.

Theorem 2. Suppose that DMU_o is under evaluation, and we define:

$$\mu_{po}^1 = max\left\{\frac{x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1,\ldots,m\right\},$$

$$\eta_{op}^1 = max\left\{\frac{y_{ro}^1}{y_{rp}^1} \middle| y_{rp}^1 + y_{ro}^1 > 0, r = 1,\ldots,s\right\},$$

$$\psi_p^1 = max\left\{\frac{\bar{z}_d}{z_{dp}} \middle| d = 1,\ldots,D\right\},$$

$$\psi_{pq}^2 = min\left\{\frac{z_{dp}}{z_{dq}} \middle| d = 1,\ldots,D\right\},$$

$$\mu_{qo}^2 = max\left\{\frac{x_{iq}^2}{x_{io}^2} \middle| x_{iq}^2 + x_{io}^2 > 0, i = 1,\ldots,m\right\},$$

$$\eta_{oq}^2 = max\left\{\frac{y_{ro}^2}{y_{rq}^2} \middle| y_{rq}^2 + y_{ro}^2 > 0, r = 1,\ldots,s\right\},$$

$$\bar{z}_d = max\{z_{dq} | q \in J\},$$

For the case: $\Gamma = CRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\eta_{oq}^2\psi_p^1, \eta_{op}^1\}\mu_{po}^1, \min_{q\in J}\eta_{oq}^2\mu_{qo}^2)$ is a Pareto optimal solution of model (4).

For the case: $\Gamma = VRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J}\mu_{po}^1, \min_{q\in J}\mu_{qo}^2)$ is a Pareto optimal solution of model (4) such that: $\eta_{op}^1 \leq 1, \eta_{qo}^2 \leq 1$ and $1 \geq \psi_{qp}^1$.

For the case: $\Gamma = NIRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\eta_{oq}^2\psi_{qp}^1, \eta_{op}^1\}\mu_{po}^1, \min_{q\in J}\eta_{oq}^2\mu_{qo}^2)$ is a Pareto optimal solution of model (4), such that: $\eta_{op}^1 \leq 1, \eta_{qo}^2 \leq 1$, $\eta_{oq}^2\psi_{qp}^1 \leq 1$ and $\eta_{oq}^2 \leq \psi_{pq}^2$.

For the case: $\Gamma = NDRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\{\eta_{oq}^2\psi_{qp}^1, \eta_{op}^1, 1\}\mu_{po}^1, \min_{q\in J} max\{1, \eta_{oq}^2\}\mu_{qo}^2)$ is a Pareto optimal solution of model (4) such that: $\eta_{oq}^2 \le \psi_{oq}^2$ and $q \in J_q$.

Proof: The details are provided in Appendix A.

Another point that should be investigated is the efficiency of centralized and decentralized supply chains and how they relate to each other. Since Chen and Yan (2011) proof the convex case, our new Theorem 3 focuses on the nonconvex case solely and it is presented as follows.

Theorem 3. The optimal solution of the nonconvex centralized model is less than or equal to the optimal solution of the nonconvex decentralized model.

Proof: The details are provided in Appendix A.

So, based on Theorem 3, if a supply chain is NC-efficient under the nonconvex centralized model, then it is NC-efficient under the nonconvex decentralized model.

4 Evaluation and Real-World Applications

4.1 Evaluation Using a Previously Studied Real-World Case

To empirically evaluate the proposed nonconvex NDEA framework, efficiency scores are calculated for 26 branches of Bank Keshavarzi in Iran in 2014, based on the dataset previously analysed by Tavakoli and Mostafaee (2019). The branches operate under a two-stage production system, in which inputs—including human resources, fixed assets, and branch numbers—are first converted into intermediate products (deposits) and subsequently into final outputs (profit). The network structure of the system, illustrated in Figure 3, represents a practical operational setting that captures the complexities of a two-stage production system, providing a suitable benchmark for assessing multi-stage efficiency.

Figure 3. Network Structure Proposed by Tavakoli and Mostafaee (2019)

The efficiency scores obtained from the proposed nonconvex NDEA framework are compared with those reported by Tavakoli and Mostafaee (2019) across the 26 branches. The detailed efficiency scores and comparisons under the CRS, VRS, NIRS, and NDRS assumptions are presented in Table 2, providing a comprehensive view of model performance and its practical implications.

Table 2. Overall efficiency scores under alternative RTS assumptions using the proposed model and Tavakoli and Mostafaee (2019)

DMUs	(Tavakoli & Mostafaee, 2019)				Proposed Model			
	CRS	VRS	NIRS	NDRS	CRS	VRS	NIRS	NDRS
DMU1	1.0000	1.0000	1.0000	1.0000	0.7967	0.6368	0.9806	0.8689
DMU2	0.8461	0.9789	0.8461	0.9568	0.5947	0.5308	0.6735	0.7237

DMU3	0.7172	1.0000	0.8829	0.7172	0.4942	0.7715	0.7841	0.5960
DMU4	0.3536	0.6746	0.3536	0.6746	0.3774	0.4486	0.4456	0.6657
DMU5	0.2798	1.0000	0.4021	1.0000	0.6399	0.7626	0.7556	0.6399
DMU6	0.2323	1.0000	0.2323	1.0000	0.3655	0.8274	0.6274	0.9218
DMU7	0.9333	1.0000	1.0000	0.9333	0.7824	0.6291	0.9942	0.8238
DMU8	0.9423	1.0000	0.9423	1.0000	0.7123	0.8480	0.9368	0.9658
DMU9	0.1950	1.0000	0.1950	1.0000	0.3363	0.7484	0.5933	0.8749
DMU10	0.4672	0.8651	0.4672	0.8651	0.4838	0.7526	0.8044	0.8442
DMU11	0.7871	1.0000	0.8434	1.0000	0.6935	0.5849	0.8000	0.6935
DMU12	0.8843	1.0000	0.8843	1.0000	0.6102	0.7550	0.7541	0.8120
DMU13	0.7983	1.0000	0.7983	0.9162	0.6923	0.6367	0.8775	0.8708
DMU14	1.0000	1.0000	1.0000	1.0000	0.6867	0.6853	0.6964	0.8310
DMU15	0.5707	1.0000	0.5707	1.0000	0.5003	0.7630	0.6760	1.0000
DMU16	0.4284	0.6488	0.4284	0.6488	0.3398	0.4203	0.4244	0.4939
DMU17	0.7905	1.0000	1.0000	0.7905	0.6504	0.5947	0.8979	0.6689
DMU18	0.7186	1.0000	0.7186	0.7631	0.7561	0.7193	0.8989	0.8011
DMU19	0.4021	0.7188	0.4021	0.7188	0.4022	0.4657	0.4836	0.5398
DMU20	0.6973	1.0000	0.7472	0.6973	0.5482	0.6800	0.6160	0.6598
DMU21	1.0000	1.0000	1.0000	1.0000	1.0000	0.8918	1.0000	1.0000
DMU22	1.0000	1.0000	1.0000	1.0000	0.7902	0.7836	0.9129	0.9129
DMU23	0.8973	1.0000	0.8973	1.0000	0.7355	0.7776	0.7320	1.0000
DMU24	1.0000	1.0000	1.0000	1.0000	0.8237	0.7515	0.9605	0.9605
DMU25	1.0000	1.0000	1.0000	1.0000	0.8814	0.7905	1.0000	1.0000
DMU26	1.0000	1.0000	1.0000	1.0000	0.6818	0.8999	1.0000	0.8599

The comparison indicates that the proposed NDEA framework provides a more realistic and discriminating assessment of branch performance. While the previous model identifies a large number of DMUs as fully efficient, the proposed model provides a finer discrimination among branches, allowing variations in operational performance to be more accurately captured. This enhanced discriminatory power supports more accurate benchmarking and facilitates evidence-based managerial decisions, such as identifying areas for improvement and allocating resources effectively.

To further validate the performance of the proposed NDEA framework and ensure the robustness of its efficiency estimates, a Li-Test is conducted (Li, 1996). This nonparametric test is particularly appropriate for paired efficiency scores, as it does not assume normality and is suitable for small sample sizes, both of which characterize nonparametric frontier results. By statistically assessing the median differences between the proposed model and the previously established benchmark, the test provides an objective measure of whether the observed variations are significant, thereby reinforcing the reliability and practical relevance of the proposed framework. The results of this analysis are summarized in Table 3, where "T" denotes the model reported by Tavakoli and Mostafaee (2019), and "PM" denotes the proposed NDEA framework.

Table 3. Statistical Comparison of Efficiency Scores Between Tavakoli's Model and the Proposed NDEA Framework Using the Li-Test (1996)

Return-to-Scale	Model	No. of Efficient DMUs	Mean ± SD	Median	Test Statistic (T)	P-Value (Asymptotic)	P-Value (Bootstrap)
VRS	T	21	0.9572 ± 0.106	1.000	86.1447	<0.0001	<0.001
VRS	PM	0	0.6983 ± 0.130	0.7499	—	—	
CRS	T	7	0.7285 ± 0.272	0.7944	20.9295	<0.0001	<0.009
CRS	PM	1	0.6298 ± 0.177	0.6661	—	—	
NIRS	T	9	0.7543 ± 0.273	0.7543	4.9770	<0.0001	<0.121
NIRS	PM	3	0.7818 ± 0.178	0.7921	—	—	
NDRS	T	15	0.9108 ± 0.127	1.000	17.8097	<0.0001	<0.006
NDRS	PM	4	0.8088 ± 0.150	0.8376	—	—	

The efficiency scores reported in Table 2 highlight substantial differences between the model of (Tavakoli & Mostafaee, 2019) and the proposed NDEA framework across the four return-to-scale assumptions. Under the VRS assumption, Tavakoli's model identifies 21 out of 26 branches as fully efficient. By contrast, the proposed framework classifies none as fully efficient, reflecting a more discriminating evaluation of performance. The mean efficiency under Tavakoli's approach (0.9572 ± 0.106) is considerably higher than that obtained from the proposed framework (0.6983 ± 0.130), with a highly significant Li-Test p-value (<0.001). This indicates that although both approaches preserve relative rankings to some extent, the newly proposed framework provides a finer, and more meaningful differentiation.

A similar pattern emerges under the CRS and NDRS assumptions. For CRS, Tavakoli's model classifies 7 branches as fully efficient, compared with only 1 under the proposed framework. Likewise, under NDRS, 15 branches are deemed fully efficient by Tavakoli's approach versus 4 by the proposed model. These statistically significant differences, together with moderate to large effect sizes, demonstrate that the proposed framework mitigates efficiency overestimation and provides a more nuanced assessment of branch performance.

Under the NIRS assumption, the Tavakoli's model identifies nine fully efficient branches, whereas the proposed model identifies three. However, the Li test indicates that the difference between the two approaches is not statistically significant ($p = 0.121$). This suggests that when non-increasing returns to scale prevail, both methods yield similar evaluations, further confirming the stability and robustness of the proposed framework under specific operational conditions. Figure 4 illustrates the distribution of efficiency scores under the two approaches, offering a visual comparison of their performance patterns across branches.

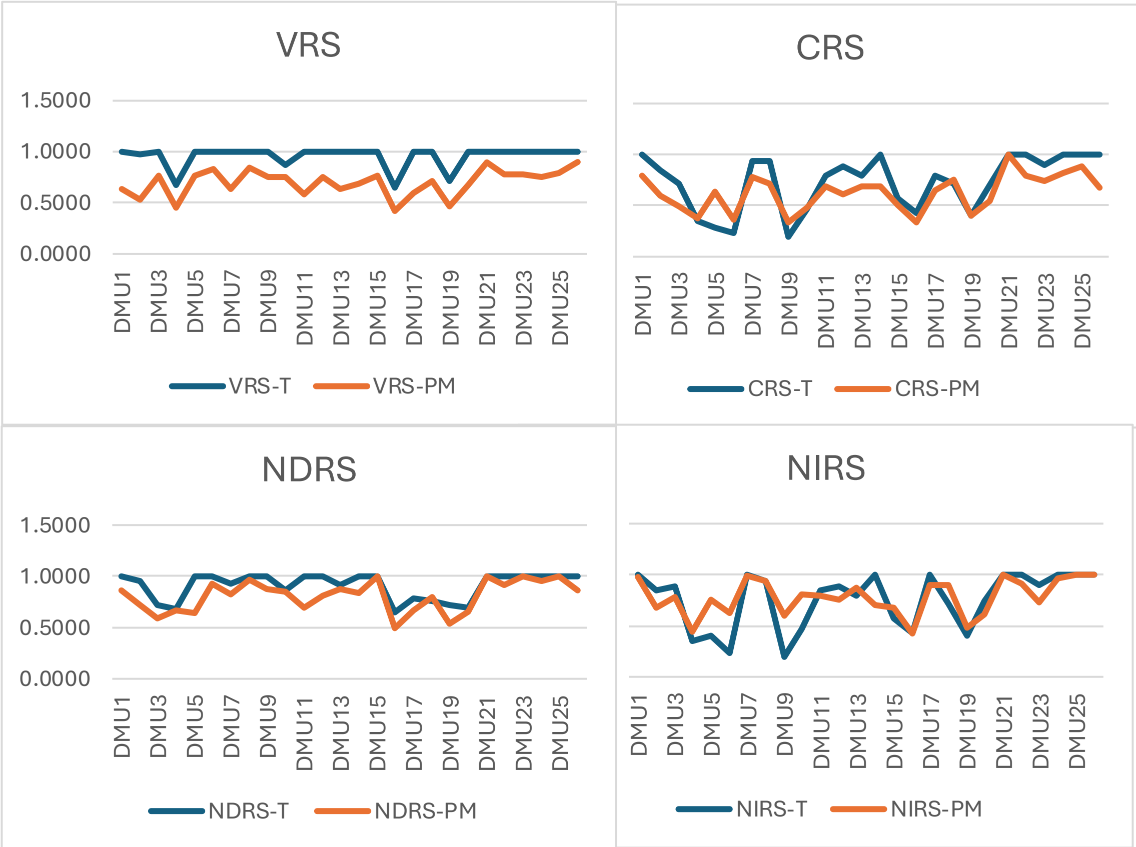

Figure 4. Distribution of Efficiency Scores under Tavakoli's Model and the Proposed NDEA Framework

Overall, the proposed NDEA framework consistently delivers a more realistic portrayal of operational efficiency by effectively capturing variations in branch performance that remain obscured under the traditional DEA specification. By reducing the number of fully efficient DMUs and generating more dispersed efficiency scores, the framework enhances discriminatory power, which is essential for practical benchmarking, performance monitoring, and resource allocation decisions. These findings underscore the superior analytical rigor and managerial relevance of our proposed approach.

4.2 Application to the Wheat Supply Chain Network

This subsection presents the main empirical application of the proposed nonconvex NDEA framework to a wheat supply chain. The model is implemented across 15 DMUs each representing the wheat supply chain of a single country. The selected countries correspond to the 15 major wheat-producing and exporting nations included in this study. Production and performance indicators are extracted from the Food and Agriculture Organization (FAO) database, ensuring consistency, comparability, and international coverage across DMUs.

The wheat supply chain is modelled as a two-stage network, as illustrated in Figure 5, which depicts the flow of inputs, intermediate products, and final outputs across the stages. In Stage 1 (Production Stage), primary agricultural inputs—agricultural labour, cultivated land area (hectares), and nitrogen fertilizer (tons)—are transformed into wheat production (tons). Wheat output constitutes the intermediate product that links the production stage to downstream activities. This intermediate flow captures the physical continuity of the supply chain and enables the decomposition of inefficiencies between upstream production performance and downstream value realization.

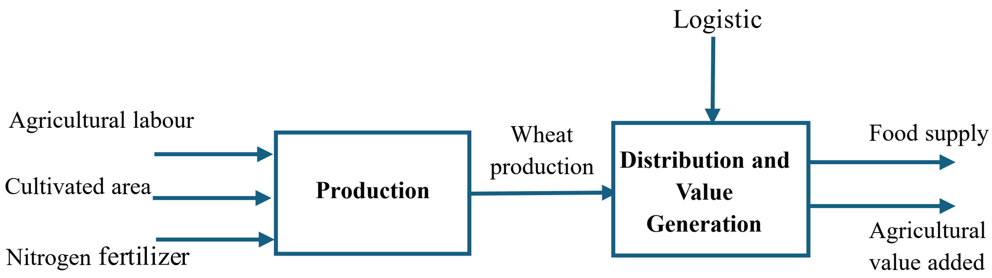

Figure 5. Wheat Supply Chain Structure

In Stage 2 (Distribution and Value Generation Stage), the intermediate product (wheat production) is combined with logistics-related inputs to generate two final outputs: food supply (measured in kilocalories per capita per day) and agricultural value added (USD). These outputs reflect both food security performance and economic contribution, enabling a comprehensive evaluation of the effectiveness of the national wheat supply chain beyond mere production volume. Appendix B (Table B1) presents the dataset for the 15 national wheat supply chains, where each DMU represents a single country. Country names have been anonymised.

This two-stage network representation enables simultaneous assessment of upstream production efficiency, downstream distribution, and value-generation efficiency, and overall system performance under both centralized and decentralized perspectives. By explicitly modelling intermediate flows and stage interactions, the framework identifies potential structural bottlenecks across national supply chains and supports stage-specific benchmarking among leading wheat-producing countries. Furthermore, the nonconvex formulation accommodates realistic production characteristics at the national level, such as input indivisibilities, scale limitations, and technological discontinuities. This is particularly relevant in global wheat systems, where land allocation decisions, fertilizer application intensities, infrastructure capacities, and policy constraints often exhibit discrete or non-linear behaviour.

Overall, this empirical application demonstrates the capability of the proposed nonconvex network DEA model to provide granular and actionable insights into performance heterogeneity, resource allocation, and efficiency improvement opportunities across strategically important wheat-producing and exporting countries, while ensuring that internationally recognised statistical databases support the selected indicators.

Building on the described dataset, the proposed nonconvex network DEA model is implemented under the VRS assumption. The resulting efficiency scores for all 15 DMUs are reported in Table 4, presenting both stage-specific efficiencies (E1 and E2) and overall network efficiency. This formulation enables a comprehensive assessment of national wheat supply chain performance by separating upstream production efficiency from downstream distribution and value-generation performance.

The adoption of the VRS specification is particularly appropriate in this context because national wheat supply chains operate at markedly different scales of production, land endowment, labour intensity, and market integration. Imposing proportional scalability would implicitly assume that all countries can expand or contract inputs and outputs without structural distortions. However, agricultural systems are typically subject to heterogeneous technologies, policy constraints, climatic conditions, and infrastructure capacities. Therefore, the VRS framework allows each country to be evaluated relative to a locally relevant scaled frontier rather than to a globally proportional benchmark.

Table 4. Stage-wise and overall efficiency results for the wheat supply chain

DMUs	VRS		
	E1	E2	OVERALL
DMU1	0.899298	0.645333	0.772315
DMU2	1	0.739946	0.869973
DMU3	0.059317	1	0.529658
DMU4	0.47106	0.71875	0.594905
DMU5	0.764139	0.57619	0.670165
DMU6	0.047393	1	0.523696
DMU7	1	1	1
DMU8	1	0.86121	0.930605
DMU9	0.27156	1	0.63578
DMU10	1	0.683616	0.841808
DMU11	0.417716	1	0.708858
DMU12	0.444296	1	0.722148
DMU13	0.563493	0.855124	0.709308
DMU14	1	0.606516	0.803258
DMU15	0.120935	0.928021	0.524478

The results reveal substantial heterogeneity across both stages. Only one unit, i.e. DMU7, achieves full efficiency in Stage 1 and Stage 2 simultaneously, thereby attaining overall efficiency equal to one. This unit represents a structurally balanced supply chain in which wheat production and downstream value realization are fully aligned under VRS. The uniqueness of this case underscores the difficulty of achieving coordinated efficiency across stages even when scale flexibility is allowed.

Stage efficiency patterns exhibit clear asymmetry. Full efficiency in Stage 1 is observed for DMU2, DMU7, DMU8, DMU10, and DMU14, whereas Stage 2 efficiency characterizes a partially different group of countries, including DMU3, DMU6, DMU7, DMU9, DMU11, and DMU12. The limited overlap between these groups indicates that production strength and downstream performance remain largely decoupled across national systems.

The dispersion in production efficiency is particularly pronounced. Some countries (notably DMU3 and DMU6) exhibit extremely low Stage 1 efficiency, while remaining fully efficient in Stage 2, suggesting that downstream systems can generate food supply and economic value efficiently despite upstream resource misallocation. Conversely, units such as DMU2 and DMU10 demonstrate full production efficiency, but only moderate downstream performance, indicating that strong agricultural productivity does not automatically translate into effective value realization.

Overall efficiency scores reflect this inter-stage imbalance. Apart from DMU7, all supply chains experience coordination losses between stages, preventing them from reaching the VRS frontier. Even high-performing units such as DMU8 and DMU2 remain below full overall efficiency due to residual inefficiencies in one component of the network. The distribution of Stage 1, Stage 2, and overall efficiency scores is illustrated in Figure 6, which visually confirms the dispersion of performance across DMUs and the model's discriminatory power under the VRS assumption.

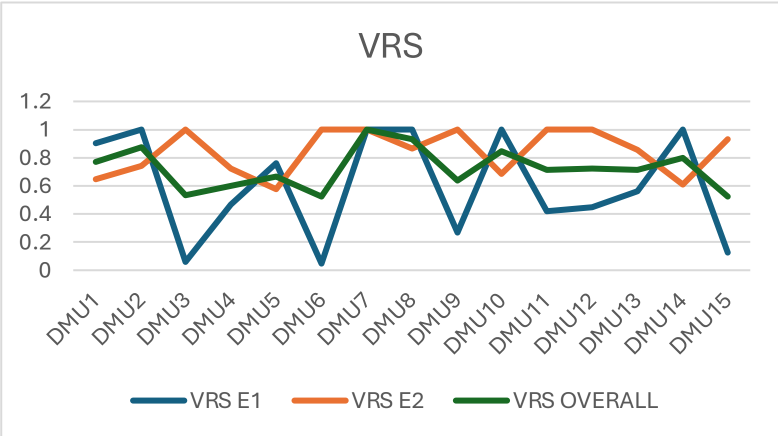

Figure 7. Dispersion of Stage 1, Stage 2, and Overall Performance Across DMUs

Collectively, the VRS findings indicate that performance disparities in national wheat supply chains are primarily structural and stage-specific rather than purely scale-driven. By relaxing the proportionality assumption, the VRS specification provides a more realistic benchmark for heterogeneous agricultural systems and yields more nuanced insights for resource allocation and policy design.

In addition to the VRS specification, the proposed nonconvex network DEA framework can also be implemented under alternative returns-to-scale assumptions, including CRS, NIRS, and NDRS. Applying the model under these alternative scale structures allows for a systematic examination of how the efficiency frontier and stage-level performance respond to different scale conditions. The corresponding efficiency results are reported in Appendix C (Table C1), while Appendix C (Figure C1) provides a visual comparison of Stage 1, Stage 2, and overall efficiencies across the alternative assumptions. This multi-scenario evaluation provides deeper insight into the structural role of scale effects in shaping national wheat supply chain performance.

5 Conclusions

This study developed a nonconvex network DEA framework to evaluate efficiency in multi-stage supply chains under both centralized and decentralized decision-making structures. By relaxing the convexity assumption, the proposed model captures technological indivisibilities and scale discontinuities that are frequently observed in real production systems, but that are overlooked in conventional network DEA models. The framework enables simultaneous assessment of overall system efficiency and stage-level performance, while also providing insights into coordination losses and structural inefficiencies across interconnected stages.

The empirical analysis of wheat supply chains revealed substantial efficiency heterogeneity across decision-making units and stages. The findings demonstrate that efficiency at individual stages does not necessarily translate into overall system efficiency, emphasizing the importance of coordination and structural balance across the supply chain. In particular, decentralized decision structures may lead to coordination losses that reduce overall performance, even when individual stages operate efficiently. These results highlight the importance of adopting system-level performance evaluation approaches rather than focusing solely on isolated components.

From a methodological perspective, the proposed model extends the network DEA literature by incorporating nonconvex technologies into multi-stage efficiency analysis under alternative decision regimes. From a managerial and policy standpoint, the framework provides a practical tool for identifying sources of inefficiency, improving resource allocation, and enhancing coordination across supply chain stages. Such insights are particularly valuable in strategic sectors, such as agricultural supply chains, where improving efficiency directly contributes to economic performance and resource sustainability.

Future research may extend this framework by incorporating dynamic structures, uncertainty, and stochastic variations in production processes, as well as by applying the model to other complex networked systems, such as energy, healthcare, and manufacturing supply chains. Furthermore, it may be useful to determine optimal RTS per stage of production: to our knowledge, this is not available in the literature.

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Electronic Companions: Online-Only Appendices

Appendix A: Proofs and Derivations

Theorem 1. Suppose that DMU_o is under evaluation, and we define:

$$\mu_{po}^{1} = max\left\{\frac{x_{ip}^{1}}{x_{io}^{1}} \middle| x_{ip}^{1} + x_{io}^{1} > 0, i = 1,\dots,m\right\}$$

$$\eta_{op}^{1} = max\left\{\frac{y_{ro}^{1}}{y_{rp}^{1}} \middle| y_{rp}^{1} + y_{ro}^{1} > 0, r = 1,\dots,s\right\}$$

$$\psi_{op}^{1} = max\left\{\frac{z_{do}}{z_{dp}} \middle| d = 1,\dots,D\right\}$$

$$\psi_{oq}^{2} = min\left\{\frac{z_{do}}{z_{dq}} \middle| d = 1,\dots,D\right\}$$

$$\mu_{qo}^{2} = max\left\{\frac{x_{iq}^{2}}{x_{io}^{2}} \middle| x_{iq}^{2} + x_{io}^{2} > 0, i = 1,\dots,m\right\}$$

$$\eta_{oq}^{2} = max\left\{\frac{y_{ro}^{2}}{y_{rq}^{2}} \middle| y_{rq}^{2} + y_{ro}^{2} > 0, r = 1,\dots,s\right\}$$

and

$$J_q = \left\{j \middle| \frac{y_{rj}^{2}}{y_{rq}^{2}} \le \frac{z_{dj}}{z_{dq}}, j = 1,..,n\right\}$$

For the case: $\Gamma = CRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\eta_{op}^{1}, \psi_{op}^{1}\}\, \mu_{po}^{1}, \min_{q\in J} \eta_{oq}^{2}\mu_{qo}^{2})$ is a Pareto optimal solution of model (5) such that: $\eta_{oq}^{2} \le \psi_{oq}^{2}$ and $q \in J_q$.

For the case: $\Gamma = VRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J}\mu_{po}^{1}, \min_{q\in J}\mu_{qo}^{2})$ is a Pareto optimal solution of model (5) such that: $\eta_{op}^{1} \le 1$, $\eta_{qo}^{2} \le 1$, $\psi_{op}^{1} \ge 1$, and $\psi_{oq}^{2} \le 1$.

For the case: $\Gamma = NIRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\eta_{op}^{1}, \psi_{op}^{1}\}\mu_{po}^{1}, \min_{q\in J}\eta_{oq}^{2}\mu_{qo}^{2})$ is a Pareto optimal solution of model (5), such that: $\eta_{op}^{1} \le 1$, $\eta_{qo}^{2} \le 1$, $\psi_{op}^{1} \ge 1$, $\psi_{oq}^{2} \le 1$, $\eta_{oq}^{2} \le \psi_{oq}^{2}$ and $q \in J_q$.

For the case: $\Gamma = NDRS$, $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} max\{\eta_{op}^{1}, \psi_{op}^{1}, 1\}\mu_{po}^{1}, \min_{q\in J} max\{1, \eta_{oq}^{2}\}\mu_{qo}^{2})$ is a Pareto optimal solution of model (5) such that: $\eta_{oq}^{2} \le \psi_{oq}^{2}$ and $q \in J_q$.

Proof:

Part *a*:

For the case: $\Gamma = CRS$, according to the constraints $\sum_{j=1}^{n} w_j^k = 1, k = 1,2,$ for an optimal solution, there exists $p \in J$ such that $w^1 = e_p$ and also there exists $q \in J$so that $w^2 = e_q$. So:

$\lambda_p^1 = \delta_p^1 > 0$ and $\lambda_j^1 = 0, j = 1,\dots,n, j \ne p$.

$\lambda_q^2 = \delta_q^2 > 0$ and $\lambda_j^2 = 0, j = 1, \dots, n, j \neq q$.

Therefore, it follows that:

$\lambda_p^1 x_{ip}^1 \leq \theta^{1*} x_{io}^1, i = 1, \dots, m \Rightarrow \frac{\lambda_p^1 x_{ip}^1}{x_{io}^1} \leq \theta^{1*}, i = 1, \dots, m \Rightarrow max\left\{\frac{\lambda_p^1 x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1, \dots, m\right\} \leq \theta^{1*} \Rightarrow \lambda_p^1 \, max\left\{\frac{x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1, \dots, m\right\} \leq \theta^{1*}$,

Thus

$\lambda_p^1 \mu_{po}^1 \leq \theta^{1*}$.

One should note that $\mu_{po}^1 < \infty$, otherwise there exists some index such as l so that: $x_{lp}^1 > 0$ and $x_{lo}^1 = 0$. As a result: $0 < \lambda_p^1 x_{lp}^1 \leq \theta^{1*} \times 0$ and this is a contradiction.

$\lambda_p^1 y_{rp}^1 \geq y_{ro}^1 \Rightarrow \lambda_p^1 \geq \frac{y_{ro}^1}{y_{rp}^1}, r = 1, \dots, S \Rightarrow \lambda_p^1 \geq max\left\{\frac{y_{ro}^1}{y_{rp}^1} \middle| y_{rp}^1 + y_{ro}^1 > 0, r = 1, \dots, s\right\} \Rightarrow \lambda_p^1 \geq \eta_{op}^1$.

Also, for constraints related to intermediate products, we have:

$\lambda_p^1 z_{dp} \geq z_{do}, d = 1, \dots, D \Rightarrow \lambda_p^1 \geq \frac{z_{do}}{z_{dp}} \Rightarrow \lambda_p^1 \geq max\left\{\frac{z_{do}}{z_{dp}} \middle| d = 1, \dots, D\right\} \Rightarrow \lambda_p^1 \geq \psi_{op}^1$

and $\psi_{op}^1 < \infty$, If $\psi_{op}^1 = \infty$, it means that: $z_{do} > 0$and $z_{dp} = 0$, then: $\lambda_p^1 \times 0 \geq z_{do} > 0, d = 1, \dots, D$. So this is a contradiction.

Using the above relations: $\lambda_p^1 \geq max\{\eta_{op}^1, \psi_{op}^1\}$. So: $max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1 \leq \theta^{1*}$.

Similarly, for stage 2, we have:$\lambda_q^2 \mu_{qo}^2 \leq \theta^{2*}, \eta_{oq}^2 \leq \lambda_q^2, \eta_{oq}^2 < \infty$.

and also:

$$\lambda_q^2 z_{dq} \leq z_{do}, d = 1, \dots, D \Rightarrow \lambda_q^2 \leq \frac{z_{do}}{z_{dq}}, d = 1, \dots, D \Rightarrow \lambda_q^2 \leq min\left\{\frac{z_{do}}{z_{dq}} \middle| d = 1, \dots, D\right\} = \psi_{oq}^2$$

As a result: $\eta_{oq}^2 \leq \psi_{oq}^2$ and $J_q \neq \emptyset$ because: $q \in J_q$. Finally: $\eta_{oq}^2 \mu_{qo}^2 \leq \theta^{2*}$subject to: $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$. So, for each $(p, q) \in J \times J$, $((\min_{p \in J} max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1, \min_{q \in J} \eta_{oq}^2 \mu_{qo}^2) \leq (\theta^{1*}, \theta^{2*}))$.

It is evident that for arbitrary $p \in J$ and also $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$ then $(\theta^1, \theta^2, w^1, w^2, \delta^1, \delta^2, \lambda^1, \lambda^2)$=$(max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1, \eta_{oq}^2 \mu_{qo}^2, e_p, e_q, \lambda_p^1, \lambda_q^2, e_p \, max\{\eta_{op}^1, \psi_{op}^1\}, e_q \eta_{oq}^2)$ is a feasible solution of model (5). This implies: $(\theta^{1*}, \theta^{2*}) \leq (max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1, \eta_{oq}^2 \mu_{qo}^2)$.

Considering that (p, q) are two arbitrary indices of $J \times J$, therefore:

$$(\theta^{1*}, \theta^{2*}) \leq (\min_p max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1, \min_q \eta_{oq}^2 \mu_{qo}^2)$$

Finally, we conclude that: $(\theta^{1*}, \theta^{2*}) = (\min_p max\{\eta_{op}^1, \psi_{op}^1\}\mu_{po}^1, \min_q \eta_{oq}^2 \mu_{qo}^2)$ such that: $\eta_{oq}^2 \leq \psi_{oq}^2$ and $q \in J_q$ ■.

Part *b*:

Now the theorem is proved for the case: $\Gamma = VRS$. According to the constraints$\sum_{j=1}^{n} w_j^k = 1, k = 1,2$, for each optimal solution, there exists $p \in J$ such that $w^1 = e_p$ and also there exists $q \in J$ so that $w^2 = e_q$. So:

$\lambda_p^1 = 1 > 0$ and $\lambda_j^1 = 0, j = 1, \ldots, n, j \neq p$

$\lambda_q^2 = 1 > 0$and $\lambda_j^2 = 0, j = 1, \ldots, n, j \neq q$

Therefore, conclude that:

$$x_{ip}^1 \leq \theta^{1*} x_{io}^1, i = 1, \ldots, m \Rightarrow \frac{x_{ip}^1}{x_{io}^1} \leq \theta^{1*}, i = 1, \ldots, m \Rightarrow max\left\{\frac{x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1, \ldots, m\right\}$$

$$\leq \theta^{1*} \Rightarrow \mu_{po}^1 \leq \theta^{1*}$$

One should note that $\mu_{po}^1 < \infty$ otherwise there exists some index such as l so that: $x_{lp}^1 > 0$ and $x_{lo}^1 = 0$. As a result: $0 < x_{ip}^1 \leq \theta^1 \times 0$, and this is a contradiction.

Also:

$y_{rj}^k \geq y_{ro}^k \Rightarrow 1 \geq \frac{y_{ro}^1}{y_{rp}^1}, r = 1, \ldots, S \Rightarrow max\left\{\frac{y_{ro}^1}{y_{rp}^1} \middle| y_{rp}^1 + y_{ro}^1 > 0, r = 1, \ldots, s\right\} \leq 1 \Rightarrow \eta_{op}^1 \leq 1.$

Similarly, for stage 2, we have: $\mu_{qo}^2 \leq \theta^2$ and $\eta_{qo}^2 \leq 1$. It is easy to prove that $\mu_{oq}^2 < \infty$.

Also, for constraints related to intermediate products, we have:

$z_{dp} \geq z_{do} \geq z_{dq}, d = 1, \ldots, D \Rightarrow max\left\{\frac{z_{dp}}{z_{do}} \middle| d = 1, \ldots, D\right\} \geq 1 \geq min\left\{\frac{z_{dq}}{z_{do}} \middle| d = 1, \ldots, D\right\} \Rightarrow \psi_{po}^1 \geq 1$ & $\psi_{qo}^1 \leq 1$.

Thus: $(\mu_{po}^1, \mu_{qo}^2) \leq (\theta^{1*}, \theta^{2*})$ subject to $\eta_{op}^1 \leq 1$, $\eta_{qo}^2 \leq 1$, $\psi_{op}^1 \geq 1$, $\psi_{qo}^2 \leq 1$. Conclude that:$(\min_{p\in J} \mu_{po}^1, \min_{q\in J} \mu_{qo}^2) \leq (\theta^{1*}, \theta^{2*})$.

Now, it is easy show that for arbitrary *p*, *q* such that: $\eta_{op}^1 \leq 1$, $\eta_{qo}^2 \leq 1$, $\psi_{op}^1 \geq 1$, $\psi_{qo}^2 \leq 1$ then, $(\theta^1, \theta^2, w^1, w^2, \delta^1, \delta^2, \lambda^1, \lambda^2)$=$(\mu_{po}^1, \mu_{qo}^2, e_p, e_q, 1, 1, e_p, e_q)$ is a feasible solution of model (5). Since (p, q) are arbitrary indices, this implies that: $(\theta^{1*}, \theta^{2*}) \leq (\min_{p\in J} \mu_{po}^1, \min_{q\in J} \mu_{qo}^2)$. Finally, we conclude that: $(\theta^{1*}, \theta^{2*}) = (\min_{p\in J} \mu_{po}^1, \min_{q\in J} \mu_{qo}^2)$ subject to $\eta_{op}^1 \leq 1, \eta_{qo}^2 \leq 1, \psi_{op}^1 \geq 1, \psi_{qo}^2 \leq 1$ ■.

Part *c*: The proof of part *c* is similar to parts *a* and *b*, so it is omitted.

Part *d*: The proof of part *d* is similar to parts *a* and *b*, so it is omitted.

Theorem 2. Suppose that DMU_o is under evaluation, and we define:

$\mu_{po}^1 = max\left\{\frac{x_{ip}^1}{x_{io}^1} \middle| x_{ip}^1 + x_{io}^1 > 0, i = 1, \ldots, m\right\},$

$\eta_{op}^1 = max\left\{\frac{y_{ro}^1}{y_{rp}^1} \middle| y_{rp}^1 + y_{ro}^1 > 0, r = 1, \ldots, s\right\},$

$\psi_p^1 = max\left\{\frac{\bar{z}_d}{z_{dp}} \middle| d = 1, \ldots, D\right\},$

$\psi_{pq}^2 = min\left\{\frac{z_{dp}}{z_{dq}} \middle| d = 1, \ldots, D\right\},$

$\mu_{qo}^2 = max\left\{\frac{x_{iq}^2}{x_{io}^2} \middle| x_{iq}^2 + x_{io}^2 > 0, i = 1, \ldots, m\right\},$

$\eta_{oq}^2 = max\left\{\frac{y_{ro}^2}{y_{rq}^2} \middle| y_{rq}^2 + y_{ro}^2 > 0, r = 1, \ldots, s\right\},$

$\bar{z}_d = max\{z_{dq} | q \in J\}$,

For the case: $\Gamma = CRS$, $(\theta^{1*}, \theta^{2*}) = (\underset{p\in J}{min}\, max\{\eta^2_{oq}\psi^1_p, \eta^1_{op}\}\mu^1_{po}, \underset{q\in J}{min}\eta^2_{oq}\mu^2_{qo})$ is a Pareto optimal solution of model (4).

For the case: $\Gamma = VRS$, $(\theta^{1*}, \theta^{2*}) = (\underset{p\in J}{min}\mu^1_{po}, \underset{q\in J}{min}\mu^2_{qo})$ is a Pareto optimal solution of model (4) such that: $\eta^1_{op} \le 1, \eta^2_{qo} \le 1$ and $1 \ge \psi^1_{qp}$.

For the case: $\Gamma = NIRS$, $(\theta^{1*}, \theta^{2*}) = (\underset{p\in J}{min}\, max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}\mu^1_{po}, \underset{q\in J}{min}\eta^2_{oq}\mu^2_{qo})$ is a Pareto optimal solution of model (4), such that: $\eta^1_{op} \le 1, \eta^2_{qo} \le 1$, $\eta^2_{oq}\psi^1_{qp} \le 1$ and $\eta^2_{oq} \le \psi^2_{pq}$.

For the case: $\Gamma = NDRS$, $(\theta^{1*}, \theta^{2*}) = (\underset{p\in J}{min}\, max\{\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}, 1\}\mu^1_{po}, \underset{q\in J}{min}\, max\{1, \eta^2_{oq}\}\mu^2_{qo})$ is a Pareto optimal solution of model (4) such that: $\eta^2_{oq} \le \psi^2_{oq}$ and $q \in J_q$.

Proof:

Part *a*:

For an optimal solution, there exists $p \in J$ such that $w^1 = e_p$ and there exists $q \in J$so that $w^2 = e_q$. So similar to Theorem 1, $\lambda^1_p\mu^1_{po} \le \theta^{1*}$, $\lambda^1_p \ge \eta^1_{op}$, $\lambda^2_q\mu^2_{qo} \le \theta^{2*}$, $\eta^2_{oq} \le \lambda^2_q$, $\eta^2_{oq} < \infty$, $\eta^1_{op} < \infty$,$\mu^1_{po} < \infty$ and $\mu^2_{qo} < \infty$. Also, for constraints related to intermediate products, we have:

$$\lambda^1_p z_{dp} \ge \lambda^2_q z_{dq}, d = 1,\dots,D \Rightarrow \lambda^1_p \ge \lambda^2_q \frac{z_{dq}}{z_{dp}} \ge \frac{\bar{z}_d}{z_{dp}}, d = 1,\dots,D \Rightarrow \lambda^1_p \ge \lambda^2_q\psi^1_p \Rightarrow \lambda^1_p \ge \eta^2_{oq}\psi^1_p$$

As a result: $max\{\eta^2_{qo}\psi^1_p, \eta^1_{op}\}\mu^1_{po} \le \theta^{1*}$ and $\eta^2_{oq}\mu^2_{qo} \le \theta^{2*}$. Since (p, q) are arbitrary indices in *J*×*J*, then $(\underset{p\in J}{min}\, max\{\eta^2_{oq}\psi^1_p, \eta^1_{op}\}\mu^1_{po}, \underset{q\in J}{min}\eta^2_{oq}\mu^2_{qo}) \le (\theta^{1*}, \theta^{2*})$.

Now, it is easy to show that for arbitrary (p , q)$\in J \times J$:

$(\theta^1, \theta^2, w^1, w^2, \delta^1, \delta^2, \lambda^1, \lambda^2)$=$(max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}\mu^1_{po}, \eta^2_{oq}\mu^2_{qo}, e_p, e_q, \lambda^1_p, \lambda^2_q,$ $e_p\, max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}, e_q\eta^2_{oq})$ is a feasible solution of model (4) so: $(\theta^{1*}, \theta^{2*}) \le (max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}\mu^1_{po}, \eta^2_{oq}\mu^2_{qo})$. Since (*p, q*) are arbitrary indices, this implies that: $(\theta^{1*}, \theta^{2*}) \le (\underset{p\in J}{min}\, max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}\mu^1_{po}, \underset{q\in J}{min}\eta^2_{oq}\mu^2_{qo})$.

Finally: $(\theta^{1*}, \theta^{2*}) = (\underset{p\in J}{min}\, max\{\eta^2_{oq}\psi^1_{qp}, \eta^1_{op}\}\mu^1_{po}, \underset{q\in J}{min}\eta^2_{oq}\mu^2_{qo})$ ■.

Part b:

It can be easily shown that: $\mu^1_{po} \le \theta^1$, $\mu^2_{qo} \le \theta^2$,$\eta^1_{op} \le 1$, $\eta^2_{qo} \le 1$, $\mu^1_{op} < \infty$ and $\mu^2_{oq} < \infty$.

Also:

$$z_{dp} \ge z_{dq}, d = 1,\dots,D \Rightarrow 1 \ge \frac{z_{dq}}{z_{dp}} \ge \frac{\bar{z}_d}{z_{dp}}, d = 1,\dots,D \Rightarrow 1 \ge \psi^1_p$$

Thus: $(\mu^1_{po}, \mu^2_{qo}) \le (\theta^{1*}, \theta^{2*})$ subject to $\eta^1_{op} \le 1, \eta^2_{qo} \le 1, 1 \ge \psi^1_p$. Conclude that: for each (p, q)$\in J$: $(\underset{p\in J}{min}\mu^1_{po}, \underset{q\in J}{min}\mu^2_{qo}) \le (\theta^{1*}, \theta^{2*})$.

Now, it is easy to show that for arbitrary (p, q)$\in J \times J$ such that: $\eta^1_{op} \le 1, \eta^2_{qo} \le 1, 1 \ge \psi^1_p$ then

$(\theta^1, \theta^2, w^1, w^2, \delta^1, \delta^2, \lambda^1, \lambda^2)$=$(\mu^1_{po}, \mu^2_{qo}, e_p, e_q, 1, 1, e_p, e_q)$ is a feasible solution of model (9). Since (p, q) are arbitrary indices, this implies that:

So, we have: $(\theta^{1*}, \theta^{2*}) = (\mu^1_{po}, \mu^2_{qo})$ such that: $\eta^1_{op} \leq 1, \eta^2_{qo} \leq 1$and $1 \geq \psi^1_p$ ■.

Part *c*: It is proving similar to parts *a* and *b*.

Part *d*: It is proving similar to parts *a* and *b*.

Theorem 3. The optimal solution of the (non)convex centralized model is less than or equal to the optimal solution of the (non)convex decentralized model.

Proof: Suppose that $(\bar{\theta}^1, \bar{\theta}^2, \bar{w}^1, \bar{w}^2, \bar{\delta}^1, \bar{\delta}^2, \bar{\lambda}^1, \bar{\lambda}^2)$ and $(\tilde{\theta}^1, \tilde{\theta}^2, \tilde{w}^1, \tilde{w}^2, \tilde{\delta}^1, \tilde{\delta}^2, \tilde{\lambda}^1, \tilde{\lambda}^2)$ are optimal solutions of models (5) and (9), respectively, it is clear that $(\bar{\theta}^1, \bar{\theta}^2, \bar{w}^1, \bar{w}^2, \bar{\delta}^1, \bar{\delta}^2, \bar{\lambda}^1, \bar{\lambda}^2)$ is a feasible solution of model (9), as a result $(\tilde{\theta}^1, \tilde{\theta}^2) \leq (\bar{\theta}^1, \bar{\theta}^2)$. Thus, the optimal solution of model (9) is less than or equal to the optimal solution of model (2) ■.

Appendix B: Input–output data for 15 supply chains in the wheat supply chain

Table B1. Input–output data for 15 supply chains in the wheat supply chain

DMUs	**Inputs Stage 1**			**Intermediate Input**	**Input Stage2**	**Final Outputs**	
	Agricultural labour	Cultivated area (ha)	Nitrogen fertilizer (t)	Wheat production (t)	Logistics	Food supply (kcal/cap/day)	Agricultural value added (USD)
DMU1	380785	9863184	1805533	14480217	3.75	577.93	26731002694
DMU2	328085	10017800	3046000	35437200	3.73	591.29	30461110689
DMU3	180263897	23383000	25885154	134256000	3.61	605.63	1.13076E+12
DMU4	726954	4519780	2090173	30181140	3.84	732.47	38671219441
DMU5	520472	2835500	1265477	22172100	4.2	523.5	28719378963
DMU6	238040174	31357020	20404000	107860510	3.18	529.73	4.99356E+11
DMU7	4619536	6004700	903185	13540500	2.85	1,193.83	32020793117
DMU8	1325916	12057071	74401	14257950	2.81	760.45	9223533982
DMU9	28582658	8804677	3533784	25247511	2.42	946.42	65655410998
DMU10	1740042	2391000	912000	12515060	3.54	802.49	15562567985
DMU11	4406996	28864312	1916418	85896326	2.76	1,007.82	59817870491
DMU12	5591546	6914632	2052685	20500000	3.15	1,183.03	48046154648
DMU13	2940018	6564500	1716082	24912350	2.83	672.07	14581346566
DMU14	342439	1387000	967000	9658000	3.99	801.94	18097445719.00
DMU15	2831606	14888140	11925955	49751180	3.89	595.09	1.62895E+11

Appendix C: CRS, NDRS and NIRS Results

Table C1. Stage and overall efficiency results for wheat supply chain units under CRS, NIRS, and NRDS

DMUs	CRS			NIRS			NDRS		
	E1	**E2**	**OVERALL**	**E1**	**E2**	**OVERALL**	**E1**	**E2**	**OVERALL**
DMU1	0.689	0.467	0.578	1.000	0.394	0.697	1.000	0.760	0.880
DMU2	1.000	0.405	0.703	1.000	0.434	0.717	1.000	0.649	0.824
DMU3	0.734	1.000	0.867	0.734	1.000	0.867	0.734	1.000	0.867
DMU4	0.975	0.488	0.731	0.975	0.522	0.748	0.975	0.630	0.802
DMU5	1.000	0.319	0.659	1.000	0.319	0.659	1.000	0.576	0.788
DMU6	0.440	1.000	0.720	0.440	0.993	0.716	0.440	1.000	0.720
DMU7	0.758	1.000	0.879	1.000	1.000	1.000	1.000	1.000	1.000
DMU8	1.000	0.646	0.823	1.000	0.692	0.846	1.000	1.000	1.000
DMU9	0.408	1.000	0.704	0.408	1.000	0.704	0.408	1.000	0.704
DMU10	0.783	0.541	0.662	1.000	0.541	0.771	1.000	0.805	0.903
DMU11	1.000	0.934	0.967	1.000	1.000	1.000	1.000	0.934	0.967
DMU12	0.570	1.000	0.785	0.728	1.000	0.864	0.616	1.000	0.808
DMU13	0.829	0.567	0.698	0.829	0.607	0.718	0.829	0.855	0.842
DMU14	0.890	0.480	0.685	1.000	0.480	0.740	1.000	0.714	0.857
DMU15	0.359	1.000	0.679	0.359	0.912	0.635	0.359	1.000	0.679
Mean	**0.7623**	**0.7231**	**0.7427**	**0.8315**	**0.7263**	**0.7788**	**0.8241**	**0.8615**	**0.8427**
Std. Deviation	**0.2280**	**0.2689**	**0.1022**	**0.2425**	**0.2665**	**0.1130**	**0.2477**	**0.1593**	**0.0993**

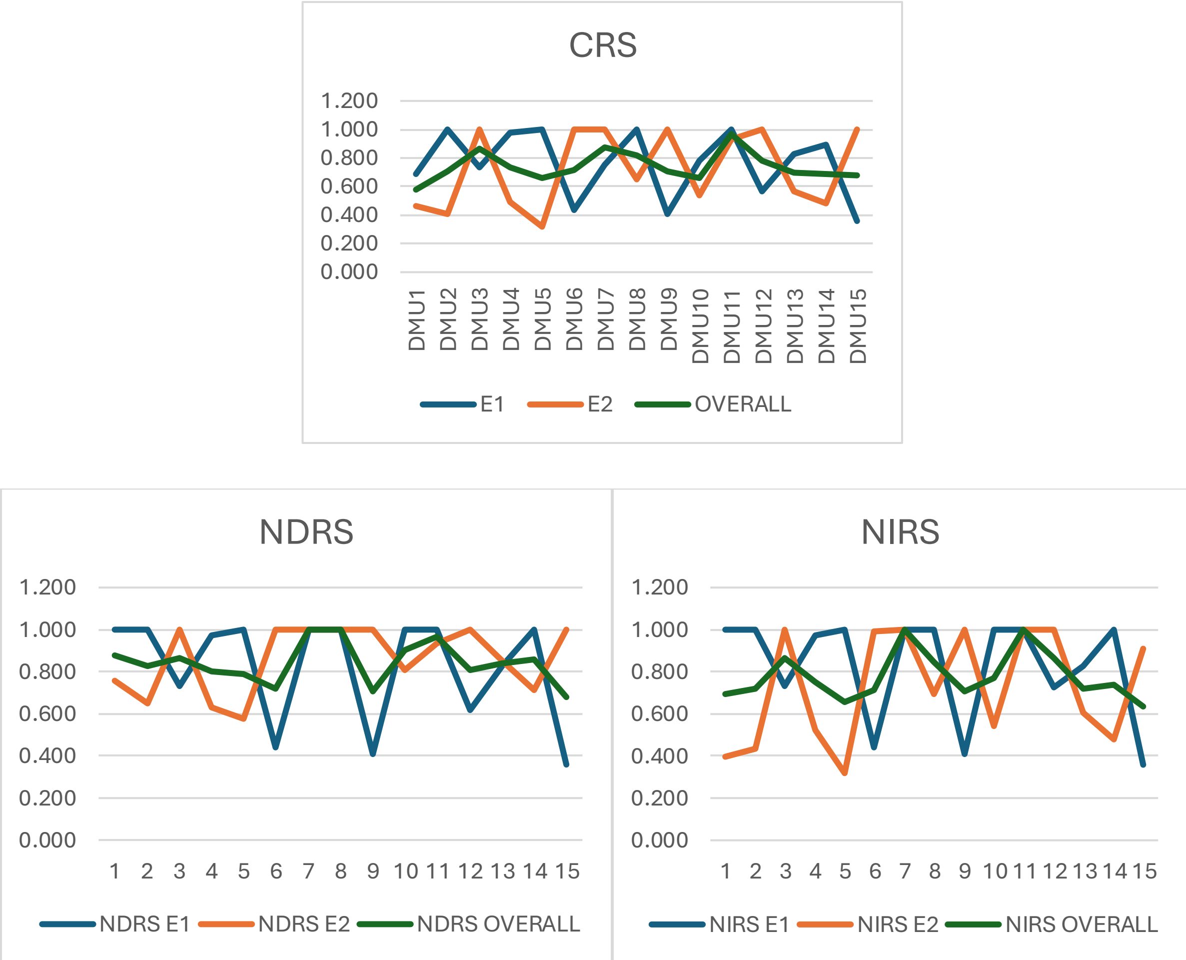

Figure C1. Dispersion of Stage 1, Stage 2, and Overall Performance Under CRS, NIRS, and NDRS

In this Appendix C we report the two-stage efficiency scores for 15 wheat supply chain decision-making units (DMUs) estimated under three returns-to-scale assumptions: Constant Returns to Scale (CRS), Non-Increasing Returns to Scale (NIRS), and Non-Decreasing Returns to Scale (NDRS). For each assumption, Stage 1 (E1) and Stage 2 (E2) scores are reported alongside an overall efficiency score, computed as the geometric mean of the two stages, with full numerical results provided in Table C1 and their dispersion illustrated in Figure C1.

As shown in Figure C1, a consistent inverse relationship between stage-level scores is observable across all three assumptions, whereby DMUs performing strongly in Stage 1 tend to underperform in Stage 2, and vice versa, reflecting a structural trade-off between upstream and downstream supply chain operations. Under CRS, overall efficiency exhibits the greatest variability and the lowest scores across DMUs, while relaxing the scale constraint to NDRS produces the most elevated overall efficiency profile, driven predominantly by improvements in Stage 2 performance. The NIRS assumption represents an intermediate case, primarily improving Stage 1 scores relative to CRS, with limited effect on Stage 2. Notably, under NDRS, DMU7 and DMU8 achieve full efficiency across both stages, attaining an overall score of 1.000, while several other DMUs reach full efficiency in at least one stage under each assumption. These patterns suggest that scale inefficiencies in the wheat supply chain are more pronounced in downstream operations, and that the choice of returns-to-scale specification materially influences the identification of efficient and inefficient units, underscoring the importance of testing multiple scale assumptions in two-stage DEA analyses.